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VOLUME I

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OF THE

LAWS OBSERVATORY

VOLUME I

NUMBERS 1-16

Prepared Under the Direction of

FREDERICK HANLEY SEARES

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LAWS OBSERVATORY
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BULLETIN No. 1

OBSERVATIONS AND ELEMENTS OF COMET *b* 1902 (PERRINE).

The following observations of Comet *b*, 1902, have been made with the 7½-inch equatorial and filar micrometer of the Laws Observatory. With the exception of the observations of October 5 and 6, the differences in right ascension of comet and comparison star were obtained by transits. Upon those dates the differences were measured directly with the micrometer.

1902	Columbia M. T.	Star	No. of Comparisons	Comet—Star		Comet's Apparent		log pΔ	
				Δα	Δδ	α	δ	α	δ
Sept. 4	12h 12m 27s	1	18 , 26	− 5s.90	− 7′ 57″.5	3h 14m 5s.69	+36° 12′ 8″.6	9.698n	0.403
6	10 0 58	2	12 , 10	−45.12	− 5 50.9	3 11 30.04	+37 6 7.5	9.758n	0.659
7	10 10 21	3	10 , 19	+ 8.87	+ 0 6.3	3 9 54.48	+37 36 39.6	9.761n	0.629
9	9 51 51	4	13 , 17	−41.22	+ 5 10.6	3 6 11.71	+38 41 28.9	9.767n	0.635
10	10 12 19	5	12 , 10	−40.82	− 3 19.2	3 3 58.21	+39 17 15.1	9.769n	0.576
12	10 15 10	6	12 , 12	−12.36	−10 34.3	2 58 48.97	+40 33 46.9	9.772n	0.529
12	10 58 59	7	— , 12	——	+23 2.6	— — ——	+40 35 5.9	——	0.408
13	10 5 51	8	— , 14	——	+ 0 31.9	— — ——	+41 14 59.7	——	0.528
26	7 20 18	9	10 , 17	+44.89	+ 2 26.9	1 11 28.36	+53 56 24.4	9.886n	0.406
27	7 33 2	10	13 , 16	+18.20	+ 5 21.5	0 52 20.73	+55 0 35.4	9.884n	0.203
28	8 18 18	11	12 , 13	+ 7.85	− 6 45.3	0 29 44.06	+55 57 19.7	9.830n	9.013n
30	7 43 38	12	12 , 10	+46.84	− 0 52.2	23 37 22.44	+57 0 13.3	9.805n	9.867n
Oct. 5	12 12 4	13	12 , 10	+35.12	− 0 58.5	20 55 36.55	+50 29 7.3	9.806	0.046
6	11 58 30	14	12 , 10	+20.47	+ 6 2.9	20 29 29.07	+47 34 20.3	9.795	0.245
7	10 41 48	15	12 , 10	− 0.27	+ 8 15.8	20 7 18.94	+44 28 28.5	9.717	0.057
9	11 14 58	16	16 , 12	+27.39	+ 2 14.0	19 28 33.37	+37 20 6.3	9.743	0.514
13	9 15 44	17	16 , 12	+ 4.05	− 4 10.9	18 38 53.83	+24 7 58.0	9.641	0.564
14	8 58 21	18	12 , 10	+ 5.32	+ 6 22.2	18 30 0.07	+21 13 59.9	9.628	0.590
15	10 27 40	19	— , 10	——	+ 6 2.8	— — ——	+18 20 39.7	——	0.703
16	8 49 34	20	12 , 10	− 6.93	−11 1.0	18 15 0.53	+16 0 12.0	9.628	0.647
17	8 45 33	21	12 , 10	−25.71	+ 3 56.7	18 8 41.35	+13 41 11.7	9.629	0.667
18	8 51 30	22	— , 6	——	+ 6 2.2	— — ——	+11 32 26.8	——	0.689
18	9 48 10	22	12 , 10	−17.80	+ 1 11.9	18 2 44.98	+11 27 36.5	9.664	0.716
21	7 50 44	23	— , 10	——	− 6 55.0	— — ——	+ 6 10 5.4	——	0.707
21	8 12 57	24	12 , 10	+24.88	+ 1 5.0	17 48 54.87	+ 6 8 36.7	9.622	0.713
23	8 3 0	25	13 , 10	−29 79	+ 0 4.8	17 41 15.86	+ 3 10 53.9	9.624	0.728
24	7 46 59	26	14 , 10	−20.26	+ 5 30.4	17 37 51.17	+ 1 52 46.1	9.617	0.733
25	7 49 5	27	13 , 13	+12.28	− 2 8.8	17 34 37.38	+ 0 38 53.7	9.624	0.738
26	7 42 50	28	15 , 12	+ 1.95	+11 31.0	17 31 34.45	− 0 29 38.3	9.625	0.742
28	7 14 33	29	15 , 13	−23.95	− 4 51.7	17 25 58.44	− 2 32 23.8	9.612	0.750

MEAN PLACES OF COMPARISON STARS FOR 1902.0

Star	α	Red. to App. Place	δ	Red. to App. Place	Authority
1	$3^h\ 14^m\ 7^s.52$	+4.07	+36° 20′ 4″.8	+ 1″.3	A. G. Lund 1719
2	3 12 10.99	+4.17	+37 11 57.0	+ 1.4	A. G. Lund 1704
3	3 9 41.37	+4.24	+37 36 31.7	+ 1.6	A. G. Lund 1680
4	3 6 48.57	+4.36	+38 36 16.3	+ 2.0	A. G. Lund 1654
5	3 4 34.61	+4.42	+39 20 32.2	+ 2.1	A. G. Lund 1633
6	2 58 56.74	+4.59	+40 44 18.5	+ 2.7	A. G. Bonn 2597
7	2 59 0.67	+4.56	+40 12 0.4	+ 2.9	A. G. Lund 1581
8	2 55 41.33	+4.66	+41 14 24.7	+ 3.1	A. G. Bonn 2559
9	1 10 37.52	+5.95	+53 53 41.9	+15.6	Comparison with star 9a
9a	1 10 9.28	+5.95	+53 54 42.4	+15.6	A. G. Cambridge, U. S. 571
10	0 51 56.55	+5.98	+54 54 55.9	+18.0	A. G. Cambridge, U. S. 429
11	0 29 30.25	+5.96	+56 3 44.2	+20.8	A. G. Helsingfors-Gotha 449
12	23 36 30.02	+5.58	+57 0 38.8	+26.7	A. G. Helsingfors-Gotha 14264
13	20 54 58.27	+3.16	+50 29 30.1	+35.7	A. G. Cambridge, U. S. 6846
14	20 29 5.79	+2.81	+47 27 42.0	+35.4	A. G. Bonn 14363
15	20 7 16.66	+2.55	+44 19 38.1	+34.6	A. G. Bonn 13855
16	19 28 3.77	+2.21	+37 17 20.4	+31.9	A. G. Lund 8492
17	18 38 47.75	+2.03	+24 11 43.4	+25.5	Comparison with star 17a
17a	18 39 6.80	+2.03	+24 7 32.5	+25.5	A. G. Berlin B 6619
18	18 29 52.74	+2.01	+21 7 13.8	+23.9	A. G. Berlin B 6540
19	18 21 20.44	+2.01	+18 14 14.4	+22.5	Comparison with star 19a
19a	18 20 35.77	+2.01	+18 12 16.5	+22.5	A. G. Berlin A 6779
20	18 15 5.45	+2.01	+16 10 51.7	+21.3	Comparison with star 20a
20a	18 15 54.24	+2.01	+16 16 14.3	+21.3	A. G. Berlin A 6733
21	18 9 5.04	+2.02	+13 36 54.9	+20.1	A. G. Leipzig I 6489
22	18 3 0.75	+2.03	+11 26 5.6	+19.0	A. G. Leipzig I 6431
23	17 48 57.19	+2.03	+ 6 16 44.2	+16.2	A. G. Leipzig II 8151
24	17 48 27.96	+2.03	+ 6 7 15.6	+16.1	A. G. Leipzig II 8141
25	17 41 43.62	+2.03	+ 3 10 34.5	+14.6	Comparison with star 25a
25a	17 39 29.76	+2.03	+ 3 0 38.3	+14.6	A. G. Albany 5895
26	17 38 9.39	+2.04	+ 1 47 1.9	+13.8	A. G. Albany 5889
27	17 34 23.07	+2.03	+ 0 40 49.4	+13.1	A. G. Nicolajew 4375
28	17 31 30.46	+2.04	— 0 41 21.7	+12.4	Comparison with star 28a
28a	17 28 36.46	+2.04	— 0 43 35.8	+12.4	A. G. Nicolajew 4358
29	17 26 20.35	+2.04	— 2 27 43.5	+11.4	Cincinnati, P. M. Stars 996

Faint comparison stars were referred to catalogue positions by chronographic and micrometer comparisons. The details of the reference are as follows:

Stars	No. of Comp.	$\Delta\alpha$	$\Delta\delta$
9— 9a	17 , 10	$+0^m\ 28^s.24$	—1′ 0″.5
17—17a	12 , 10	—0 19.05	+4 10.9
19—19a	10 , 10	+0 44.67	+1 57.9
20—20a	11 , 8	—0 48.79	—5 22.6
25—25a	10 , 9	+2 13.86	+9 56.2
28—28a	10 , 10	+2 54.00	+2 14.1

There is evidence of proper motion in right ascension for star No. 24. The position given above is the unmodified catalogue position. The substitution of the weighted mean position derived below, which includes proper motion, would diminish the printed right ascension by 0ˢ.17.

The following table exhibits the catalogue positions, systematic corrections and weights employed in the determination of the proper motion and a mean position:

Catalogue	Epoch	No. of Obs.	Cat. α for 1902.0	Systematic Correction	Corrected α	Weight	α	Residuals
Lalande 32707	1790	—	$17^h\ 48^m\ 28^s.92$	+0ˢ.20	29ˢ.12	0.3	27ˢ.94	+0ˢ.15
Poulkova 2521	1844.9	4	28.29	+0.06	28.35	2.0	27.75	—0.04
Munich I 14889	53.1	4	28.43	—0.15	28.28	1.0	27.77	—0.02
Quetelet 7229	66.9	3	28.01	+0.07	28.08	1.0	27.71	—0.08
Rogers 803	78.8	6	28.12	—0.01	28.11	2.0	27.87	+0.08
A. G. Leipzig II 8141	84.4	2	27.96	0.00	27.96	1.0	27.77	—0.02

The epoch of the Lalande position is assumed, inasmuch as the *Histoire Céleste* is not accessible.

The values of the weighted right ascension and proper motion for 1902.0 resulting from a least square solution are:

$$\alpha = 17^h\ 48^m\ 27^s.79 \qquad \mu = -0^s.0105.$$

The column headed α contains the seconds of right ascension for the individual catalogue positions reduced to 1902.0 and corrected for systematic correction and proper motion.

The elements printed below have been obtained by an adaptation of the method developed by Professor Leuschner in his paper entitled: *A Short Method of Determining Orbits from Three Observations*, which forms Part I of Vol. VII of the Publications of the Lick Observatory. The formulas there derived lend themselves readily to the calculation of elements for the case in which approximations are already known. This adaptation is suggested by Leuschner on page 19 of his paper, but the range of application of his formulas is limited on account of the non-convergence of the series for the quantities f and g for large values of the intervals. The limits of application of the method can be extended considerably by replacing the series for f and g by closed expressions which are functions of the elements. It is assumed that approximate values of the elements are known, consequently, values for f and g sufficiently accurate for a first approximation can be obtained no matter what the intervals may be. The same formulas may be used for the computation of accurate values of f and g when the corrected elements have been obtained. This does not permit of an application of the method to all cases, however, for the reason that the series for f and g in ascending powers of the intervals enter not only into the ephemeris formulas but also into the differential formulas of Group VII, page 17, of Leuschner's paper, and for extreme values of the intervals the neglected terms become appreciable. Theoretically it is possible to replace these quantities even here by expressions the applicability of which is independent of the intervals; but the differential formulas would become extremely complicated, and the change would serve no useful purpose, for the differential formulas as they stand can be used for most cases which are likely to arise in practice. Fortunately, only certain coefficients in the differential formulas are affected, and usually these are needed accurate to only four or five places of decimals. Moreover, the neglected terms are multiplied into the corrections which must be applied to the radius vector and its derivative, as obtained from the approximate elements, in order that the corrected values thus deduced may represent the observations upon which the calculation is based. With close approximations for the elements, therefore, the neglected terms will not seriously affect the result even when the intervals approach 40 or 50 days, unless the radius vector itself should happen to be considerably less than unity. With intervals of this magnitude the computation of f and g from the series accurate to six or seven places, as is necessary, for g at least, in the final evaluation of the ephemeris formulas of Group VII, would be extremely laborious if not impossible; and, as close approximations for the elements will generally be known for just those cases in which it is

desirable to use large intervals, the extension of the limits of application of the method is a real one.

Should it happen that the approximations for the elements are not sufficiently near the true value to render the neglected terms of the differential formulas entirely inappreciable, a second approximation for the corrections to the fundamental data $x_\circ'$, $y_\circ'$, $z_\circ'$, $\rho_\circ$, will usually reduce the residuals for the dates of the calculation to zero. That is to say, the slow convergence of the series entering into the differential formulas is counteracted by the more rapid convergence of the successive approximations for the corrections to the fundamental data.

The quantities f and g are defined by the relations:

$$\left.\begin{aligned} x_\prime &= f_\prime x_\circ + g_\prime x_\circ' & x_{\prime\prime\prime} &= f_{\prime\prime\prime} x_\circ + g_{\prime\prime\prime} x_\circ' \\ y_\prime &= f_\prime y_\circ + g_\prime y_\circ' & y_{\prime\prime\prime} &= f_{\prime\prime\prime} y_\circ + g_{\prime\prime\prime} y_\circ' \\ z_\prime &= f_\prime z_\circ + g_\prime z_\circ' & z_{\prime\prime\prime} &= f_{\prime\prime\prime} z_\circ + g_{\prime\prime\prime} z_\circ' \end{aligned}\right\} \quad (1)$$

$g_\prime$ and $g_{\prime\prime\prime}$ have a definite geometrical meaning. $g_\prime$ is the negative ratio of triangle to sector included between the first and second positions multiplied by the interval $\theta_{\prime\prime\prime} = k\,(t_{\prime\prime} - t_\prime)$; $g_{\prime\prime\prime}$ is the positive value of the corresponding ratio for the second and third positions multiplied by the interval $\theta_\prime = k\,(t_{\prime\prime\prime} - t_{\prime\prime})$. We have therefore:

$$\left.\begin{aligned} g_\prime &= -\frac{[r_\prime\, r_{\prime\prime}]}{\sqrt{p}} \\ g_{\prime\prime\prime} &= +\frac{[r_{\prime\prime}\, r_{\prime\prime\prime}]}{\sqrt{p}} \end{aligned}\right\} \quad (2)$$

Expressions for $f_\prime$ and $f_{\prime\prime\prime}$ may be derived by the method of Kühnert, (Oppolzer, *Bahnbestimmung der Kometen und Planeten*, Vol. I, p. 97). The resulting formulas are

$$\left.\begin{aligned} f_\prime &= 1 - \frac{g_\prime^2}{2r_\prime\, r_{\prime\prime}^2 \cos^2 \tfrac{1}{2}(v_{\prime\prime} - v_\prime)} \\ f_{\prime\prime\prime} &= 1 - \frac{g_{\prime\prime\prime}^2}{2r_{\prime\prime\prime}\, r_{\prime\prime}^2 \cos^2 \tfrac{1}{2}(v_{\prime\prime\prime} - v_{\prime\prime})} \end{aligned}\right\} \quad (3)$$

In outline, the application of the adaptation of Leuschner's method is as follows:

With approximate elements calculate the heliocentric rectangular coördinates for the dates, corrected for aberration, of the three positions which are to form the basis of the computation. For the date of the second position calculate the geocentric distance $\rho_\circ$, and then

$$\begin{aligned} x_\circ &= \rho_\circ \cos\delta_{\prime\prime} \cos\alpha_{\prime\prime} - X_{\prime\prime} \\ y_\circ &= \rho_\circ \cos\delta_{\prime\prime} \sin\alpha_{\prime\prime} - Y_{\prime\prime} \\ z_\circ &= \rho_\circ \sin\delta_{\prime\prime} - Z_{\prime\prime} \end{aligned}$$

in which $\alpha_{\prime\prime}$ and $\delta_{\prime\prime}$ are the *observed* coördinates of the second position.

With the approximate elements calculate $g_\prime$ and $g_{\prime\prime\prime}$ from (2) and $f_\prime$ and $f_{\prime\prime\prime}$ from (3).

The first approximations for $x_\circ'$, $y_\circ'$, $z_\circ'$ may be obtained from (1), written in the form

$$\left.\begin{aligned} x_\circ' &= \frac{x_\prime - f_\prime x_\circ}{g_\prime} = \frac{x_{\prime\prime\prime} - f_{\prime\prime\prime} x_\circ}{g_{\prime\prime\prime}} \\ y_\circ' &= \frac{y_\prime - f_\prime y_\circ}{g_\prime} = \frac{y_{\prime\prime\prime} - f_{\prime\prime\prime} y_\circ}{g_{\prime\prime\prime}} \\ z_\circ' &= \frac{z_\prime - f_\prime z_\circ}{g_\prime} = \frac{z_{\prime\prime\prime} - f_{\prime\prime\prime} z_\circ}{g_{\prime\prime\prime}} \end{aligned}\right\} \quad (4)$$

Substitute the various quantities thus derived into the ephemeris formulas of page 16 of Leuschner's paper. They are

$$\left.\begin{aligned} \rho\cos\delta\cos\alpha &= X + fx_\circ + gx_\circ' = \xi \\ \rho\cos\delta\sin\alpha &= Y + fy_\circ + gy_\circ' = \eta \\ \rho\sin\delta &= Z + fz_\circ + gz_\circ' = \zeta \end{aligned}\right\} \quad (5)$$

into which the subscripts referring to the first and third positions are to be successively introduced. Compare the resulting values of α and δ with the first and third observed positions and form the residuals in the sense, observation minus computation.

Group VII, of Leuschner's formulas page 17, are then used for the determination of corrections to $\rho_\circ$, $x_\circ'$, $y_\circ'$, $z_\circ'$, which, when applied will reduce the residuals to zero. Neglecting accumulated errors of computation, the middle place must be perfectly represented in all cases, for, from the manner in which $x_\circ$, $y_\circ$ and $z_\circ$ have been obtained, the calculated middle position must coincide with the observed place, whatever the corrections to $\rho_\circ$ and the resulting corrections to $x_\circ$, $y_\circ$ and $z_\circ$ may be.

With the corrected values of $x_\circ'$, $y_\circ'$ $z_\circ'$, and $x_\circ$, $y_\circ$, $z_\circ$, compute the improved elements from Group VIII of Leuschner's formulas.

The recalculation of (2), (3) and (5) will show whether or not the approximate elements were sufficiently accurate for a single approximation to reduce the residuals to zero. Usually a single approximation will be sufficient, but if it is not, a second is quickly

made inasmuch as the values of many of the quantities involved remain unchanged for the successive approximations.

The method is very convenient and can be used with extreme facility for those cases which lie intermediate between a preliminary orbit and the definitive least square solution.

For the determination of the elements of Comet *b*, 1902, three normal places were formed. The first is based upon observations made at Berlin, Heidelberg and Jena on Sept. 2. The second and third depend upon the Laws Observatory positions of Sept. 26, 27, 28, 30, and Oct. 24, 25, 26, 28. The normal places, referred to 1902.0, are,

Berlin M. T.	α	δ	O—C.
1902 Sept. 2.50000	49° 7′ 25″.3	+35° 13′ 13″.0	— 5″.0 + 0″.1
28.63703	7 24 43.3	+55 57 3.1	+26.6 — 2.1
Oct. 26.61079	262 53 7.3	— 0 29 26.6	+14.6 +19.9

Strömgren's elements, A. N. 3821, were used for the calculation of approximations for $g_{,}$, $g_{,,,}$, $f_{,}$, $f_{,,,}$, $x_{\circ}$, $y_{\circ}$, $z_{\circ}$, $x_{\circ}'$, $y_{\circ}'$, $z_{\circ}'$ and $\rho_{\circ}$. The residuals given by these elements for the normal dates are appended to the normal positions although they do not enter directly into the computation.

The residuals obtained from (5) for the evaluation of the differential formulae, were

	I.	III.
$\Delta\alpha$	—11″.4	+11″.0
$\Delta\delta$	+ 6.4	+11.1

They are so small that the case here presented offers no real test of the performance of the method. A single approximation gave the following elements:

T = 1902, Nov. 23.90094 Berlin M. T.
$\log q$ = 9.602596
$\log e$ = 9.999709

Equator 1902.0.	Ecliptic 1902.0.
ω' = 219° 39′ 9″.2	ω = 153° 1′ 34″.4
Ω' = 112 17 25.3	Ω = 49 20 11.0
i' = 160 47 57.1	i = 156 20 56.1

The representation of the three normal places, in the sense observation minus computation, is:

	I.	II.	III.
$\cos\delta\Delta\alpha$	—0″.9	0″.0	+0″.7
$\Delta\delta$	0.0	0.0	+0.1

The corrections for the reduction of the elements from 1902.0 to 1903.0 are:

$\Delta\omega'$ = —23″.13 $\Delta\omega$ = + 0″.98
$\Delta\Omega'$ = +24.22 $\Delta\Omega$ = +51.16
$\Delta i'$ = —18.55 Δi = + 0.27

The rectangular heliocentric coördinates are given by

$$\left.\begin{aligned} x &= [9.978899]\, r.\sin(196^\circ\ 11'\ 16''.1 + v) \\ y &= [9.996594]\, r.\sin(288\ \ 29\ \ 22.7 + v) \\ z &= [9.517037]\, r.\sin(219\ \ 39\ \ 9.2 + v) \end{aligned}\right\} 1902.0$$

$$\left.\begin{aligned} x &= [9.978890]\, r.\sin(196\ \ 10\ \ 25.4 + v) \\ y &= [9.996590]\, r.\sin(288\ \ 28\ \ 38.5 + v) \\ z &= [9.517149]\, r.\sin(219\ \ 38\ \ 46.0 + v) \end{aligned}\right\} 1903.0$$

Columbia, Missouri, 1902, Dec. 15.

F. H. Seares.

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BULLETIN No. 2

OBSERVATIONS OF COMETS.

The following observations of comets have been made with the 7½-inch equatorial and filar micrometer of the Laws Observatory. Throughout the entire series the differences in right ascension of comet and comparison star have been obtained by transits. The magnifying power used was 140.

Date	Columbia M. T.	Star	No. of Comparisons	Comet—Star		Comet's Apparent		log p Δ	
				Δα	Δδ	α	δ	π	δ
1902-03				Comet 1902 III (*b* 1902).					
Nov. 7	6h 22m 32s	1	12 , 10	+0m 14s.66	−21′ 24″.7	17h 1m 39s.99	− 9° 59′ 42″.4	9.629	0.770
Feb. 21	10 1 13	2	11 , 10	−0 52.77	+ 6 32.7	6 51 59.42	−17 19 42.1	9.175	0.857
1903				Comet 1903 I (*a* 1903).					
Jan. 21	7 39 40	3	—, 6	———	+ 2 39.9	— — ———	+ 2 49 1.9	———	0.731
25	7 23 15	4	16 , 15	−0 46.17	+ 8 48.3	23 4 38.30	+ 3 48 16.2	9.630	0.726
30	6 53 28	5	13 , 10	+0 33.92	+ 9 56.3	10 43.96	+ 5 8 5.2	9.618	0.718
Feb. 1	6 52 16	6	15 , 11	−0 51.16	− 8 45.7	13 24	+ 5 42 2	9.622	0.716
4	6 43 17	7	17 , 10	+0 35.30	− 6 59.2	17 31.07	+ 6 35 26.5	9.621	0.710
5	7 24 56	8	12 , 10	+0 16.50	− 4 14.9	18 59.32	+ 6 54 16.5	9.651	0.723
8	5 22 30	9	18 , 10	+0 16.56	−14 53.3	23 25.63	+ 7 50 59.3	9.577	0.692
16	6 30 36	10	12 , 8	−0 56.38	− 5 37.7	36 36.94	+10 36 44.1	9.638	0.696
18	7 9 40	11	11 , 10	+0 23.45	− 7 8.6	40 9.97	+11 20 19.1	9.662	0.713
21	7 28 12	12	12 , 10	−1 19.48	+ 8 22.0	45 45.12	+12 28 20.7	9.669	0.722
22	7 16 10	13	15 , 10	−0 18.20	− 5 34.6	47 40.21	+12 50 49.3	9.668	0.716
28	7 18 30	14	20 , 10	+0 8.53	− 8 18.3	23 59 39.03	+15 5 27.4	9.675	0.719
Mar. 1	7 11 48	14	8 , —	+2 10.37	— ——	0 1 40.87	— — ——	9.675	——
1	7 41 3	15	4 , 12	+0 21.99	− 8 39.6	1 43	+15 26 58	9.675	0.734
1903				Comet 1903 II (*d* 1902).					
Jan. 25	14 2 28	16	15 , 9	−0 14.73	− 4 42.3	6 45 17.48	+11 41 42.4	9.576	0.660
29	13 15 40	17	16 , 12	−1 11.08	+10 1.2	43 0.88	+13 1 21.3	9.532	0.633
29	14 9 36	18	12 , 10	−1 2.22	− 8 40.9	43 0.20	+13 2 0.2	9.617	0.662
Feb. 1	12 40 45	19	14 , 10	+0 39.19	− 3 19.3	41 29.64	+14 0 27.1	9.491	0.611
4	12 57 31	20	14 , 10	−0 41.89	+ 4 39.3	40 7.67	+15 0 15.8	9.556	0.617
1904				Comet *a* 1904 (Brooks).					
Apr. 17	9 39 55	21	11 , 12	−0 38.31	− 1 57.1	16 55 49.60	+44 51 32.4	9.806_n	0.521
18	8 56 18	22	15 , 15	−1 17.27	− 8 8.2	53 7.61	+45 27 56.1	9.813_n	0.609
20	8 40 14	23	—, 2	— ——	− 7 37.3	— ——	+46 40 56.7	———	0.606
20	14 15 47	23	11 , 11	−0 41.43	+ 0 58.6	46 34.63	+46 49 32.6	9.005_n	0.070_n
22	8 32 54	24	4 , 4	−2 38.72	+ 9 25.0	41 4.58	+47 52 10.1	9.833_n	0.581

Date	Columbia M. T.	Star	No. of Comparisons	Comet—Star		Comet's Apparent		log p Δ	
				Δα	Δδ	α	δ	α	δ
22	10h 17m 50s	25	10 , 10	+0m 30.28	+ 9′ 18.″1	16h 40m 49.s42	+47° 54′ 43.″3	9.791n	0.186
22	10 55 32	26	10 , 10	+4 0.81	+ 1 54.5	40 44.52	+47 55 39.5	9.750n	9.907
26	8 19 58	27	25 , 25	+0 16.48	+ 1 53.8	27 28.64	+50 5 55.1	9.852n	0.515
26	9 4 21	28	20 , 20	+0 40.52	— 3 15.4	27 21.91	+50 6 57.2	9.840n	0.355
26	12 2 50	28	12 , 12	+0 13.68	+ 0 45.8	26 55.07	+50 10 58.4	9.568n	9.990n
27	8 26 42	29	14 , 14	+0 34.51	+ 7 17.1	23 48.05	+50 37 40.6	9.856n	0.480
27	9 2 44	30	20 , 20	+1 13.82	— 1 38.6	23 43.33	+50 38 26.4	9.844n	0.342
28	10 42 27	31	15 , 14	—0 8.21	— 0 56.4	19 44.06	+51 11 4.8	9.733n	8.602n
28	11 17 36	31	25 , 14	—0 13.98	— 0 8.7	19 38.29	+51 11 52.5	9.662n	9.778n
28	11 56 31	32	15 , 13	+0 37.84	+ 5 28.7	19 31.69	+51 12 51 5	9.546n	0.045n
29	9 1 23	33	17 , 12	+0 2.31	+ 5 9.4	16 10.10	+51 38 50.3	9.846n	0.255
29	11 29 17	33	— , 3	— ——	+ 8 11.3	— ——	+51 41 52.2	———	9.973n
30	8 15 5	34	16 , 16	—0 27.71	— 9 14.0	12 23.33	+52 6 42.9	9.868n	0.305
30	8 40 43	35	10 , 10	+0 30.81	— 5 12.7	12 18.81	+57 7 13.7	9.857n	0.155
30	8 53 40	35	9 , —	+0 28.84	— ——	12 16.84	— — ——	9.849n	———
30	9 11 13	34	10 , —	—0 35.68	— ——	12 15.36	— — ——	9.844n	———
May 1	8 12 19	36	18 , 15	+0 10.43	+ 2 59.8	8 24.28	+52 34 33.0	9.870n	0.389
2	8 27 47	37	15 , 15	+2 7.45	+ 2 28.5	4 16.16	+53 1 40.8	9.864n	0.276
3	8 40 52	38	15 , 10	—0 47.79	— 1 3.2	16 0 5.31	+53 27 42.0	9.855n	0.137
3	11 9 44	39	15 , 15	+2 26.03	+ 5 1.6	15 59 38.33	+53 30 20.6	9.597n	0.127n
7	8 49 30	40	17 , 10	+1 1.12	+ 3 6.5	42 43.70	+55 □ 11.5	9.829n	9.214
8	8 17 14	41	23 , 10	+0 21.01	+ 4 0.8	38 20.57	+55 20 4.4	9.858n	9.808
8	8 40 46	42	18 , 10	—0 14.13	+ 8 38.0	38 16.35	+55 20 20.4	9.833n	9.304
9	9 6 10	43	20 , 10	+2 39.96	+ 2 33.5	33 39.30	+55 39 50.1	9.789n	9.765n
9	9 52 52	44	23 , 10	—0 12.98	— 9 44.4	33 30.82	+55 40 25.9	9.694n	0.140n
10	9 24 9	45	16 , 10	—1 5 76	+ 3 43.2	29 0.97	+55 58 3.8	9.743n	0.043n
11	9 11 28	46	15 , 15	+0 37.15	+11 25.4	24 26.75	+56 14 35.5	9.755n	0.024n
14	8 33 20	47	16 , 10	—0 47.61	+11 35.7	10 35	+56 57 34	9.787n	9.966n
14	9 44 22	48	12 , 9	—1 4.91	+ 0 53.9	15 10 21	+56 58 14	9.609n	0.304n
18	13 29 13	49	17 , 11	—0 37.45	+ 7 27.7	14 50 54.43	+57 40 55.3	9.702	0.240n
18	13 57 21	50	12 , 10	+0 36.36	+ 4 15.1	50 49.24	+57 41 3.4	9.767	0.107n
19	8 43 19	51	19 , 19	—1 0.29	— 1 27.6	47 12.23	+57 46 54.1	9.676n	0.277n
19	13 19 36	52	20 , 19	+1 3.80	+ 3 39.3	46 18.52	+57 48 12.7	9.701	0.248n
20	8 20 59	53	21 , 10	—0 10.98	— 8 3.3	42 40.94	+57 53 19.1	9.715n	0.231n
20	13 55 30	53	20 , 20	—1 15.76	— 6 38.9	41 36.16	+57 54 43.5	9.799	0.013n
21	8 32 6	54	24 , 14	+0 33.43	— 8 39.9	38 3.35	+57 58 51.6	9.662n	0 301n
24	8 28 44	55	18 , 18	—1 10.69	— 1 39.1	24 36	+58 9 13	9.590n	0.361n

MEAN PLACES OF COMPARISON STARS FOR BEGINNING OF YEAR.

Star	α	Red. to App. Place	δ	Red. to App. Place	Authority
1	17h 1m 23.s29	+2.s04	— 9° 38′ 25.″1	+ 7.″4	B. D. —9°.4501. Comparison with star 1a
1a	17 1 41.88	+2.04	— 9 39 34.5	+ 7.4	Schj. 6097
2	6 52 50.50	+1.69	—17 25 54.1	—20.7	B. D. —17°.1699. Wash. A. G. Zones.
3	22 59 8.05	—0.30	+ 2 46 19.3	+ 2.7	A. G. Albany 7960
4	23 5 24.74	—0.27	+ 3 39 25.4	+ 2.5	A. G. Albany 7997

Star	α	Red. to App. Place	δ	Red. to App. Place	Authority
5	23^h 10^m $10^s.31$	$-0^s.27$	$+4^\circ$ 58′ 6″.7	+2″.2	A. G. Albany 8028
6	14 15	−0.28	+ 5 50 46	+ 2.1	B. D. $+5^\circ.5156$
7	16 56.05	−0 28	+ 6 42 23.8	+ 1.9	A. G. Leipzig II 11618
8	18 43.09	−0.27	+ 6 58 29.6	+ 1.8	½ (Schj. 9637 + Schj. 9638)
9	23 9.32	−0.25	+ 8 5 50.8	+ 1.8	A. G. Leipzig II 11652
10	37 33.55	−0.23	+10 42 20.7	+ 1.1	A. G. Leipzig I 9407
11	39 46.75	−0.23	+11 27 26.7	+ 1.0	A. G. Leipzig I 9422
12	47 4 80	−0.20	+12 19 57.9	+ 0.8	A. G. Leipzig I 9464
13	47 58.58	−0.17	+12 56 23.1	+ 0.8	A. G. Leipzig I 9469
14	23 59 30.67	−0.17	+15 13 45.4	+ 0.3	A. G. ½ (Berlin A 9773 + Leip. I 9540)
15	0 1 21	−0.17	+15 35 37	+ 0.2	B. D. $+15^\circ.4938$
16	6 45 30.26	+1.95	+11 46 37.7	−13.0	A. G. Leipzig I 2530
17	44 10.01	+1.95	+12 51 32.7	−12.6	A. G. Leipzig I 2515
18	44 0.47	+1.95	+13 10 53.7	−12.6	A. G. Leipzig I 2511
19	40 48.52	+1.93	+14 3 58.7	−12.3	Comparison with star 19a
19a	40 33.85	+1.93	+14 8 37.9	−12.3	A. G. Leipzig I 2476
20	6 40 47.65	+1.91	+14 55 48.8	−12.3	A. G. Leipzig I 2477
21	16 56 26.34	+1.57	+44 53 36.7	− 7.2	A. G. Bonn 10878
22	54 23.27	+1.61	+45 36 11.3	− 7.0	A. G. Bonn 10847
23	47 14.37	+1.69	+46 48 40.6	− 6.6	A. G. Bonn 10765
24	43 41.53	+1.77	+47 42 51.3	− 6.2	A. G. Bonn 10729
25	40 17.36	+1.78	+47 45 31.3	− 6.1	B. D. $+47^\circ.2373$. Comp. with 25a and 25b
25a	39 42.41	+1.78	+47 49 14.6	− 6.1	Comparison with star 26
25b	38 35.83	+1.78	+47 38 48.9	− 6.1	A. G. Bonn 10681
26	36 41.92	+1.79	+47 53 51.0	− 6 0	A. G. Bonn 10666
27	27 10.23	+1.93	+50 4 6.3	− 5.0	B. D. $+50^\circ.2299$. Comparison with star 28
28	26 39.46	+1.93	+50 10 17.6	− 5.0	B. D. $+50^\circ.2297$. Comparison with star 28a
28a	24 38.01	+1.93	+50 9 48.8	− 5.0	A. G. Cambridge, U. S. 5003
29	23 11.58	+1.96	+50 30 28.1	− 4.6	A. G. Cambridge, U. S. 4998
30	22 27.55	+1.96	+50 40 9.6	− 4.6	A. G. Cambridge, U. S. 4994
31	19 50.27	+2.00	+51 12 5.4	− 4.2	A. G. Cambridge, U. S. 4982 } See notes.
32	18 51.85	+2.00	+51 7 27.0	− 4.2	A. G. Cambridge, U. S. 4976 } See notes.
33	16 5.77	+2.02	+51 33 44 8	− 3.9	A. G. Cambridge, U. S. 4960
34	12 48.98	+2.06	+52 16 0.3	− 3.4	A. G. Cambridge, U. S. 4950
35	11 45.94	+2.06	+52 12 29.8	− 3.4	A. G. Cambridge, U. S. 4947
36	8 11.77	+2.08	+52 31 36.2	− 3 0	Comparison with stars 36a and 36b
36a	8 16 97	+2.08	+52 23 7.2	− 3.0	A. G. Cambridge, U. S. 4937
36b	5 32.05	+2.10	+52 34 56.8	− 3.0	A. G. Cambridge, U. S. 4926
37	2 6.60	+2.11	+52 59 14.9	− 2.6	A. G. Cambridge, U. S. 4911
38	16 0 50.96	+2.14	+53 28 47.4	− 2.2	A. G. Cambridge, U. S. 4903
39	15 57 10.16	+2 14	+53 25 21.2	− 2.2	A. G. Cambridge, U. S. 4884
40	41 40.36	+2.22	+54 57 5.6	− 0.6	B. D. $+55^\circ.1774$. Comparison with star 40a
40a	40 13.24	+2.21	+54 50 21.4	− 0 6	A. G. ½ (Camb. 4822 + Hels.-Goth. 8474).
41	37 57.32	+2.24	+55 16 3.8	− 0.2	A. G. Helsingfors-Gotha 8455
42	38 28.24	+2.24	+55 11 42.6	− 0 2	Comparison with star 42a
42a	37 14.38	+2.23	+55 10 51.2	− 0.2	A. G. Helsingfors-Gotha 8452
43	30 57.10	+2.24	+55 37 16.3	+ 0 3	A. G. Helsingfors-Gotha 8408
44	33 41.55	+2.25	+55 50 10.0	+ 0.3	A. G. Helsingfors-Gotha 8428

Star	α	Red. to App. Place	δ	Red. to App. Place	Authority
45	$15^h\ 30^m\ 4^s.48$	$+2^s.25$	$+55°\ 54'\ 19''.9$	$+\ 0''.7$	A. G. Helsingfors-Gotha 8402
46	23 47.36	+2.24	+56 3 8.9	+ 1.2	A. G. Helsingfors-Gotha 8373
47	11 20	+2.23	+56 45 56	+ 2.6	Comparison with star 47a
47a	11 9	+2.22	+56 38 39	+ 2.6	B. D. $+56°.1791$
48	15 11 24	+2.23	+56 57 17	+ 2.7	Comparison with star 47
49	14 51 29.71	+2.17	+57 33 23.2	+ 4.4	A. G. Helsingfors-Gotha 8172
50	50 10.71	+2.17	+57 36 43.9	+ 4.4	Comparison with star 49
51	48 10.35	+2.17	+57 48 16.8	+ 4.9	A. G. Helsingfors-Gotha 8149
52	45 12.57	+2.15	+57 44 28.6	+ 4.8	A. G. Helsingfors-Gotha 8117. See notes.
53	42 49.78	+2.14	+58 1 17.3	+ 5.1	B. D. $+58°.1527$ Comparison with star 53a
53a	42 18.76	+2.14	+58 12 45.6	+ 5.1	A. G. Helsingfors-Gotha 8107
54	37 27.81	+2.11	+58 7 26.1	+ 5.4	A. G. Helsingfors-Gotha 8071
55	25 45	+2.02	+58 10 46	+ 6.5	B. D. $+58°.1504$

Faint comparison stars were referred to catalogue positions by chronographic and micrometer comparisons. The details of the reference are as follows:

Star	No. of Comp.	$\Delta\alpha$	$\Delta\delta$
1— 1a	17 , 5	$-0^m\ 18^s.59$	$+\ 1'\ 9''.4$
19—19a	15 , 7	+0 14.67	— 4 39.2
25—25a	10 , 10	+0 35.06	— 3 42.8
25—25b	10 , 10	+1 41.42	+ 6 41.9
25a—26	10 , 10	+3 0.49	— 4 36.4
27—28	23 , 16	+0 30.77	— 6 11.3
28—28a	13 . 6	+2 1.45	+ 0 28.8
36—36a	19 , 5	—0 5.11	+ 8 30.5
36—36b	17 , 10	+2 39.64	— 3 21.3
40—40a	16 , 5	+1 27.13	+ 6 44.2
42—42a	14 , 6	+1 13.86	+ 0 51.4
47—47a	23 , 10	+0 11.18	+ 7 16.8
48—47	13 , 6	+0 3.95	+11 20.7
50—49	15 , 7	− 1 19.00	+ 3 20.7
53—53a	11 , 5	+0 31.02	—11 28.3

NOTES.

COMET 1902 III (b 1902).

The observations have been corrected for differential refraction.

Nov. 7—Observation made during twilight. Comet low, in the west. Star 1a was nearer the comet than the comparison star actually used, but, owing to the slight difference in right ascension between 1a and the comet, it was not possible to use it as comparison star. The right ascension for this date is probably uncertain on account of the large $\Delta\delta$.

Feb. 21—An unusually fine night. Comet very faint, 11½ mag. Measures very difficult; especially in right ascension, on account of the impossibility of reducing the illumination of the longitudinal wire sufficiently. The comet was searched for on several nights, both preceding and following this date, but was not seen again.

COMET 1903 I (a 1903).

The observations have been corrected for differential refraction.

The telegraphic announcement of discovery was re-

ceived on the afternoon of Jan. 20, but clouds prevented an observation that night.

Jan. 21.—Smoke from the University power house prevented the completion of this observation. Continuously cloudy weather until Jan. 25.

COMET 1903 II (*d* 1902).

The observations have been corrected for differential refraction.

Jan. 25—Comet very faint 11 mag. Measures difficult, haze at end of observation.

Jan. 29—Fine night. Comet small, nearly round, with 12 mag. nucleus.

COMET *a* 1904 (BROOKS)

The observations have not been corrected for differential refraction.

The telegraphic announcement of discovery was received on the afternoon of April 17.

Apr. 17—Light haze. Comet apparently 10 mag. with 11 mag. nucleus. Measures interrupted by haze.

Apr. 20—Clouds prevented the completion of the first observation of this date. At the time of the second observation the comet was near the zenith, and consequently, difficult to observe. The measures are good, however.

Apr. 22—The positions of star 25 as obtained from 25a and 25b are given equal weight.

April 27, 28, 29—Observations difficult on account of moon.

Apr. 28—The apparent declinations of the comet as derived from stars 31 and 32 are discordant by about 10″. An examination of the observations made it seem probable that there must be large errors in the star places. The difference in declination of the two stars was then measured directly with the result

$$\Delta\delta = 4' \ 46''.0$$

The corresponding difference in the catalogue positions is

$$\Delta\delta = 4' \ 38''.4$$

Both stars 31 and 32 were then referred to Nos. 4985, 4987, and 4989 of the A. G. Camb. U. S. Catologue. The results are as follows:

		STAR 31	STAR 32
From	4985	+51° 12′ 11″.6	+51° 7′ 24″.8
	4987	11.5	24.7
	4989	8.7	21.9
	Means	10.6	23.8
	Catalogue	5.4	27.0

The right ascensions appear to be consistent. It is evident, however, that the positions of these stars should be investigated carefully before the observations are used.

May 1—The right ascensions of star 36 as obtained from 36a and 36b are given equal weight. The declination from 36b is given double weight.

May 11—Observations difficult on account of haze.

May 18—High humidity. Stars badly defined.

May 19—The right ascension of the comparison star, B. D. +57°.1534=Helsingfors-Gotha 8117, is 1^m too small, both in the B. D. (2nd ed.) and in the Helsingfors-Gotha catalogue. The position is correctly given by the B. D. charts, (2nd ed.).

May 20—Slight haze during first observation.

Instead of diminishing in brightness as predicted, the comet seems to have slightly gained in light. The nucleus at least, is brighter and better defined than when discovered. Observations on May 26 (two days before full moon) were far more easily obtained than those made a month ago. Upon exceptionally clear nights it has been possible to trace the tail for a distance of 3′ or 4′.

Columbia, Missouri, 1904, May 26.

F. H. Seares.

LAWS OBSERVATORY
UNIVERSITY OF MISSOURI

BULLETIN No. 3

OBSERVATIONS OF COMET *a* 1904 (BROOKS).

The following observations of Comet *a*, 1904, form a continuation of the series published in Bulletin No. 2. As in the preceeding series, the differences in right ascension have been obtained by transits. The observations have not been corrected for differential refraction. During the period covered by the present series, the comet has slowly dimmished in brightness, until at the present date it is too faint for further observation with the 7½-inch equatorial of this observatory.

1904	Columbia M. T.	Star	No. of Comparisons	Comet—Star		Comet's Apparent		log p Δ	
				Δα	Δδ	α	δ	α	δ
May 26	8h 28m 3s	1	—, 8	— ——	+13′ 10″.3	— — ——	+58° 11′ 37″.1	— —	0.393n
26	8 59 19	2	17, 8	+0 43.09	+ 9 40.7	14 15 51.06	+58 11 36.3	9.348n	0.436n
27	8 29 6	3	8, 5	−1 12.02	+ 5 3.9	11 43.10	+58 11 28.3	9.481n	0.409n
27	14 45 15	4	21, 10	−0 3.59	+ 9 23.1	14 10 35.94	+58 11 14.7	9.921	0.115
30	12 44 9	5	12, —	+0 15.33	— ——	13 58 45.77	—— — ——	9.819	——
30	12 44 9	6	—, 8	— ——	−18 29.3	— ——	+58 5 31.4	——	9.833n
30	13 3 44	7	5, 5	−0 51.69	+ 8 41.3	58 41.58	+58 5 32.6	9.848	9.664n
June 1	12 10 46	8	13, 8	−0 14.37	+ 3 22.4	51 6.56	+57 58 7.5	9.786	0.070n
1	12 32 4	9	15, 8	+0 13.28	+ 5 8.5	51 3.50	+57 58 5.1	9.823	9.889n
5	11 59 2	10	25, 10	−0 5.60	− 5 56.8	36 46.95	+57 35 44.6	9.814	9.897n
5	12 44 16	10	21, 10	−0 12.38	− 6 9.5	36 40.17	+57 35 31.9	9.873	9.195
5	13 26 16	10	20, 10	−0 18.32	− 6 20.1	36 34.23	+57 35 21.3	9.908	0.054
6	11 20 42	11	18, 8	−0 28.03	− 7 7.7	33 30.21	+57 28 59.6	9.755	0.126n
6	11 51 12	12	16, 8	+0 22.11	− 6 45.7	33 25.69	+57 28 47.8	9.811	9.899n
7	11 35 38	13	20, 8	−0 34.20	+ 3 53 9	30 11.51	+57 21 27.7	9 796	9.967n
7	12 29 36	14	14, 9	+1 53.37	− 3 21.7	30 4.71	+57 21 12.6	9.870	9.262
8	11 39 19	15	19, 10	−1 21.53	− 3 55.5	27 0.74	+57 13 34.7	9.812	9.849n
8	13 37 18	16	16, 10	+0 52.59	+ 3 57.6	26 44.72	+57 12 51.2	9.919	0.294
9	12 14 50	17	22, 10	+0 16.38	+ 6 59.8	23 51.12	+57 5 0.5	9.866	9.245
9	12 47 0	18	16, 10	−1 17.03	+ 5 34.9	23 46.82	+57 4 52.2	9.895	9.962
10	12 45 40	19	19, 8	−1 1.31	− 1 25.4	20 47.85	+56 56 5.4	9.897	0.032
10	14 4 57	20	14, 8	−2 22.68	+ 6 56.6	20 38.04	+56 55 31.1	9.923	0.482
11	11 37 7	21	16, 10	−0 13.92	+ 1 58.0	18 2.15	+56 47 21.8	9.833	9.440n
12	11 11 59	22	17, 8	−0 37.50	+10 58.1	15 17.06	+56 38 14.5	9.805	9.802n
12	11 53 [illegible]	23	15, 9	+2 8.67	− 0 56.3	15 13.02	+56 37 57.5	9.860	9.345
13	12 39 5	24	c, 10	+0 0.26	+ 7 13.7	12 24.42	+56 27 53.2	9.898	0.136
13	12 59 41	24	22, 10	−0 1.90	+ 7 4.7	12 22.26	+56 27 44.2	9.909	0.290
16	11 51 59	25	17, 10	+1 16.02	+ 1 15.6	4 52.63	+55 57 35.8	9.875	9.953
16	12 12 50	26	17, —	−0 8.03	— ——	13 4 50.29	—— — ——	9.890	——

1904	Columbia M. T.	Star	No. of Comparisons	Comet—Star		Comet's Apparent		log p Δ	
				Δα	Δδ	α	δ	α	δ
June 16	12h 31m 54s	26	—, 8	— ——	+ 0′ 37″.6	— — ——	+55° 57′ 17″.7	——	0.253
21	10 33 21	27	19, 10	—0 4.27	+ 2 38.0	12 53 59.00	+55 3 9.0	9.814	8.976n
21	10 50 10	27	23, 10	—0 5.91	+ 2 29.4	53 57.36	+55 3 0.4	9.835	9.412
22	11 46 48	28	14, 10	—0 11.58	+13 45.5	51 54.73	+54 51 3.2	9.884	0.227
23	11 31 45	28	16, 10	—2 3.75	+ 2 22.1	50 2.56	+54 39 39.8	9 877	0.173
25	10 50 7	29	24, 10	+0 4.22	— 4 48.9	46 27.58	+54 16 13.0	9.886	0.349
29	9 53 30	30	20, 10	+0 17.02	+ 4 9.3	40 13.54	+53 30 4.0	9.805	9.302
30	9 18 6	30	20, 10	—1 6.44	— 7 21.6	38 50.08	+53 18 33.1	9.758	9.544n
July 1	9 5 19	31	20, 10	—0 6.66	— 7 54.5	37 28.55	+53 6 49.6	9.743	9.659n
3	9 24 36	32	19, 10	+0 11.62	— 1 41.7	36 8.59	+52 54 53.3	9.782	8.205
8	10 18 51	33	24, 8	+0 10.04	— 2 24.8	29 20.89	+51 44 7.0	9.851	0.285
8	10 39 28	34	22, 8	+0 20.64	— 2 28.8	29 20.03	+51 43 57.4	9.860	0.381
9	10 30 53	35	11, 5	—0 13.29	— 5 57.0	28 23.00	+51 32 28.8	9.857	0.368
10	10 58 55	36	14, 13	+1 18.37	— 9 58.4	27 28.42	+51 20 52.4	9.863	0.493
12	11 17 21	37	15, 10	+1 24.60	+ 7 5.2	25 47.28	+50 57 52.3	9.860	0.579
16	12 37 33	38	15, 12	—1 9.87	+ 2 43.7	22 52.75	+50 12 49.1	9.798	0.784
18	10 49 35	39	14, 10	+0 32.31	— 6 4.2	21 43.22	+49 51 57.1	9.850	0.588
Aug. 1	9 7 8	40	18, 10	—0 35.26	— 9 36.1	16 38.64	+47 33 24.4	9.827	0.490
16	9 12 34	41	17, 17	+1 13.34	+ 6 11.4	15 59.93	+45 31 7.7	9.807	0.674

MEANS PLACES OF COMPARISON STARS FOR 1904.0

No.	α	Red. to App. Place	δ	Red. to App. Place	Authority
1	14h 16m 6s.94	+1s.94	+57° 58′ 19″.6	+ 7″.2	Comparison with star 2a
2	15 6.03	+1.94	+58 1 48.4	+ 7.2	Comparison with star 2a
2a	19 4.61	——	+58 3 2.6	——	A. G. Helsingfors-Gotha 7444
3	12 53.16	+1.92	+58 6 16.9	+ 7.5	Comparison with star 2
4	14 10 37.61	+1.92	+58 1 44.1	+ 7.5	Comparison with star 2
5	13 58 28.70	+1.74	+58 15 58.3	+ 8.4	B. D. +58°1481. Comparison with star 5a
5a	59 26.97	——	+58 28 18.5	——	A. G. Helsingfors-Gotha 7806
6	59 0.44	+1.74	+58 23 52.3	+ 8.4	Comparison with star 5a
7	59 31.55	+1.72	+57 56 43.0	+ 8.3	B. D. +58°1483. Comp. with stars 7a and 7b
7a	58 46.52	——	+57 41 14.8	——	A. G. Helsingfors-Gotha 7801
7b	54 32.52	——	+57 55 5.1	——	A. G. Helsingfors-Gotha 7772
8	51 19.25	+1.68	+57 54 36.3	+ 8.8	Comparison with stars 7b and 9a
9	50 48.54	+1.68	+57 52 47.8	+ 8.8	Comparison with stars 7b and 9a
9a	54 23.93	——	+57 49 56.8	——	A. G. Helsingfors-Gotha 7771
10	36 51.08	+1.47	+57 41 31.9	+ 9.5	A. G. Helsingfors-Gotha 7657
11	33 56.80	+1.44	+57 35 57.5	+ 9.8	B. D. +57°1453. Comparison with star 10
12	33 2.14	+1.44	+57 35 23.7	+ 9 8	Comparison with star 10
13	30 44.34	+1.37	+57 17 24.0	+ 9.8	A. G. Helsingfors-Gotha 7622
14	28 9.99	+1.35	+57 24 24.5	+ 9.8	A. G. Helsingfors-Gotha 7607
15	28 20.95	+1.32	+57 17 20.2	+10.0	A. G. Helsingfors-Gotha 7608
16	25 50.84	+1.29	+57 8 43.7	+ 9.9	Comparison with star 18

No.	α	Red. to App. Place	δ	Red. to App. Place	Authority
17	$13^h\ 23^m\ 33^s.47$	$+1^s.27$	$+56°\ 57'\ 52''.6$	$+10''.1$	Comparison with star 18
18	25 2.58	+1.27	+56 59 7.2	+10 1	A. G. Helsingfors-Gotha 7583
19	21 47.93	+1.23	+56 57 20.5	+10.3	Comparison with star 17
20	22 59.48	+1.24	+56 48 24.3	+10 2	B. D. $+57°.1439$. Comparison with star 20a
20a	25 20.45	———	+56 43 43.2	———	A. G. Helsingfors-Gotha 7588
21	18 14.85	+1.22	+56 45 13.5	+10.3	Comparison with star 20
22	15 53.44	+1.12	+56 27 6.2	+10.2	A. G. Helsingfors-Gotha 7525
23	13 3.25	+1.10	+56 38 43.6	+10.2	B. D. $+56°.1648$. Comp. with 23a and 24
23a	13 50.28	———	+56 54 19.8	———	A. G. Helsingfors-Gotha 7513
24	12 23.09	+1.07	+56 20 29.2	+10.3	A. G. Helsingfor-Gotha 7503
25	3 35.59	+1.02	+55 56 9.8	+10.4	B. D. $+56°.1641$. Comp. with star 26
26	4 57.30	+1.02	+55 56 30.1	+10.4	Comparison with star 26a
26a	13 11 23.77	———	+55 49 25.4	———	A. G. Helsingfors-Gotha 7496
27	12 54 2.50	+0.77	+55 0 20.7	+10.3	B. D. $+55°.1565$. Comp. with 27a and 27b
27a	54 20.46	———	+55 15 27.4	———	A. G. Helsingfors-Gotha 7388
27b	55 32.00	———	+54 49 4.6	———	A. G. ½ (Hels. 7398+Camb. 4187).
28	52 5.59	+0.72	+54 37 7.5	+10.2	A. G. ⅛ (Hels. 7378+Camb. 4172).
29	46 22.77	+0 59	+54 20 51.8	+10.1	Comparison with star 29a
29a	45 12.00	———	+54 29 27.7	———	A. G. Cambridge U. S. 4151
30	39 56.00	+0.52	+53 25 44.9	+ 9.8	Comparison with star 30a
30a	42 58.87	———	+53 23 8.0	———	A. G. Cambridge U. S. 4144
31	37 34.72	+0.49	+53 14 34.2	+ 9.9	B. D. $+53°.1563$. Comparison with star 30
32	35 56.53	+0.44	+52 56 25.3	+ 9.7	Comparison with star 32a
32a	40 40.36	———	+52 53 41.2	———	A. G. Cambridge U. S. 4134
33	29 10.60	+0.25	+51 46 22.8	+ 9.0	A. G. Cambridge U. S. 4098
34	28 59.14	+0.25	+51 46 17.2	+ 9.0	A. G. Cambridge U. S. 4097
35	28 36.07	+0.22	+51 38 16.9	+ 8.9	A. G. Cambridge U. S. 4095
36	26 9.86	+0.19	+51 30 42.2	+ 8.6	A. G. Cambridge U. S. 4092
37	24 22.50	+0.18	+50 50 38.9	+ 8.2	A. G. Cambridge U. S. 4085
38	24 2.48	+0.14	+50 9 57.6	+ 7.8	A. G. Cambridge U. S. 4083
39	21 10.84	+0.07	+49 57 53.8	+ 7.5	A. G. Cambridge U. S. 4072
40	17 14.01	—0.11	+47 42 55.0	+ 5.5	A. G. Bonn 8469
41	14 46.81	—0.22	+45 24 53.5	+ 2.8	A. G. Bonn 8450

Faint comparison stars were referred to catalogue positions by chronographic and micrometer comparisons. The details of the reference are as follows:

Stars	No. of Comp.	$\Delta\alpha$	$\Delta\delta$
1—2a	10 , 5	$-2^m\ 57.67$	$-\ 4'\ 43''.0$
2—2a	10 , 5	−3 58.58	− 1 14.2
3—2	9 , 5	−2 12.87	+ 4 28 5
4—2	10 , 5	−4 28.42	− 0 4.3
5—5a	10 , 10	−0 58.27	−12 20.2
6—5a	7 , 10	−0 26.53	− 4 26.2
7—7a	10 , 5	+0 45.00	+15 28.0

Stars	No. of Comp.	$\Delta\alpha$	$\Delta\delta$
7—7b	9 , 5	$+4^m\ 59^s.05$	+ 1′ 38″.2
8—7b	11 , 5	—3 13.27	— 0 27.2
8—9a	11 , 5	—3 4.68	+ 4 38.0
9—7b	11 , 5	—3 43.99	— 2 15.7
9—9a	11 , 5	—3 35.38	+ 2 49.5
11—10	10 , 5	—2 54.28	— 5 34.4
12—10	10 , 5	—3 48.94	— 6 8.2
16—18	18 , 10	+0 48.26	+ 9 36.5
17—18	16 , 6	—1 29.11	— 1 14.6
19—17	12 , 5	—1 45.54	— 0 32.1
20—20a	10 , 5	—2 20.97	+ 4 41.1
21—20	10 , 5	—4 44.63	— 3 10.8
23—23a	9 , 4	—0 46.98	—15 34.0
23—24	8 , 4	+0 40.11	+18 12.3
25—26	19 , 5	—1 21.71	— 0 20.3
26—26a	10 , 5	—6 26.47	+ 7 4.7
27—27a	40 , 5	—0 17.78	—15 6.9
27—27b	27 , 5	—1 29.69	+11 16.3
29—29a	13 , 8	+1 10.77	— 8 35.9
30—30a	10 , 5	—3 2.87	+ 2 36.9
31—30	10 , 5	—2 21.18	—11 10.7
32—32a	10 , 5	—4 43.83	+ 2 44.1

The right ascension of the first observation of June 13 depends upon a coincidence of the comet with the hour circle passing through the comparison star. The printed value

$$\Delta\alpha = + 0^s.26,$$

results from the application of a correction for non-perpendicularity of the wires of the micrometer.

A comparison of the later observations of the series with the the ephemeris of Aitken and Maddrill, Lick Observatory Bulletin No. 56, gives the following series of corrections to that ephemeris:

Date	$\Delta\alpha \cos\delta$	$\Delta\delta$	Date	$\Delta\alpha \cos\delta$	$\Delta\delta$
June 22	— 1.8	+ 2″	July 8	— 1.9	0″
23	— 1.8	+ 2	9	— 2.2	— 1
25	— 1.3	+ 1	10	— 1.7	+ 5
29	— 2.0	+ 1	12	— 1.7	— 2
30	— 1.9	+ 1	16	— 2.1	— 1
July 1	— 1.8	+ 1	18	— 2.0	— 2
2	— 1.9	+ 3	August 1	— 2.5	— 6
8	— 1.9	0	16	— 2.4	— 7

The relatively large residual in declination for July 10 is perhaps due to proper motion. The epoch of the position of the comparison star, which is Camb. U. S. 4092, is 1871.4, and as this position depends upon six observations it is probably well determined. Upon the other hand, the position of the comet depends upon thirteen measurs in declination with a probable error for the mean difference in declination of $\pm$ 0.″4. The comet was very faint and the measures are more or less irregular, but the maximum residual for the individual settings is only 3.″6. From the appearance of the comet, it would scarcely seem possible that a systematic error, large enough to account for the above residual, could have entered into the measures.

Columbia, Missouri, 1904 Aug. 19. *F. H. Seares.*

LAWS OBSERVATORY
UNIVERSITY OF MISSOURI

BULLETIN NO. 4.

OBSERVATIONS OF COMET *e* 1904 (BORRELLY).

The following observations of Comet *e* 1904 have been made with the 7½-inch equatorial and filar micrometer of the Laws Observatory. The differences in right ascension of comet and comparison star were obtained by transits. The magnifying power used was 140. The observations have not been corrected for differential refraction.

1904-5	Columbia M. T.	Star	No. of Comparisons	Comet—Star		Comet's Apparent		log p Δ		Red. to App. Place	
				Δα	Δδ	α	δ	μ	δ		
Dec. 31	7h 36m 26s	1	12, —	+0m 3s.11	— ——	1h 16m 24s.72	—— — ——	9.085	——	+2s.86	——
31	8 4 5	2	11, 12	+2 16.07	+1 21.7	16 26.06	— 8 11 6.3	9.237	0.803	+2.86	+9.9
Jan. 3	10 53 12	3	26, 10	+0 4.95	—6 39.0	20 37.2	— 5 38.0	9.624	0.759	—0.13	—8.7
4	8 30 28	4	20, 20	—1 7.80	+3 29.7	21 53.5	— 4 54.5	9.375	0.777	—0.12	—8.6
19	8 39 56	5	10, 10	+0 51.97	+5 26.7	46 15.95	+ 6 54 34. 9	9.490	0 686	—0.07	—6.3
23	7 16 40	6	8, —	+0 52.86	— ——	53 39.26	—— — ——	9.266	——	—0.05	——
23	8 0 20	6	10, 10	+0 56.04	+1 44.7	53 42.44	+ 9 52 6.4	9.418	0.648	—0.05	—5.7
25	8 9 0	7	23, 10	—0 7.05	—3 22.0	1 57 37.53	+11 19 22.7	9.453	0.636	—0.05	—5 4
27	7 9 12	8	15, 15	+0 57.03	+5 2.0	2 1 32.50	+12 43 4.2	9.272	0.600	—0.06	—5.3
29	9 11 3	9	21, 10	—0 11.09	+2 30.0	5 48.52	+14 10 34.1	9.887	0.641	—0.04	—5.1
Feb. 6	10 26 10	10	12, 12	—1 8.68	+2 29.9	23 22.48	+19 31 30.2	9.675	0.670	—0.02	—4.1
5	10 53 55	11	12, 12	+0 39.07	—2 8.2	2 23 24.95	+19 32 23.6	9.683	0.695	—0.02	—4.1

MEAN PLACES OF COMPARISON STARS FOR BEGINNING OF YEAR.

No.	α	δ	Authority.
1	1h 16m 18s.75	—— — ——	Comparison with star 2
2	14 7.13	—8 12 37.9	A. G. Wien-Ottakring 263
3	20 32.4	—5 31.2	B. D.—5°.254
4	23 1.4	—4 57.9	B. D.—5.261
5	45 24.05	+6 49 14.5	A. G. Leipzig II 690
6	52 46.45	+9 50 27.4	A. G. Leipzig II 746
7	1 57 44.63	+11 22 50.1	A. G. Leipzig I 608
8	2 0 35.53	+12 38 7.5	A. G. Leipzig I 622
9	5 59.65	+14 8 9.2	A. G. Leipzig I 645
10	24 31.18	+19 29 4.4	A. G. Berlin A 684
11	22 45.90	+19 34 35.9	A. G. Berlin A 680

NOTES.

As seen in the 7½-inch equatorial the comet appeared nearly round with a diameter of about 1′. When first observed it was of the eleventh magnitude.

Dec. 31—No nucleus visible, but much brighter near center.

Jan. 4—Slight haze at beginning of observation, but very clear later. Comet well seen. Nucleus of 11^m. Measures good.

Jan. 19—Comet extremely faint on account of nearly full moon. Settings very irregular. This observation should receive but little weight.

Jan. 23—Nucleus suspected. Observations interrupted by clouds.

Jan. 25—Fine night. Nucleus well defined; 11½ mag.

Jan. 27—Nucleus easily seen. Measures difficult on account of fluctuation in illumination of micrometer wires.

Jan. 29—Nucleus not so easily seen as on Jan. 27. Short tail suspected.

Feb. 6—Comet very faint. Nucleus seen only occasionally.

PHOTOMETRIC OBSERVATIONS OF *NOVA GEMINORUM* Ch. 2387.

Systematic observations of *Nova Geminorum* were prevented by almost continuously cloudy and hazy weather during the months of April and May, 1903. Even the few observations printed below were obtained with considerable difficulty on account of floating clouds and haze.

The measures were obtained with a Pickering stellar photometer, alternate settings being made upon the *nova* and the comparison star. The comparison star was B. D. +30°1306, which is No. 11 of Hagen's *Chart and Catalogue for Observing Nova Geminorum.* Its magnitude as given by Hagen is 8.6, but the value 9.02 derived by Parkhurst[1], has been used in the reduction of the observations.

The following table shows the results of the observations. Column four contains the difference in brightness of the *nova* and comparison star expressed in divisions of the wedge. Column five gives the same difference, expressed in magnitudes, on the assumption that one division of the wedge corresponds to $0^m.127$. The last column contains the probable errors of the accepted magnitudes given in column six.

The publication of these observations has been postponed several months in order that they might appear in a special bulletin devoted to photometric investigations; but these investigations are not yet finished, and the results are printed here rather than delay them longer.

1903	Sid. Time	No. of Comp.	Nova—*c*	*Δm*	Magnitude	P. E.
March 31	$12^h\ 7^m$	10	—4.21	—0.53	8.49	$\pm 0^m.037$
April 1	9 37	10	—1.80	—0.23	8.79	0.030
1	11 18	4	—2.05	—0.26	8.76	0.060
6	9 50	15	+1.39	+0.18	9.20	0.023
8	9 30	15	—0.19	—0.02	9.00	0.030
11	11 41	10	+3.74	+0.47	9.49	0.022
30	10 54	10	+7.44	+0.94	9.96	±0.027

[1]Parkhurst, J. A., *Popular Astronomy*, 1903, p. 330.

SPECIAL TIME SIGNALS FROM THE U. S. NAVAL OBSERVATORY.

At the request of the Superintendent of the U. S. Naval Observatory, the special time signals of September 8 and December 31, 1904, were received at the Laws Observatory and carefully compared with the standard clock. The single set of September 8, and all four sets of December 31 were received upon the chronograph along with the beats of the local clock. Observations for the determination of the error of the clock were made on the nights the signals were received. In neither case can the accidental error of the clock correction exceed 0.03 or 0.04. The longitude used in the reduction of the comparisons is the accepted value $6^h\ 9^m\ 18.33 \pm 0.04$ west from Greenwich, determined by Professors Updegraff and Pritchett in 1891.

The following are the central standard times at which the final beats of the Washington clock were received.

1904, September 8,	$10^h\ 59^m$	59.46
December 31,	10 59	59.37
	11 59	59.33
	12 59	59.40
	13 59	59.42

According to letters received from the Superintendent of the Naval Observatory the correction to the midnight signal of September 8, as sent from Washington was −0.05 while that for December 31 was 0.00.

The signals were therefore received too early by 0.59, 0.63, 0.67, 0.60, and 0.58 respectively.

Columbia, Missouri, 1905, Feb. 7. *F. H. Seares.*

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LAWS OBSERVATORY
UNIVERSITY OF MISSOURI

BULLETIN NO. 5

THE POLARIS VERTICAL CIRCLE METHOD OF DETERMINING TIME AND AZIMUTH

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COLUMBIA, MISSOURI
PRESS OF E. W. STEPHENS
April, 1905

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INTRODUCTION

The determination of time and azimuth by the observation of transits of stars across the vertical circle passing through Polaris has long been known as a method of extreme accuracy. Hansteen, Schumacher, Bessel, and Hansen have all remarked upon the precision which can be obtained in this manner, and have developed formulae for the practical application of the method. But the labor of reduction in the use of these formulae is considerable, and consequently they have found no general application, although it has always been admitted that the operations involved in the observations are simpler and more expeditious than those occurring in the usual method of meridian time determination. Moreover, with the vertical circle method, a merely nominal increase in labor leads to a simultaneous determination of the azimuth, a fact which, for field operations, is of no small importance.

In outline, the observations by the vertical circle method are as follows: Three or four minutes before the transit of a time star, the instrument is directed toward Polaris and adjusted in azimuth until that object is bisected by the middle thread of the reticle. Without changing the azimuth of the instrument, the telescope is then rotated about the horizontal axis and the transit of the time star observed. The time of transit and the instant of bisection, together with the inclination of the horizontal axis, are the observed data. When combined with the known positions of the stars and the latitude of the place, these suffice for the formation of an equation involving the clock correction and the collimation constant as unknowns. A repetition of the process, with the instrument reversed, supplies a second equation, which, combined with the first, leads to the evaluation of both the unknowns. If the readings of the horizontal circle, corresponding to the bisection of Polaris and to the settings on a mark, be added to the above data, the observations will then be complete for the determination of the azimuth of the mark.

The advantages of the method are as follows: First, the constancy of the azimuth and collimation is assumed for only four or five minutes, whereas, with the usual method, they are supposed to remain unchanged for at least half an hour; second, no preliminary adjustment in the plane of the meridian is necessary; third, observations complete in themselves and sufficient for the determination of the instrumental constants can be obtained in a very brief period; and finally, as mentioned above, there is the possibility of a simultaneous determination of time and azimuth with only a very slight increase in labor. It would seem, therefore, that the vertical circle method is peculiarly adapted for use with unstable instruments, and for all cases in which shortness of observing program is a desideratum.

The most serious objection to the method has always been the amount of time required for the reduction of the observations. This has been relieved in a measure by the method proposed by Döllen, who, through the introduction of approximate formulæ and the ingenious tabulation of certain quantities for each time star, was able to reduce materially the labor of calculation, and to present the matter in a really practical form. The investigations of Döllen were almost a life work and the greater part of the literature of the vertical circle method

consists of his writings. His principal memoirs were published in 1863 and 1874[1]. His later investigations are contained principally in the introductions to the special series of Ephemerides[2], which he prepared for the facilitation of the observations and reductions.

The theoretical developments necessary for the solution of the problem can be carried out in two quite different ways. One is a direct method and involves the solution of a series of trigonometric equations without any assumption as to the values of the unknowns. The other is indirect and is based upon the method of successive approximations, but the convergence is so rapid that ordinarily only a single approximation is necessary for the determination of the clock correction. Döllen's first memoir follows the direct method, while in that of 1874 he uses the indirect solution. In the Ephemerides, the publication of which began in 1886, he returned to his original method of treatment to which he adheres in all his later papers. The reductions are very simple when the Ephemerides are at hand, and require less time than is necessary for the usual method of time determination. The Ephemerides give, for a considerable number of time stars, certain constants and the time of transit, T, of each star across a vertical circle passing through Polaris, in so far as this quantity is independent of the latitude. The clock correction is given at once by forming the difference between T, reduced to the given latitude, and the observed time of transit, corrected for level and collimation and for a term arising from the fact that the vertical circle chosen for the Ephemerides does not coincide, in general, with that over which the transit has been observed. These special ephemerides replace entirely the usual star list, but like the latter, must be specially calculated for each year so that their formation entails a large amount of labor. Although convenient in actual use, the Ephemerides present a disadvantage in that they do not apply to the southern hemisphere. Further, the simplicity gained through the introduction of approximate formulæ has been carried to such an extent that in certain cases an uncertainty of $0^s.03$ or $0^s.04$ may exist in the calculated clock correction.

It was to remedy these defects and certain others that in 1899 Harzer[3] rediscussed the whole question in a most minute and painstaking manner, and arrived at a solution which, so far as precision is concerned, leaves nothing to be desired, for the formulæ are so arranged that the sum of the neglected terms never exceeds a few thousandths of a second of time. His treatment

[1]Döllen, W. *Das Durchgangsinstrument im Verticale des Polarsterns, eine Festschrift. Die Zeitbestimmung vermittelst des tragbaren Durchgangsinstruments im Verticale des Polarsterns,* St. Petersburg, 1863. *Zweite Abhandlung,* St. Petersburg, 1874.

[2]Döllen, W. *Zeitsternephemeriden auf das Jahr 1886 fur die Zeitbestimmung vermittelst des tragbaren Durchgangsinstruments im Verticale des Polarsterns,* St. Petersburg, 1886.

Sternephemeriden auf das Jahr . . . zur Bestimmung von Zeit und Azimut mittelst des tragbaren Durchgangsinstruments im Verticale des Polarsterns, 1887-94; St. Petersburg, 1886-90; Berlin, 1891; Dorpat, 1893.

From 1895 to 1899 the Ephemerides were published by the Russian Astronomical Society under the title: *Ephémérides des étoiles pour la détermination de l'heure et de l'azimut au moyen d'un instrument des passages portatif établi dans le plan vertical de la polaire pour . . .* Their publication was discontinued with the year 1899. See *Astronomischer Jahresbericht,* 1900, Ref. 478 and 479.

The introduction to the Ephemerides for 1890 is particularly important.

In this connection see also, Döllen, *Zeitbestimmung zum Allgemeinbedarfe,* A. N. Bd. 114, S. 289.

[3]Harzer, Paul, *Ueber die Zeitbestimmung im Verticale des Polsternes. Publicationen der Sternwarte in Kiel. X,* Leipzig, 1899. In the following pages references to this memoir will be indicated by, *Harzer,* p—.

follows the direct method, but his final form for reduction is radically different from that of Döllen. The quantities tabulated in the Ephemerides of Döllen are independent of the latitude, but are of such a nature that their values must be given for intervals of ten days in order to take into account the variation in the star places due to precession, nutation, annual aberration, and proper motion. Harzer, on the other hand, makes his solution depend upon quantities which, for any given time star, may be considered constant for a long period of years, but which are not independent of the latitude. The quantities of Döllen must be given anew for each year, but are universal in their application to the northern hemisphere; those of Harzer are practically constant, but must be specially calculated for each place of observation.

It is difficult to estimate the relative amount of labor involved in the complete calculations for the two processes, as that depends largely upon the extent to which the vertical circle method finds application; but if both the Ephemerides of Döllen and the Harzer Star Constants are at hand, then there is no question but that the former afford the more expeditious method of reduction. Nevertheless, such a comparison is manifestly unfair to the solution of Harzer, which is of a far higher degree of precision than that due to Döllen.

In spite of the obvious advantages and the elaborate investigations that have been published, the vertical circle method has never received the appreciation which it merits, and, outside of Russia, has found only a limited application. In America the method is almost unknown. Döllen's original memoir was translated and appeared as a publication of the Bureau of Navigation[1], and his method of treatment has been succinctly explained in an article by Comstock[2]. These, together with an entirely different development by Stone[3], form, so far as I can learn, the entire American literature on the subject. The method has been tentatively used at various American observatories, but I am aware of no place where it finds constant application in the regular determination of time.

That such are the circumstances seems to be due to two causes: first, the developments are radically different from those of the usual theory of the transit; second, the calculations are generally supposed to be much longer than they really are, for although the computation for each star is longer than in the usual method, the observation of a smaller number of time stars usually suffices for the determination of the clock correction to a given degree of precision.

It has therefore seemed worth while to discuss the matter anew, and to attempt a development of the theory which should parallel somewhat closely that of the usual meridian method. Any simplification which it is possible to introduce, either in the manner of development or in the arrangement of formulæ for calculation, is certainly to be desired, especially in view of the fact that the publication of Döllen's Ephemerides has been discontinued. I believe that the

[1]Abbe, C., *The Portable Transit Instrument in the Vertical of the Pole Star, Translated from the Original Memoir of Wm. Döllen.* Bureau of Navigation, Washington, 1870.

[2]Comstock, G. C., *Determination of Time and Azimuth from Transits over the Vertical of the Pole Star.* Bull. Univ. Wisconsin, Science Series, v. I, p. 81. Madison, 1895.

[3]Stone, O., *On the Extra-Meridian Determination of Time by Means of a Portable Transit Instrument.* Cincinnati Society of Natural History, v. II, p. 211, Cincinnati, 1880.

following method of treatment presents such a simplification, both in the theoretical development and in the practical application of the vertical circle method.

The instant of transit of a star across the middle thread of the reticle is affected by three instrumental constants, namely, azimuth, level, and collimation. In the usual notation these are represented by a, b, and c, respectively. Let A be the western point of intersection of the rotation axis produced, with the surface of the celestial sphere. Let P and Z be the celestial pole and the zenith of the place of observation, and let S designate the position of a star at the instant of its transit. The relations between the parts of the spherical triangles APZ and APS make it possible to determine the effect of each instrumental constant upon the time of transit, and lead to the evaluation of the hour angle, τ, of the point S. τ represents the combined effect of all three instrumental constants, and, when added to the observed time, gives the apparent time of meridian transit. The evaluation of τ is the principal point in the reduction of transit observations made in accordance with the usual procedure, and the equation for its determination is extremely simple in form on account of the smallness of the constants a, b, and c. But in the vertical circle method the constants b and c are the same as in the ordinary method of observation. It is only a which presents any essential difference, and, even for the vertical circle method, this constant is always a moderately small quantity, since it can never exceed the azimuth of Polaris. For latitudes as high as 60°, a limit which includes all important observatories, the azimuth of Polaris is never greater than 2° 30′. It would therefore seem probable that there can be found for τ an expression entirely analogous in form to that occurring in the usual theory, and only slightly more complicated since the instrumental azimuth has been only moderately increased. Obviously, an analogous treatment of this sort would present a certain advantage over the solutions of Döllen and Harzer, for the similarity of the development to the well-known theory of the transit would permit an instant appreciation of the spirit of the method.

The determination of the azimuth of the instrument presents no difficulty, for this constant differs from the azimuth of Polaris only by small terms of the order of the level and collimation. The azimuth of Polaris is readily calculated when an approximation for the clock correction is known, and the effect of level and collimation is always easily included. Even when the clock correction is entirely unknown, no serious difficulty is encountered, for a simple differential formula gives the correction to the final result, arising from the fact that, in the first approximation, the error of the clock has been assumed equal to zero. The similarity of the development proposed to that already in use does not end with the formulæ which determine the error of the clock, for, as will be seen in the following, the final equation for azimuth is no other than that most frequently employed in precise determinations of this quantity.

It will be observed that what is proposed involves a return to the indirect method of solution for which developments and suggestions have been given by both Döllen and Bruns. None of these investigations are accessible to me, and therefore, I am unable to state in what particular my own method of treatment approximates to those already in existence. But I feel certain that their arrangement of formulæ is not the most fortunate possible, for Döllen, in his Ephemerides, returned to the direct solution of his first memoir, and Harzer, in discussing this

phase of the question, dismisses it with the statement[1] that the indirect solution is impracticable unless extensive and accurate tables for the azimuth of Polaris are at hand; whereas, it will be shown in the following that the clock correction can be obtained by the indirect method, with all the precision of Harzer, by an expenditure of labor in calculation only two-thirds as great as that required for his formulæ, and, moreover, without the use of special tables of azimuth.

What has been so persistently and eloquently urged by Döllen with regard to the advantages of the vertical circle method for the simultaneous determination of time and azimuth in triangulation operations,[2] applies equally well to the approximate determination of these quantities, which is sufficient in the work of the engineer. Thus far, the vertical circle method seems to have found no application in operations of this class, although its simplicity and directness make it of great value where the engineer's transit is the instrument used. Part II of this paper contains a development of the very simple formulæ which suffice for the case in which an uncertainty of $0^s.2$ or $0^s.3$ is permissible in the error of the timepiece. The formulæ are supplemented by examples illustrating the arrangement of the calculation and showing how close an agreement can be obtained between the results from individual stars observed with very small instruments.

The vertical circle method thus modified is the simplest with which I am familiar for the approximate determination of time and azimuth, and, in so far as the clock correction is concerned, is capable of giving results of a precision far beyond that of any method depending upon a direct reading of the circles. Moreover, it frees the observer completely from the necessity of waiting for the elongation of a circumpolar in order to make accurate observations for azimuth.

[1] *Harzer*, p. 6: "Soll einmal das indirecte Verfahren festgehalten werden, was nur bei der Existenz bequemer und scharfer Tafeln für das Azimut des Polsternes annehmbar erscheint, so ist mir das nach Herrn Bruns Vorschrift in Leipzig geübte Verfahren zweckmässiger vorgekommen, als das von Döllen in der zweiten Abhandlung festgehaltene."

[2] In this connection see, Herz, N., *Einige Bemerkungen zur Zeitbestimmung nach der Döllenschen Methode.* A. N. Bd. 156, S. 49.

PART I

THE PRECISE DETERMINATION OF TIME AND AZIMUTH

1. Methods of Observing

The exact arrangement of the observing program for the practical application of the vertical circle method will vary somewhat with the details of construction of the instrument used. Any instrument provided with a set of vertical threads, capable of rotation about a horizontal axis and of adjustment in azimuth throughout a range of a few degrees will answer the purpose. For precise determinations a sensitive striding level is also necessary, although, as will appear in Part II, an ordinary engineer's transit fitted with the usual plate levels will give results of a surprising degree of accuracy. If, besides the fixed threads, there be a movable micrometer wire perpendicular to the axis of rotation, the precision of the results can be increased and the use of the instrument considerably facilitated. Again, if the instrument can be quickly reversed without endangering the constancy of the azimuth, it will be possible, by a proper arrangement of the threads or by the use of a transit micrometer, to reverse upon each star, and thus make the observation of each time star independent of all the others, through a complete elimination of the collimation.

The most satisfactory arrangement of the striding level is that which permits its remaining constantly in position on the instrument. The combination of readings, taken before and after reversal, will then give the desired value of the inclination without any correction for inequality of pivots, although correction for irregularity in form must be applied in the usual manner. In any case, it is desirable to leave the level in position whenever possible and to take the readings of the bubble as near the instants of observation of Polaris and of the time star as possible. Döllen recommends that, with the usual arrangement in which the level must be removed during the reversal of the instrument, the level itself be not reversed during any given observation and the inclination be determined by combining readings taken in a single position with the error of adjustment as determined at the beginning and end of the series. It is needless to remark that this procedure is unwise unless the construction of the level is such as to give assurance of the constancy of its adjustment throughout a considerable period of time, a condition which is by no means always satisfied.

The variation in the details of construction of the instrument gives rise to three different arrangements of the observing program, which, in the following sections, will be referred to as Methods I, II, and III, respectively.

I. Given an instrument provided with a set of fixed threads, but having no movable micrometer wire. Adjust in azimuth until the image of Polaris is bisected by the middle thread and note the sidereal time of the instant of bisection. Rotate about the horizontal axis and observe the transit of a time star across all the threads. Let the sidereal time of the mean of the transits be θ_m. Reverse the instrument and repeat the process with a second time star. Such a

group of four observations, consisting of two settings on Polaris and of the transits of two time stars, forms a set, and suffices for the determination of the clock correction and the constants of azimuth and collimation. In selecting the time stars, only those should be chosen which culminate as near the zenith as possible, in order that any uncertainty in the determination of the azimuth may have a minimum effect upon the observation of the transit.

II. Given an instrument as in I, but provided with a movable micrometer thread parallel to the fixed threads of the reticle. Adjust in azimuth until Polaris is near the middle thread, then with the micrometer make two or three settings on the star in quick succession, noting the time of each. The mean of the settings, m_0, compared with the coincidence reading, m, gives the distance, f_0, of the star from the middle thread corresponding to the mean of the times, θ_0. A time star is then observed, as in I, and the process repeated with the instrument reversed in order to complete the set. The quantity f_0 should not exceed 1′ or 2′ in order to secure the proper convergence in the formulæ of reduction.

III. This method, due to Harzer, is an important modification of II. It requires a slight variation in procedure, according as the time star is observed across fixed threads, or by means of a registering transit micrometer.

a) The instrument has a movable micrometer thread as in II, and further, two sets of fixed threads situated at some distance from the collimation plane. Polaris is observed as in II. The transit of the time star across the first set of threads is observed and the instrument quickly reversed in order to observe its transit across the same threads as it recedes from the center of the field. Then, without changing the azimuth, the telescope is directed toward Polaris and two or three settings are made with the micrometer as before. Let m_1 and m_2 be the means of the micrometer readings before and after reversal; let θ_1 and θ_2 be the corresponding means of the clock times; and let θ_m be the mean of all of the transits of the time star.

b) The instrument is provided with a registering transit micrometer. The observations are the same as in IIIa. The details of the use of the micrometer, and the interpretation of certain constants appearing in the reduction, are fully explained in the memoir of Harzer.

Method III permits of a complete elimination of the collimation and makes the observation of each time star complete in itself. Evidently, everything depends upon the possibility of reversing the instrument quickly without introducing any change in the azimuth. That this can be done with a properly constructed instrument seems to be amply demonstrated by the experience of careful observers. For the majority of existing instruments, it will not be possible to make use of this method without some modification of the details of construction, and, for many instruments, not even then on account of the difficulty of reversal. The azimuth of the instrument can be much more accurately determined in both methods II and III than in I, consequently they are to be preferred on account of the greater range of zenith distance permissible in selecting the time stars.

All things considered, Method II is probably the one which would find the widest application at the present time. The reductions are but little longer, if any, than those necessary for III,

while the fact that the constancy of the azimuth is not endangered by a reversal, is an undeniable advantage in the case of the average instrument.

The question of the most advantageous distribution of the time stars, and of the probable error of the final result, has been very minutely examined by Harzer[1]. In so far as the conditions of observation are the same, the results obtained by him apply equally well to his own method of reduction and to that presented here, since in both methods the errors due to the introduction of approximate formulæ are small as compared with those arising from other sources. The conclusions refer to the results of observations made by Methods III. An investigation of the variation of the probable error with the declination shows that, for latitude 54° 20′, the minimum probable error corresponds to a declination of about 39° and the variation is such that for IIIa all stars included within the zone $\delta = 11.^{\circ}6$ to $\delta = 55.^{\circ}0$ may be given unit weight. For IIIb the corresponding limits are $\delta = 13.^{\circ}8$ and $\delta = 53.^{\circ}6$. For four settings of the micrometer upon Polaris and twenty transits of the time star observed by IIIa, the probable error of the clock correction derived from a single star within these limits is $0.^{s}0141$. For forty transits observed by IIIb, the probable error is $0.^{s}0131$. For the two methods, the probable errors of the means from three stars are $0.^{s}0081$ and $0.^{s}0076$, respectively. In all cases these results are assumed to be free from systematic personal error. Similar results can be derived for any other latitude, but those given in the memoir of Harzer are sufficient to indicate their character for latitudes which do not differ too greatly from 54° 20′. In general, it may be said that, for intermediate latitudes, the selection of time stars should be confined to the zone included between the zenith and the celestial equator and the results from all such stars should be given unit weight.

2. Development of the Fundamental Formulæ

For the northern hemisphere we adopt the convention that a and b shall be positive when the western end of the axis of rotation lies above and to the south of the east and west line; that c shall be positive when, upon rotation of the instrument about the horizontal axis, the small circle traced upon the celestial sphere by the middle thread of the reticle lies east of the collimation plane; and that τ, the hour angle of the time star at the instant of its transit across the middle thread, shall be positive when measured from the meridian toward the east. If further, we represent by $90^{\circ}-m$ and n the hour angle and declination, respectively, of the point A; by α and δ the right ascension and declination of a time star, S; and by φ the latitude of the place; the parts of the spherical triangles PAZ and PAS may be expressed as follows:

Side	$PA = 90^{\circ} - n$	Angle	$ZPA = 90^{\circ} - m$
	$ZA = 90^{\circ} - b$		$ZPS = \tau$
	$ZP = 90^{\circ} - \varphi$		$APS = 90^{\circ} + \tau - m$
	$AS = 90^{\circ} + c$		$PZA = 90^{\circ} + a$
	$PS = 90^{\circ} - \delta$		$PSA = \omega$

The application of the three fundamental formulæ of spherical trigonometry to the triangle PAS gives,

[1] *Harzer*, p. 45, *sqq.*

$$\begin{aligned} \sin n &= \sin b \sin \varphi - \cos b \cos \varphi \sin a, \\ \sin m \cos n &= \sin b \cos \varphi + \cos b \sin \varphi \sin a, \\ \cos m \cos n &= \cos b \cos a. \end{aligned} \tag{1}$$

The application of the first of the fundamental formulæ to the triangle PAS gives

$$-\sin c = \sin n \sin \delta - \cos n \cos \delta \sin (\tau - m). \tag{2}$$

The solution of the problem requires a transformation of equations (1) and (2) which will express τ explicitly in terms of the latitude, the declination of the star, and the instrumental constants a, b, and c. Further, let S_0 represent the position of Polaris at the instant of bisection by the middle thread of the reticle; let a_0 be its azimuth, measured from the north point positive toward the west; let τ_0 be its hour angle, positive when measured from the meridian toward the east; and let z_0 and π be its zenith distance and north polar distance, respectively. Then in the triangle PZS we have the relations

$$\begin{aligned} \cos z_0 &= \cos \pi \sin \varphi + \sin \pi \cos \varphi \cos \tau_0, \\ \sin z_0 \cos a_0 &= \cos \pi \cos \varphi - \sin \pi \sin \varphi \cos \tau_0, \\ \sin z_0 \sin a_0 &= -\sin \pi \sin \tau_0. \end{aligned} \tag{3}$$

Equations (3) determine the azimuth of Polaris at the instant its hour angle is τ_0.

Expanding equation (2) and eliminating m and n by (1) we derive

$$\begin{aligned} -\sin c = &-\sin a \cos b \cos \varphi \sin \delta + \sin a \cos b \sin \varphi \cos \delta \cos \tau \\ &+ \sin b \sin \varphi \sin \delta + \sin b \cos \varphi \cos \delta \cos \tau \\ &- \cos a \cos b \cos \delta \sin \tau. \end{aligned} \tag{4}$$

Replacing $\cos \tau$ by $1 - 2 \sin^2 \tfrac{1}{2}\tau$, (4) assumes the form

$$\begin{aligned} -\sin c = &\sin a \cos b \sin (\varphi - \delta) - 2 \sin a \cos b \cos \delta \sin \varphi \sin^2 \tfrac{1}{2}\tau \\ &+ \sin b \cos (\varphi - \delta) - 2 \sin b \cos \delta \cos \varphi \sin^2 \tfrac{1}{2}\tau \\ &- \cos a \cos b \cos \delta \sin \tau. \end{aligned}$$

Transposing and dividing by $\cos a \cos b \cos \delta$ and introducing at the same time the customary notation,

$$A = \sin (\varphi - \delta) \sec \delta, \qquad B = \cos (\varphi - \delta) \sec \delta, \qquad C = \sec \delta, \tag{5}$$

we find

$$\begin{aligned} \sin \tau = A \tan a + \frac{B \tan b}{\cos a} + \frac{C \sin c}{\cos a \cos b} \\ - 2 \sin^2 \tfrac{1}{2}\tau \, \frac{\sin b \cos \varphi + \cos b \sin \varphi \sin a}{\cos a \cos b} \end{aligned} \tag{6}$$

Equation (4) is equally true for the case in which Polaris is the star considered, whence we find

$$-\sin c = -\sin a \cos b (\cos\varphi \cos\pi - \sin\varphi \sin\pi \cos\tau_0) + \sin b (\sin\varphi \cos\pi + \cos\varphi \sin\pi \cos\tau_0) - \cos a \cos b \sin\pi \sin\tau_0.$$

By (3) this reduces to

$$-\sin c = -\sin a \cos b \sin z_0 \cos a_0 + \cos a \cos b \sin z_0 \sin a_0 + \sin b \cos z_0,$$

or finally,

$$-\sin c = \cos b \sin z_0 \sin(a_0 - a) + \sin b \cos z_0, \qquad (7)$$

which will serve as a relation between the instrumental azimuth and the azimuth of Polaris. Again, from the second and third of (3)

$$\tan a_0 = -\frac{\tan\pi \sec\varphi \sin\tau_0}{1 - \tan\pi \tan\varphi \cos\tau_0} \qquad (8)$$

Writing

$$g = \tan\pi \sec\varphi, \qquad h = \tan\pi \tan\varphi,$$

$$G = \frac{1}{1 - h\cos\tau_0} \qquad (9)$$

(8) assumes the form

$$\tan a_0 = -gG \sin\tau_0. \qquad (10)$$

Equations (6), (7), (9), and (10) are entirely rigorous and contain the solution of the problem. Equation (10) determines the azimuth of Polaris, which, corrected by the difference $a - a_0$ obtained from (7), gives the azimuth of the instrument. The substitution of a into (6) leads to an equation of condition in which the clock correction and the collimation are the only unknowns, for τ is defined by the relation

$$\tau = a - \theta - \Delta\theta,$$

where θ is the sidereal clock time of the transit of the time star and $\Delta\theta$ the error of the timepiece. The observation of a second time star, with the instrument reversed, leads to a second equation of condition in $\Delta\theta$ and c, which, combined with the first, permits of a determination of both unknowns. If the instrument is reversed upon each star in accordance with the proposal of Harzer, the term in c in equation (6) will disappear, and the substitution of a will leave an equation in which the error of the clock is the only unknown.

Equations (6) and (7) can be simplified for purposes of numerical calculation by the introduction of approximate formulæ, since the instrumental constants, and hence also τ, are always

small quantities. In practice the constants b and c need never exceed one second of time, and we therefore assume

$$|b| = |c| \lessapprox 1^s,$$

where, as usual, a quantity enclosed in vertical lines signifies the absolute magnitude of that quantity. For $\pi = 1^\circ 12'$ and $\varphi = 60^\circ$ we find from (10) as the maximum value of a_0, sensibly $144'$. We therefore assume in round numbers

$$|a| \lessapprox 150'.$$

For the purpose of comparison of terms it will be convenient to refer to the following series of numbers which gives the values of the successive powers of the azimuth.

a	a^2	a^3	a^4	a^5	a^6	
$150'$	$392''.6$	$17''.1$	$0''.75$	$0''.03$	$0''.001$	(11)
600^s	$26^s.2$	$1^s.14$	$0^s.05$	$0^s.002$	$0^s.0001$	

It appears that b and c are of the third order with respect to the azimuth, and that, generally speaking, a term of the fifth order with respect to this quantity will not sensibly affect the final result; but in order to test the accuracy of the approximate formulæ the fifth order terms will be retained and subjected to a special examination.

Before passing to a discussion of the approximations to be introduced let us consider for a moment the maximum values of the coefficients A, B, and C in equation (6). It is desirable that A should never exceed unity, in order that the uncertainty in a may not appear increased in τ. The limits for δ within which this is true are

$$\tfrac{1}{2}\varphi + 45^\circ > \delta > \tfrac{1}{2}\varphi - 45^\circ.$$

The corresponding limiting zenith distances are $45^\circ - \tfrac{1}{2}\varphi$ and $45^\circ + \tfrac{1}{2}\varphi$. The range for the selection of time stars is therefore ample, and in practice the values of A can always be kept within the limit here set. B is at a maximum for $\varphi = \delta$ and δ a maximum, which, for latitude 60°, gives $B = 2$. C increases with $|\delta|$, but, in general, stars whose declination exceeds 60° will not be observed on account of their slowness of motion. We therefore assume as the maximum, $C = 2$.

With these values of the coefficients in mind, a reference to (11) and to the limiting values for b and c shows that the errors introduced by writing in (6)

$$\cos b = \cos c = 1,$$

and

$$\tan b = b, \qquad \sin c = c,$$

are entirely inappreciable. Further, noting that

$$\frac{1}{\cos a} = 1 + \tfrac{1}{2}a^2 + \tfrac{5}{24}a^4 + \cdots, \qquad (12)$$

equation (6) takes the simplified form

$$\begin{aligned}\sin\tau = A\tan a + Bb + Cc - 2\sin^2\tfrac{1}{2}\tau\sin\varphi\tan a \\ + \tfrac{1}{2}a^2(Bb + Cc) - 2b\sin^2\tfrac{1}{2}\tau\cos\varphi,\end{aligned} \tag{13}$$

which is correct to terms of the fifth order inclusive. Now

$$2\sin^2\tfrac{1}{2}\tau = 2\left\{\frac{\tau}{2} - \frac{\tau^3}{48} + \cdots\right\}^2 = \frac{\tau^2}{2} - \frac{\tau^4}{24} + \cdots.$$

This quantity appears multiplied by the factor $\sin\varphi\tan a$ in the third order term in (13). The error introduced by writing in this term

$$2\sin^2\tfrac{1}{2}\tau = \frac{\tau^2}{2}$$

is of the order $\frac{1}{24}a^5$. Again, if we write in the left number of (13)

$$\sin\tau = \tau - \frac{\tau^3}{6}$$

the error will be of the order of $\frac{1}{120}a^5$. Neither of the errors thus committed can sensibly affect the final result. Introducing these approximations, we find

$$\tau = A\tan a + Bb + Cc + \frac{\tau^3}{6} - \frac{\tau^2}{2}\sin\varphi\tan a + \epsilon_1, \tag{14}$$

where ϵ_1, the sum of the fifth order terms, is given by the relation

$$\epsilon_1 = \tfrac{1}{2}a^2(Bb + Cc) - \tfrac{1}{2}a^2bA^2\cos\varphi. \tag{15}$$

We may write

$$\tau = A\tan a + s, \tag{16}$$

where s is the sum of the terms of the third and higher orders. We then derive, correct to the fifth order inclusive,

$$\begin{aligned}\frac{\tau^2}{2} &= \frac{A^2}{2}\tan^2 a + sA\tan a, \\ \frac{\tau^3}{6} &= \frac{A^3}{6}\tan^3 a + \frac{s}{2}A^2\tan^2 a.\end{aligned} \tag{17}$$

The substitution of (17) into (14) gives, to the same degree of accuracy,

$$\tau = A\tan a + Bb + Cc + D\tan^3 a + \epsilon_2, \tag{18}$$

where

$$D = \tfrac{1}{6}A^3 - \tfrac{1}{2}A^2\sin\varphi, \tag{19}$$

and where

$$\epsilon_2 = \epsilon_1 + s(\tfrac{1}{2}A^2 - A\sin\varphi)\tan^2 a. \tag{20}$$

It will be shown later that the fifth order terms can be neglected so that the equation for τ becomes

$$\tau = A \tan a + Bb + Cc + D \tan^3 a . \qquad (21)$$

The resemblance of this expression to

$$\tau = A a + Bb + Cc ,$$

which is the equation of Mayer in the usual theory of the transit, is sufficiently evident.

We now proceed to a determination of the instrumental azimuth:

From (7) it is seen that the difference $a - a_0$ is of the same order as b and c, that is, of the third order in a. We have, therefore, with sufficient approximation,

$$- c \operatorname{cosec} z_0 = a_0 - a + b \cot z_0 ,$$

whence

$$a = a_0 + B_0 b + C_0 c , \qquad (22)$$

in which

$$B_0 = \cot z_0 , \qquad C_0 = \operatorname{cosec} z_0 . \qquad (23)$$

It will be observed that the same value of the inclination appears in both equations (18) and (22); in other words, we assume that the inclination remains constant for the short interval separating the observations of Polaris and the time star, and in practice, we use the mean of the values of b corresponding to the two instants of observation. It is quite possible, however, to keep the level observations separate and to obtain in the final formulæ each of the observed values multiplied by its own proper coefficient. But if the interval separating the observations of Polaris and the time star be small, the variation in the inclination will be slight, even for instruments only moderately stable. Moreover, if sudden changes of considerable magnitude should occur in the level during the observation of any pair of stars, one will usually regard with suspicion the final result from that pair, on account of the inherent probability of a simultaneous change in the azimuth. If, however, it should happen that the zenith distance of the time star is small, an uncertainty in the azimuth would not seriously affect the error of the timepiece, and it would then be desirable to treat separately the two values of the inclination. But such cases will arise only rarely, and we may usually look upon the above outlined procedure as sufficient.

Neither equations (18) nor (22) are complete as they stand, for there remains yet to be added the effect of diurnal aberration. For (18) the required term is $-0^s.021 \cos \varphi \sec \delta$, while for (22) it is $-0^s.021 \cos \varphi \operatorname{cosec} z_0$.

In both equations (18) and (22), c is the distance of the middle thread from the collimation plane, since the hour angles τ and τ_0 refer to the instants of transit across this thread. For the time star, the observations give in all cases, θ_m, which is the mean of the separate transits. The mean time of transit across the middle thread is found from θ_m by the addition of a small correction, ν, which is always easily calculated. In Method I, Polaris is observed when on the middle

thread, so that the observed time, θ_0, leads at once to the desired hour angle, τ_0. There is no difficulty, therefore, in applying (18) and (22) to the data derived by this method of observing. But in Method II, Polaris is not on the middle thread, in general, at the instant θ_0, and the equations can not be directly applied to the data of observation. Two methods of procedure are open. Either we must apply to θ_0 a correction which will reduce it to the time of transit across the middle thread, as in the case of the time star, or else we must replace c in equation (22) by $c + f_0$, where f_0 is the distance of Polaris from the middle thread at the instant θ_0. The calculation of the reduction to the middle thread is much more complicated for Polaris than for the time star, and for observations made near elongation may become quite impossible. On the other hand, the quantity f_0 can always be easily obtained from the micrometer settings. It is therefore better to replace c by $c + f_0$ in (22) and use θ_0 unchanged for the caluclation of τ_0. What has been said of Method II applies equally well to Method III. It is only necessary to remark that in this case it is simpler to refer f_0 to the collimation plane, since, in Method III, transits over the middle thread are not observed and, in consequence, ν must represent the reduction from mean thread to collimation plane. Formulae for the calculation of f_0 and ν are derived in Sections 3 and 4. Equations (18) and (23) therefore become

$$a = a_0 + bB_0 + cC_0 + f_0C_0 - 0^s.021\, C_0 \cos\varphi\,, \qquad (24)$$

$$\tau = A \tan a + bB + cC - 0^s.021\, C \cos\varphi + D \tan^3 a + \varepsilon_2\,, \qquad (25)$$

where it is to be remembered that for Method I, $f_0 = 0$, while for Method III, we must write $c = 0$.

Equation (24) may be written in the form

$$a = a_0 + \sigma\,, \qquad (26)$$

where σ is the sum of all the terms of the right members of (24) excepting the first. From (26) we find

$$\tan a = \frac{\tan a_0 + \tan\sigma}{1 - \tan\sigma \tan a_0}$$

and introducing approximations,

$$\tan a = \frac{\tan a_0 + \sigma}{1 - \sigma \tan a_0} = (\tan a_0 + \sigma)(1 + \sigma \tan a_0) = \tan a_0 + \sigma \sec^2 a_0\,, \qquad (27)$$

in which the error is of the order of $\sigma^2 \tan a_0$. The largest term in σ is f_0C_0, and, admitting that this may have a value as great as $5'$, which is extreme and easily avoidable in practice, the error in (27) will be only one one-thousandth of a second of time, even when a_0 has its maximum value. Equation (27) is, therefore, practically rigorous for the present discussion. Noting that

$$\sec^2 a_0 = 1 + a_0^2 + \cdots,$$

we find from (27)

$$A\tan a = A\tan a_\circ + AB_\circ b + AC_\circ + AC_\circ f_\circ \sec^2 a_\circ - 0^s.021\, AC_\circ \cos\varphi + \varepsilon_3, \tag{28}$$

where the fifth order terms are given by

$$\varepsilon_3 = Aa_\circ^2 (B_\circ b + C_\circ c). \tag{29}$$

Again, from (27), with all requisite accuracy,

$$D\tan^3 a = D\tan^3 a_\circ. \tag{30}$$

The acceptance of equation (30) involves the neglect of the term $3\sigma D\tan^2 a_\circ$. Theoretically, this is of the fifth order, or even larger. Practically, however, the chance is very slight that the different factors should all have, at the same time, the extreme values necessary to make this term of importance, and in consequence, the term can be omitted. Introducing now the constants

$$B' = AB_\circ + B, \qquad C' = AC_\circ + C, \qquad F = AC_\circ \sec^2 a_\circ, \tag{31}$$

and writing

$$x = -0^s.021\, C'\cos\varphi + D\tan^3 a_\circ, \tag{32}$$

we find upon substituting (28) and (30) into (25),

$$\tau = A\tan a_\circ + B'b + C'c + x + f_\circ F + \varepsilon, \tag{33}$$

where

$$\varepsilon_4 = \varepsilon_2 + \varepsilon_3, \tag{34}$$

includes all of the fifth order terms which are likely to be of importance.

The investigation of the magnitude of these terms now requires our attention. From (15), (18), (20), (29), and (34), we obtain

$$\varepsilon_4 = \tfrac{1}{2} a^2 (Bb + Cc) - \tfrac{1}{2} a^2 bA^2 \cos\varphi + a^2 A (B_\circ b + C_\circ c) + (Bb + Cc + D\tan^3 a)(\tfrac{1}{2} A^2 - A\sin\varphi)\tan^2 a. \tag{35}$$

An inspection of (35) shows that, in general, the largest values of ε_4 will occur for extreme northern latitudes. It will be sufficient, therefore, to calculate ε_4 for various values of the declination, upon the assumption $\varphi = 60°$, in order to determine the maximum error which is likely to occur in practice as a result of the neglected terms. We may replace $\tan a$ by a in (35) and write $b = c = a^3$. Further, we may suppose

$$B_\circ = \cot z_\circ = \tan\varphi_\circ, \qquad C = \operatorname{cosec} z_\circ = \sec\varphi,$$

whereupon, (35) assumes the form

$$\varepsilon_4 = a^5 [0.50\, B \pm 0.50\, C - 0.25\, A^2 + (1.73 \pm 2.00)\, A + (B \pm C + 0.17\, A^3 - 0.43\, A^2)(0.50\, A^2 - 0.87\, A)], \quad (36)$$

where the double sign precedes those terms which contain the collimation as a factor and which, in consequence, change their algebraic signs with a reversal of the instrument. For $\varphi = 60°$, we find from (11), $a^5 = 0^s.002$, and writing

$$\mathrm{I} = 0.50\, B - 0.25\, A^2 + 1.73\, A + (B + 0.17\, A^3 - 0.43\, A^2)(0.50\, A^2 - 0.87\, A),$$
$$\mathrm{II} = 2.00\, A + 0.50\, C + C(0.50\, A^2 - 0.87\, A),$$

(36) takes the form

$$\varepsilon_4 = 0^s.002\,(\mathrm{I} \pm \mathrm{II}).$$

The variation of the expressions I and II with the declination is exhibited by the following table:

δ	I	II	δ	I	II
− 15°	+ 1.6	+ 2.1	+ 45°	+ 1.0	+ 1.1
− 5	+ 1.5	+ 1.9	+ 55	+ 0.9	+ 1.0
+ 5	+ 1.4	+ 1.8	+ 60	+ 1.0	+ 1.0
+ 15	+ 1.3	+ 1.6	+ 65	+ 1.3	+ 1.1
+ 25	+ 1.2	+ 1.5	+ 70	+ 2.0	+ 2.0
+ 35	+ 1.1	+ 1.2	+ 75	+ 4.2	+ 5.2

For observations made in accordance with Method III, in which the instrument is reversed upon each star, the terms II disappear, and the expression for the fifth order terms reduces to

$$\varepsilon_4 = 0^s.002\,\mathrm{I}.$$

For Methods I and II, there will be compensation of the terms II when the separate results from any given pair are combined to form the mean value of the clock correction. The neutralization will be the more complete the nearer the declinations of the two time stars approach equality. An examination of the table reveals the fact that, for the extreme conditions assumed for this discussion, the error appearing in the final value of the clock correction for any set, as a result of the neglected terms, cannot exceed three or four thousandths of a second of time, while for the great majority of cases occurring in practice this error will be of the order of one one-thousandth of a second or even less. We may conclude, therefore, that, in general, the second place of decimals in the value of τ derived by the formulæ of this section will be certain, insofar as the uncertainties in this quantity may be dependent upon the formulæ themselves.

The tabular values of I and II have been derived upon the assumption that both a and b are positive. Other combinations of algebraic sign would have modified somewhat the numerical values of the table, but would not have changed the general conclusion.

The reduction of the observations in accordance with the procedure explained in this paper presupposes a knowledge of the approximate value of the error of the timepiece. Ordinarily, this quantity will be known to such a degree of accuracy that the uncertainty in the calculated value, $\Delta\theta$, arising from the error in the assumed value, $\Delta\theta_0$, will be quite inappreciable. Nevertheless, cases will occasionally arise in which the requisite knowledge of $\Delta\theta_0$ is wanting. It will, therefore, be convenient to have a simple method of correcting $\Delta\theta$ for the error due to the uncertainty in $\Delta\theta_0$. Differentiating equation (33) with respect to τ_0 and neglecting small terms, we find

$$d\tau = A \tan a_0 (\cot \tau_0 + \sin \varphi \tan a_0)\, d\tau_0 ,$$

and further, since

$$\tau = a - \theta - \Delta\theta ,$$

we have

$$d\tau = -d\Delta\theta , \qquad d\tau_0 = -d\Delta\theta_0 ,$$

The second term in the expression for τ is of the second order and usually it can be neglected. In case this term should be of importance it will be simpler to recalculate equation (33), using for $\Delta\theta_0$ in the second approximation the value of $\Delta\theta$ obtained from the first, rather than to attempt to calculate both terms of the differential formula. Only four or five numbers in the calculation for $\Delta\theta$ need be changed and the corrections can be written in the margin alongside the first approximation. The equation for the differential correction of $\Delta\theta$ is therefore

$$d\Delta\theta = A \tan a_0 \cot \tau_0 \, (\Delta\theta - \Delta\theta_0) , \tag{37}$$

or

$$d\Delta\theta = -gAG \cos \tau_0 \, (\Delta\theta - \Delta\theta_0) .$$

The alternative form is useful for cases in which τ_0 approaches the critical values of 0^h and 12^h.

The fundamental equations for the determination of the clock correction are therefore

$$\begin{aligned} \tau_0 &= a_0 - \theta_0 - \Delta\theta_0 , \\ \tan a_0 &= -g G \sin \tau_0 , \\ \tau &= A \tan a_0 + B'b \pm C'c + \varkappa + Ff_0 , \\ \Delta\theta &= a - \theta - \tau , \\ d\Delta\theta &= A \tan a_0 \cot \tau_0 \, (\Delta\theta - \Delta\theta_0) , \end{aligned} \tag{38}$$

in which the constants are defined by equations (5), (9), (23), (31), and (32).

The formulae thus far developed have been arranged with special reference to the precise determination of time, such as is necessary for longitude operations or for the regular routine time service of observatories. Usually there is no necessity for a determination of azimuth, either approximate or precise, simultaneously with the precise determination of time. On the other hand, it is of the greatest advantage to be able to determine the value of the clock correction, approximately at least, in connection with precise determinations of azimuth, such as occur in triangulation operations. As a consequence, no expression for the determination of azimuth appears in equations (38), and as a further consequence, it is desirable to rearrange the formulae of this section so as to apply to observations in which the determination of azimuth is the matter of prime importance. The adaptation is readily made. Equations (24) and (25) lead at once to the desired result. In fact equation (24), with the exception of the term in f_o, is precisely that which forms the basis of the reduction in the standard methods of precise azimuth determination. Moreover, observations for time, made simultaneously with those for the determination of azimuth, will be used ordinarily only for a proper reduction of the azimuth observations; and since, for this purpose, a moderate degree of precision will suffice, micrometric settings on Polaris will not be necessary. Pointings on that object can, therefore, be made in accordance with some one of the various methods employed in regular azimuth observations. This will cause the term in f_o in (24) to disappear, and will bring both observations and reductions into exact agreement with the standard form.

The application of (24) to the observations of Polaris, made before and after reversal, gives the values of the instrumental azimuth; and a comparison of these with the settings on the mark leads in the usual way to a determination of the collimation and the azimuth of the mark. The value of a derived from (24), substituted into (25) and combined with the observations of the time stars, gives the corresponding value of τ, and, finally, the value of the clock correction. Equation (25) also gives a second value for the collimation constant, which must agree with that from (24) within the errors of observation.

The exact arrangement of the observing program will depend upon the degree of precision desired for the azimuth. In order to simplify the reductions, it is customary in determining azimuth to combine the mean of several settings on Polaris, made in quick succession, with the mean of the times and to treat this as a single observation. Naturally, this cannot be done when observations are made simultaneously for the determination of time. In this case, usually a single setting will be made on Polaris, both before and after reversal, which will lead very simply to a determination of both time and azimuth. If, however, several settings have been made, only the last of each series before and after reversal, will be combined with the observations of the time stars for the determination of time. These will also give a value for the azimuth which can be combined with that derived from the remainder of the settings for the formation of the final result. In primary operations, it will be advisable to separate entirely the observations for azimuth and time in order to preserve the symmetry and uniformity of the sets for the determination of azimuth; but, even in this case, the advantages of the vertical circle method are scarcely less, for

the observer has always at hand a direct and simple method for the determination of time which can be applied at once to the instrument in use without further preparation or adjustment.

Since, as already remarked, a moderate degree of precision in the clock correction suffices for the reduction of the azimuth observations, it will be possible to neglect the effect of diurnal aberration and the term $D \tan^3 a$ in equation (25). The necessary formulæ are, therefore,

$$\begin{aligned} a &= a_0 + bB_0 \pm cC_0 - 0^s.021\, C_0 \cos\varphi\,, \\ \tau &= A \tan a + bB \pm cC\,, \\ \Delta\theta &= a - \theta - \tau\,. \end{aligned} \qquad (39)$$

The values of the constants B_0 and C_0 can be taken from a small table with τ_0 as argument. If the number of observations made at any given station is too small to warrant the calculation of such a table, these constants can be determined with the aid of the expression

$$z_0 = (90^\circ - \varphi) + \varepsilon_0\,,$$

where ε_0 is to be obtained from Table IV[1] on the last page of the *American Ephemeris and Nautical Almanac*, with the argument τ_0. The value of A must be specially calculated, usually to four or five places of decimals, on account of the magnitude of the factor $\tan a$. B and C can be taken from the sources indicated in the footnote on page 54, but usually, it will be quite as simple to calculate them directly at the same time that A is determined. Their values will be needed to two places of decimals, only.

3. Reduction of Polaris to the Middle Thread and to the Collimation Plane

In Method I, Polaris is observed when bisected by the middle thread; the distance f_0 is zero, and, consequently, the reduction is zero.

In Method II, the distance of Polaris from the middle thread is obtained directly from the micrometer settings by means of the formula

$$f_0 = R\,(m - m_0)\,,$$

in which R is the value of one revolution of the micrometer, and m_0 and m, the means of the settings on the star and the middle thread, respectively. f_0 is to be counted positive when Polaris is east of the middle thread. The calculation of the term $f_0 F$ requires no explanation other than the remark that F can be tabulated as a star constant.

In Method III, we have given the micrometer readings m_1 and m_2 corresponding to the settings made on Polaris at the times θ_1 and θ_2. It is required to find the distance of Polaris from the collimation plane at the instant θ_0. We have

[1] See *infra*, p. 56.

$$2\theta_0 = \theta_2 + \theta_1. \tag{40}$$

Write

$$2\gamma = \theta_2 - \theta_1, \tag{41}$$

and we obtain

$$\theta_1 = \theta_0 - \gamma, \qquad \theta_2 = \theta_0 + \gamma. \tag{42}$$

Further, we have

$$a_0 - \theta_0 - \Delta\theta = \tau_0,$$

whence we derive

$$\begin{aligned} a_0 - \theta_1 - \Delta\theta &= \tau_0 + \gamma, \\ a_0 - \theta_2 - \Delta\theta &= \tau_0 - \gamma. \end{aligned} \tag{43}$$

Now let $a_1, a_0, a_2, z_1, z_0, z_2, f_1, f_0, f_2$, be the azimuth of Polaris, its zenith distance, and its distance from the collimation plane, respectively, at the times $\theta_1, \theta_0, \theta_2$. Further, let a represent the azimuth of the instrument, as before. Then by (7) we may write

$$\begin{aligned} E_1 &= -f_1 - b\cos z_1 = \sin z_1 \sin(a_1 - a), \\ E_0 &= -f_0 - b\cos z_0 = \sin z_0 \sin(a_0 - a), \\ E_2 &= -f_2 - b\cos z_2 = \sin z_2 \sin(a_2 - a), \end{aligned} \tag{44}$$

in which the symbols E_1, E_0, and E_2 are introduced in order to secure brevity in the transformations to follow. The angles f_1, f_0, and f_2 have been written in place of their sines; but since in practice they will not exceed 5′, it is evident from (11) that equations (44) are practically rigorous. A similar approximation might have been introduced for the differences of the azimuths, but, in view of the transformations to be made, they are more convenient as they stand. From (43) and from the second and third of (3) we find

$$\begin{aligned} \sin z_1 \sin a_1 &= -\sin\pi \sin(\tau_0 + \gamma), \\ \sin z_0 \sin a_0 &= -\sin\pi \sin\tau_0, \\ \sin z_2 \sin a_2 &= -\sin\pi \sin(\tau_0 - \gamma), \end{aligned} \tag{45}$$

$$\begin{aligned} \sin z_1 \cos a_1 &= \cos\pi\cos\varphi - \sin\pi\sin\varphi\cos(\tau_0 + \gamma), \\ \sin z_0 \cos a_0 &= \cos\pi\cos\varphi - \sin\pi\sin\varphi\cos\tau_0, \\ \sin z_2 \cos a_2 &= \cos\pi\cos\varphi - \sin\pi\sin\varphi\cos(\tau_0 - \gamma). \end{aligned} \tag{46}$$

Multiplying each of (45) by $\cos a$, and each of (46) by $\sin a$, and subtracting the latter series of products from the former, pair by pair, we find upon comparison with (44),

$$\begin{aligned} E_1 &= -\cos a \sin\pi \sin(\tau_0+\gamma) - \sin a \cos\pi \cos\varphi + \sin a \sin\pi \sin\varphi \cos(\tau_0+\gamma), \\ E_0 &= -\cos a \sin\pi \sin\tau_0 \quad\quad - \sin a \cos\pi \cos\varphi + \sin a \sin\pi \sin\varphi \cos\tau_0, \\ E_2 &= -\cos a \sin\pi \sin(\tau_0-\gamma) - \sin a \cos\pi \cos\varphi + \sin a \sin\pi \sin\varphi \cos(\tau_0-\gamma). \end{aligned} \tag{47}$$

One-half the sum of the first and last of (47) minus the second gives

$$\begin{aligned} \tfrac{1}{2}(E_1+E_2) - E_0 = {} & \sin\pi \cos a \sin\tau_0 (1-\cos\gamma) \\ & - \sin\pi \sin a \cos\tau_0 (1-\cos\gamma)\sin\varphi, \end{aligned}$$

$$\tfrac{1}{2}(E_1+E_2) - E_0 = 2\sin\pi \sin^2\tfrac{1}{2}\gamma (\cos a \sin\tau_0 - \sin a \cos\tau_0 \sin\varphi). \tag{48}$$

Applying the second fundamental formula of spherical trigonometry to the polar of the triangle whose vertices are the pole, Polaris, and the zenith, we find

$$-\sin q_0 \cos\pi = \cos a_0 \sin\tau_0 - \sin a_0 \cos\tau_0 \sin\varphi, \tag{49}$$

where q_0 is the parallactic angle of Polaris at the time θ_0. The right hand member of (49) differs from the expression in parenthesis in (48) by a quantity of the order of $a - a_0$, that is, of the order of f_0. Hence the error introduced by substituting (49) for this expression will be of the order of $2f_0 \sin\pi \sin^2\frac{1}{2}\gamma$. In the example given by Harzer, which is reproduced in Section 7 of this paper, the values of f_c for the three observations are $+0''.7$, $-0''.4$, and $+0''.5$, respectively. We shall be entirely safe, therefore, if we adopt for f_0 the limit chosen by Harzer, namely, $|f_0| \lessapprox 5''$. Suppose further, $2\gamma = \theta_2 - \theta_1 \lessapprox 10^m$
We then find

$$2f_0 \sin\pi \sin^2\tfrac{1}{2}\gamma \lessapprox 0^s.0001,$$

whence the practically rigorous equation

$$\tfrac{1}{2}(E_1+E_2) - E_0 = -2\sin\pi \cos\pi \sin q_0 \sin^2\tfrac{1}{2}\gamma. \tag{50}$$

Now, from (44),

$$\tfrac{1}{2}(E_1+E_2) - E_0 = -\tfrac{1}{2}(f_1+f_2) + f_0 - b[\tfrac{1}{2}(\cos z_1 + \cos z_2) - \cos z_0]. \tag{51}$$

By Taylor's theorem we find

$$\begin{aligned} \cos z_1 &= f(\tau_0+\gamma) = f(\tau_0) + \gamma f'(\tau_0) + \tfrac{1}{2}\gamma^2 f''(\tau_0) + \cdots, \\ \cos z_0 &= f(\tau_0), \\ \cos z_2 &= f(\tau_0-\gamma) = f(\tau_0) - \gamma f'(\tau_0) + \tfrac{1}{2}\gamma^2 f''(\tau_0) - \cdots. \end{aligned} \tag{52}$$

From (52) we form

$$\tfrac{1}{2}(\cos z_1 + \cos z_2) - \cos z_0 = \tfrac{1}{2}\gamma^2 f''(\tau_0) = \tfrac{1}{2}\gamma^2 \frac{d^2}{d\tau_0^2}\cos z_0.$$

The last term of (51) is, therefore,

$$\tfrac{1}{2} b\gamma^2 \frac{d^2}{d\tau_\circ^2}\cos z_\circ .$$

For the limits $|b| \leqq 1^s$, $\gamma \leqq 5^m$, the coefficient of the derivative has a maximum value of $\pm 0^s.0003$; moreover, as is readily seen, the derivative itself contains $\sin\pi$ as a factor. This term is, therefore, entirely inappreciable.

To connect the equations which have been developed with the observed data we proceed as follows: We have

$$f_1 = iR(m_1 - m), \qquad f_2 = iR(m_2 - m),$$

in which R is the value of one revolution of the micrometer, m_1 and m_2, the means of the micrometer settings on Polaris, and m, the reading when the micrometer thread is in the collimation plane. The factor i is equal to $+1$ or -1 according as increasing micrometer readings carry the movable thread toward the east or toward the west. Since in Method III, the instrument is reversed during the observation of each star, the factor i must have the value $+1$ in one of the above equations, and the value -1 in the other. Hence we have

$$\tfrac{1}{2}(f_1 + f_2) = \pm \tfrac{1}{2} R(m_1 - m_2),$$

in which the positive sign is to be used when, for the initial position of the instrument, increasing micrometer readings carry the micrometer thread toward the east. Writing now

$$F_\circ = \tfrac{1}{2}(f_1 + f_2), \tag{53}$$

we obtain from (50) and (51)

$$f_\circ = F_\circ - 2\sin\pi\cos\pi\sin q_\circ \sin^2\tfrac{1}{2}\gamma, \tag{54}$$

in which the error is of the order of one ten-thousandth of a second of time.

If we replace $2\sin^2\frac{1}{2}\gamma$ by $\frac{1}{2}\gamma^2$, note that

$$\sin\pi\sin q_\circ = \sin\pi_1\cos\delta_1,$$

where the right member is expressed in Harzer's notation, and allow for further differences in notation, (54) takes the form of the equation derived by Harzer[1] for the reduction of Polaris to the collimation plane. The expression given by Harzer is sufficiently accurate for practical purposes, but the more precise form of (54) is retained on account of the simplicity with which it combines with the terms of the fundamental equation for τ.

In the triangle whose vertices are at the pole, Polaris, and the zenith, we have the relation

$$\sin\pi\sin q_\circ = \sin a_\circ \cos\varphi,$$

[1] *Harzer*, p. 39.

which substituted into (54) gives

$$f_\circ = F_\circ - 2\cos\pi\cos\varphi\sin a_\circ \sin^2\tfrac{1}{2}\gamma . \qquad (55)$$

The correction which must be applied to τ in Method III in order to reduce Polaris to the collimation plane is, therefore, of the form

$$Ff_\circ = FF_\circ - 2F\cos\pi\cos\varphi\sin a_\circ\sin^2\tfrac{1}{2}\gamma ,$$

in which

$$F = A\operatorname{cosec} z_\circ \sec^2 a_\circ .$$

But from the second of (3) and from (9) we find

$$\cos\pi\cos\varphi\operatorname{cosec} z_\circ \sec a_\circ = G .$$

Hence,

$$F\cos\pi\cos\varphi = AG\sec a_\circ .$$

Substituting this relation into the above expression for $Ff_\circ$, we obtain

$$Ff_\circ = FF_\circ - 2GA\tan a_\circ \sin^2\tfrac{1}{2}\gamma . \qquad (56)$$

Again, since

$$G = 1 + h\cos\tau_\circ + h^2\cos^2\tau_\circ + \cdots ,$$

it is evident that the error introduced by writing $G = 1$ will always be very small. For extreme values of A and $\tan a_\circ$, and for $\gamma = 5^m$, the uncertainty is 0.003. But since the normal value of γ is about 3^m, and since the maximum values of A and $\tan a_\circ$ will occur simultaneously with exceptional values of γ only in very rare instances, we may write without hesitation

$$Ff_\circ = FF_\circ - 2A\tan a_\circ \sin^2\tfrac{1}{2}\gamma . \qquad (57)$$

Substituting (57) into the fundamental equation for τ, we find for Method III

$$\tau = A\tan a_\circ\cos\gamma + B'b + x + FF_\circ . \qquad (58)$$

It is to be noted that the preceding discussion assumes that f_1 and f_2 are the true distances of Polaris from the collimation plane. The measured distances are affected by refraction, and, in the most refined observations, care should be taken that this effect is eliminated. Harzer has shown[1] that the true distances from the collimation plane can be obtained by multiplying the apparent distances by the factor

$$1 + k = 1.00028.$$

[1] *Harzer*, p. 32.

The matter can be attended to once for all by including the factor $1 + k$ in the value of the micrometer revolution, which will give the value of the *apparent* micrometer revolution. This having been done, the true declinations are then to be used throughout all the formulæ. If, however, the value of one revolution of the micrometer has been determined from transits of stars without correction for refraction, the value thus derived will, in itself, be the apparent micrometer revolution, and will not require multiplication by the factor $1 + k$.

4. Reduction of the Time Star to the Middle Thread and to the Collimation Plane

Given the thread intervals, the position of the instrument, the coördinates of a star, and the mean of the times of transit across the separate threads: to find the corrections which, applied to this mean, will give the time of transit across the middle thread, and also the instant of passage through the collimation plane.

Let c be the distance of the middle thread from the collimation plane, and let i_s be the distance of any side thread from the middle thread. Let θ be the time of transit across the side thread, and let ν_s be the interval required for the star to pass from this thread to the middle thread. The hour angle of the star when on the middle thread will then be

$$\tau = \alpha - \theta_s - \nu_s - \Delta\theta\,,$$

while its hour angle at the instant of transit across the side thread will be

$$\tau + \nu_s = \alpha - \theta_s - \Delta\theta\,.$$

Equation (2) then gives

$$\begin{aligned} -c &= \sin n \sin\delta - \cos n \cos\delta \sin(\tau - m), \\ -(i_s + c) &= \sin n \sin\delta - \cos n \cos\delta \sin(\tau - m + \nu_s). \end{aligned} \tag{59}$$

Expanding the second of (59) and subtracting the result from the first, we obtain

$$i_s = \cos n \cos\delta \cos(\tau - m) \sin\nu_s - 2 \cos n \cos\delta \sin(\tau - m) \sin^2 \tfrac{1}{2}\nu_s\,.$$

Since ν_s is always less than 15′, we have with sufficient accuracy

$$\nu_s = i_s \sec n \sec\delta \sec(\tau - m) + \tfrac{1}{2}\nu_s^2 \tan(\tau - m). \tag{60}$$

Now, from the triangle *PAS*, we find[1]

$$\begin{aligned} \sin n &= -\sin c \sin\delta + \cos c \cos\delta \cos\omega\,, \\ \cos n \sin(\tau - m) &= \sin c \cos\delta + \cos c \sin\delta \cos\omega\,, \\ \cos n \cos(\tau - m) &= \cos c \sin\omega\,, \end{aligned} \tag{61}$$

whence

$$\tan(\tau - m) = \tan c \cos\delta \operatorname{cosec}\omega + \sin\delta \cot\omega\,.$$

[1] See *supra*, p. 34

The product of the first term of the right member of this expression into the factor $\frac{1}{2}\nu_s^2$ is entirely inappreciable. Substituting, therefore,

$$\cos n \cos(\tau - m) = \sin \omega,$$
$$\tan(\tau - m) = \cot \omega \sin \delta,$$

into (60), we find

$$\nu_s = i_s \sec \delta \operatorname{cosec} \omega + \tfrac{1}{2}\nu_s^2 \sin \delta \cot \omega,$$

or finally

$$\nu_s = i_s \sec \delta \operatorname{cosec} \omega + \tfrac{1}{2} i_s^2 \sin \delta \sec^2 \delta \cos \omega \operatorname{cosec}^3 \omega, \qquad (62)$$

in which the error will never exceed 0^s.001. If we introduce the constants I and I', defined by the relations,

$$I = \sec \delta \operatorname{cosec} \omega, \qquad I' = \tfrac{1}{2} \sin \delta \sec^2 \delta \cos \omega \operatorname{cosec}^3 \omega, \qquad (63)$$

(62) becomes

$$\nu_s = i_s I + i_s^2 I'. \qquad (64)$$

If now

$$\theta_1, \theta_2, \theta_3, \ldots\ldots \theta_q,$$

be the times of transit across the threads whose intervals are

$$i_1, i_2, i_3, \ldots\ldots i_q;$$

and if the corresponding reductions to the middle thread be

$$\nu_1, \nu_2, \nu_3, \ldots\ldots \nu_q,$$

we find from (64)

$$\theta = \theta_1 + \nu_1 = \theta_1 + i_1 I + i_1^2 I',$$
$$\theta = \theta_2 + \nu_2 = \theta_2 + i_2 I + i_2^2 I',$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots,$$
$$\theta = \theta_q + \nu_q = \theta_q + i_q I + i_q^2 I',$$

The time of transit across the middle thread is therefore given by

$$\theta = \theta_m + \nu = \theta_m + i_m I + \frac{1}{q} I' \Sigma i^2, \qquad (65)$$

where θ_m is the mean of the transits, i_m the mean of the thread intervals, and q the number of threads.

Again, if we let i_s be the distance of any side thread from the collimation plane, and ν_s the interval required for the star to pass from that thread to the collimation plane; and, further, if τ be the hour angle of the star when in the collimation plane, we find similarly to (59),

$$0 = \sin n \sin \delta - \cos n \cos \delta \sin(\tau - m),$$
$$-i_s = \sin n \sin \delta - \cos n \cos \delta \sin(\tau - m + \nu_s).$$

These equations lead again to (65). In other words, the form of reduction to the collimation plane is the same as that to the middle thread. The only difference is that, in the former case, the thread intervals must be measured from the collimation plane, while in the latter, they are to be referred to the middle thread.

For Methods I and II, the reduction is to be made to the middle thread; for IIIa, to the collimation plane. For Method IIIb, in which a Repsold registering micrometer is used, the intervals, i, require a special interpretation.[1]

For the majority of instruments, applied to Methods I and II, the second term of (65) can be neglected without sensible error. Moreover, we can generally write

$$I = \sec\delta = C, \tag{66}$$

unless some of the threads have been missed. Again, in Method IIIa, if no threads have been missed, $i_m = 0$, and we have simply

$$\theta = \theta_m + \nu = \theta_m + \frac{1}{q} I' \Sigma i^2 . \tag{67}$$

For purposes of calculation it will be convenient to express I and I' as functions of the azimuth. From the first of (61) and the first of (1)

$$\cos\omega = b\sin\varphi\sec\delta + c\tan\delta - \cos\varphi\sec\delta\sin a .$$

It can easily be shown that the small terms containing b and c arising in ν from the substitution of $\cos\omega$ into I and I' will be quite inappreciable in the final result. Therefore, we write

$$\cos\omega = -\cos\varphi\sec\delta\sin a .$$

Further, write

$$\operatorname{cosec}\omega = (1 - \cos^2\omega)^{-\frac{1}{2}} = (1 - y^2)^{-\frac{1}{2}},$$
$$= 1 + \tfrac{1}{2}y^2 + \tfrac{3}{8}y^4 + \cdots .$$

But since we have

$$\sec y = 1 + \tfrac{1}{2}y^2 + \tfrac{5}{24}y^4 + \cdots ,$$

we may write

$$\operatorname{cosec}\omega = \sec y , \tag{68}$$

where

$$y = \cos\omega . \tag{69}$$

The error thus introduced is of the fourth order with respect to the azimuth. It is easily seen that $\operatorname{cosec}^3\omega$ differs from unity by a quantity of the second order. Hence, if we write $\operatorname{cosec}^3\omega = 1$, the resulting error in ν will be of the order of i^2a^3 which is negligible. Introducing these results into (63), we have, finally,

[1] *Harzer*, p. 38 *sqq*.

$$I = \sec\delta \sec y\,, \quad I' = \tfrac{1}{2} y \tan\delta \sec\delta\,,$$
$$y = -\cos\varphi \sec\delta \sin a\,. \qquad (70)$$

The remarks at the end of Section 3 concerning the effect of refraction apply equally well here. In order to free the observations from the effect of refraction, the apparent thread intervals and the true declinations are to be used throughout all the formulæ, just as in the case of the meridian circle. The apparent thread intervals may be obtained from the true values by multiplying by the factor 1.00028. If the thread intervals have been obtained from star transits without correction for refraction, the values thus derived are already the apparent intervals, and require no further correction in using them for the reduction of observations.

5. Collection of Formulæ and Calculation of the Star Constants

Collecting all the formulæ which are necessary for the reduction of observations for time made by the vertical circle method we have from (5,) (19,) (23), (31), (32), (70), (65), (9), (38), and (58)

$$A = \sin(\varphi - \delta)\sec\delta\,, \quad B = \cos(\varphi - \delta)\sec\delta\,, \quad C = \sec\delta\,,$$
$$B_\circ = \cot z_\circ\,, \quad C_\circ = \operatorname{cosec} z_\circ\,, \quad D = \tfrac{1}{6}A^3 - \tfrac{1}{2}A^2 \sin\varphi\,,$$
$$B' = AB_\circ + B\,, \quad C' = AC_\circ + C\,, \quad F = AC_\circ \sec^2 a_\circ\,,$$
$$x = -0^s.021\, C' \cos\varphi + D\tan^3 a_\circ\,,$$
$$\tfrac{1}{2}y = -\tfrac{1}{2} C \cos\varphi \sin a_\circ\,, \quad I = C\sec y\,, \quad I' = \tfrac{1}{2} y C \tan\delta\,,$$
$$\nu = i_m I + \frac{1}{q} I' \Sigma i^2\,, \quad \theta = \theta_m + \nu\,,$$
$$g = \tan\pi \sec\varphi\,, \quad h = \tan\pi\tan\varphi = g\sin\varphi\,,$$
$$\tau_\circ = a_\circ - \theta_\circ - \Delta\theta_\circ\,,$$
$$G = \frac{1}{1 - h\cos\tau_\circ}$$
$$\tan a_\circ = -gG\sin\tau_\circ\,.$$

For Method II compute

$$f_\circ = R(m - m_\circ)\,.$$

$f_\circ$ is positive when Polaris is east of the middle thread.

$$\Delta\theta' = a - \theta - A\tan a_\circ - B'b - x - Ff_\circ\,,$$
$$c = \frac{\Delta\theta'_w - \Delta\theta'_e}{C'_w + C'_e}$$
$$\Delta\theta = \Delta\theta' \pm cC'\,.$$

For Method I the formulæ are the same as for Method II, excepting that $f_0 = 0$.
For Method III compute

$$F_0 = \tfrac{1}{2} R (m_1 - m_2), \qquad 2\gamma = \theta_2 - \theta_1.$$

F_0 is positive when for the initial position of the instrument increasing readings carry the micrometer thread toward the east.

$$\Delta\theta = \alpha - \theta - A \tan a_0 \cos\gamma - B'b - x - FF_0.$$

At first glance, it would seem that the calculation necessary for reduction by these formulæ is considerable, but a moment's consideration will show that the matter is much simpler than it appears. In the first place, the formulæ are written down with extreme detail in order that no step in the process may remain obscure. Further, it will be observed that a number of the quantities involved are independent of the time, excepting insofar as they may be affected by precession, nutation, annual aberration, and proper motion. These quantities are, therefore, practically constant for a long period of years, and their values can be tabulated for each time star for any given latitude. Finally, it will be seen that auxiliary tables can be utilized for the simplification of the computation so that the time required for the determination of the constants for any given time star is reduced to a minimum.

We may assume that the transit factors A, B, and C are known, since every observatory possesses tables which give their values, either directly for each star, or else for equidistant values of the declination. Even in the case of field operations at stations for which special tables have not been calculated, these constants may readily be obtained from the general tables of Davidson[1] or Albrecht. It may be remarked at this point that it is not advisable to tabulate the values of A which appear in the term $A \tan a_0$ in the expression for τ, since A must be known to five places of decimals on account of the magnitude of the factor $\tan a_0$. The changes in the declination arising from precession, nutation, annual aberration, and proper motion would soon render such a table obsolete, and any attempt to tabulate the variations in A due to these causes will, in general, occasion more labor than the direct calculation of the constant itself. Nevertheless, this will not prevent the computer from taking advantage of the fact that A is practically constant for several days at a time, a circumstance which allows of a simplification in the reduction of observations of the same stars made upon successive nights, as, for example, in longitude operations.

The values of the constant D may be taken from a very small table with the declination as argument.

[1]Davidson, Geo., *The Star Factors A, B, C, for Reducing Transit-Observations.* U. S. Coast Survey, Washington, 1874. The values of the factors are given in these tables to three places of decimals. Two-place tables are contained in the following: *Report of the Superintendent of the U. S. Coast Survey . . . . 1866,* p. 64; *1880,* p. 220; *1897-8,* p. 308; Washington, 1869, 1882, 1899.

The remaining constants all depend upon the azimuth and zenith distance of Polaris at the instant of transit of the time star across the vertical circle passing through that object; but the azimuth and zenith distance of Polaris are simple functions of its hour angle, its declination, and the latitude of the place. It is possible, therefore, to tabulate these quantities and their functions with the hour angle as argument. Moreover, there is a simple relation connecting the coördinates of the time star with the hour angle of Polaris at the instant of transit, so that, given the coördinates for any time star, the corresponding hour angle of Polaris at the instant of transit of that time star can be determined. With this as an argument, the various functions of the azimuth and zenith distance of Polaris required for the different constants can at once be taken from the tables. The tables themselves will assume a very compact form; for, in the first place, the range of variation in the azimuth and zenith distance of Polaris is slight; and secondly, three places of decimals in the tabular values will be sufficient, since the maximum value of the factors into which the constants are to be multiplied will not greatly exceed one second of time.[1]

The azimuth of Polaris is given by equations (9) and (10). From the second of (3), from (9) and (23) we find

$$C_0 = \operatorname{cosec} z_0 = g' G \cos a_0 ,$$

in which

$$g' = \sec \pi \sec \varphi .$$

These formulæ give the true zenith distance. But it is the apparent zenith distance which is required. To determine the effect of refraction, write for the apparent zenith distance

$$z = z_0 - r ,$$

in which the refraction is given with sufficient accuracy by

$$r = 1' \cot \varphi .$$

We then find, approximately,

$$\operatorname{cosec} z = \operatorname{cosec} (z_0 - r) = \operatorname{cosec} z_0 (1 + r \cot z_0) .$$

But in the term $r \cot z_0$ we may replace z_0 by $90^\circ - \varphi$, whence

$$\operatorname{cosec} z = 1.0003 \operatorname{cosec} z_0 .$$

Similarly, we derive

$$\cot z = \cot z_0 + 0.0003 \sec \varphi \operatorname{cosec} \varphi .$$

[1]For method III, the value of f_0 may be such as to require four places of decimals in the constant F. But for this case, as explained on page 63, it will be simpler to form the values of F, as needed, from the numbers which occur in the regular process of reduction.

It therefore appears that the effect of refraction is negligible. Collecting the formulæ for the azimuth and zenith distance of Polaris we have

$$g' = \sec\pi \sec\varphi\,, \qquad g = g' \sin\pi\,,$$

$$h = g \sin\varphi\,, \qquad G = \frac{1}{1 - h\cos\tau_\circ} \tag{71}$$

$$\tan a_\circ = \sin a_\circ = -g\,G \sin\tau_\circ\,,$$

$$C_\circ = \operatorname{cosec} z_\circ = g'G\cos a_\circ\,,$$

$$B_\circ = \cot z_\circ\,,$$

in which g, g', and h are constants whose values are to be computed but once. G can be found by means of addition and subtraction logarithms[1], or better still, its values can be taken directly from specially constructed tables[2] which are within the reach of almost every computer.

The values of $B_\circ$ and $C_\circ$ can also be calculated by an entirely different method which will serve as a control upon the results obtained by means of equations (71). Upon the last page of each volume of the *American Ephemeris and Nautical Almanac*, there is printed in Table IV the difference in zenith distance of Polaris and the pole point for every five minutes of hour angle of Polaris. If we designate this difference by $\varepsilon_\circ$, the zenith distance of Polaris for the hour angle $\tau_\circ$ will be given by

$$z_\circ = (90^\circ - \varphi) + \varepsilon_\circ\,, \tag{72}$$

where $\varepsilon_\circ$ is to be taken from the table with the argument $\tau_\circ$. The values of $\varepsilon_\circ$ given in Table IV correspond to latitude 45°, but the variation in this quantity arising from a variation in the latitude is so slight that the tabular values may be used without hesitation for all cases which are likely to occur in practice.

To connect the coördinates of the time star with the corresponding hour angle of Polaris, we proceed as follows: The hour angle of the time star at the instant of its observation is approximately aA, where a is the azimuth of the instrument, and A the usual transit factor. If α be the right ascension of the time star, the sidereal time of its observation, T, will be given by

$$T = \alpha - aA\,.$$

But the azimuth of the instrument is sensibly equal to the azimuth of Polaris at the instant the telescope is directed upon that object. Hence, we may replace a by $a_\circ$. The average amount

[1]Comstock, G. C., *A Text-Book of Field Astronomy for Engineers*, p. 152, New York, 1902.

[2]Albrecht, Th., *Formeln und Hülfstafeln für geograpische Ortsbestimmungen*, Leipzig, 1894.
Valentiner, *Handwörterbuch der Astronomie*, Bd. I, S. 440, Breslau, 1897.
Hayford, in *Report of Superintendent U. S. Coast and Geodetic Survey . . . . 1897—8*, p. 399, Washington, 1899.
These tables give the values of $\log \frac{1}{1-a}$ with the argument $\log a$.

by which the observation of Polaris precedes the observation of the time star will vary somewhat with different instruments, and with the method of observation. The Ephemerides of Döllen are based upon the assumption that this interval is equal to four minutes, and experience has shown that this is sufficiently near the truth. We therefore assume that Polaris is observed at the sidereal time

$$T - 4^{m} = \alpha - \alpha_{\circ} A - 4^{m},$$

whence we have for the corresponding hour angle of Polaris

$$\tau_{\circ} = \alpha_{\circ} - (T - 4^{m}) = \alpha_{\circ} - \alpha + \alpha_{\circ} A + 4^{m}.$$

We may write, therefore, for the purposes of calculation

$$\begin{aligned} \tau_1 &= \alpha_{\circ} + 4^{m} - \alpha, \\ \tau_{\circ} &= \tau_1 + \alpha_{\circ} A, \\ T &= \alpha - \alpha_{\circ} A. \end{aligned} \qquad (73)$$

In using these formulæ a first approximation for $\alpha_{\circ}$ is taken from the tables with the argument τ_1. The value of $\alpha_{\circ}$ thus obtained leads to a first approximation for $\tau_{\circ}$, which is the hour angle of Polaris corresponding to the time star whose right ascension is α and whose azimuth factor is A. When necessary, this value of $\tau_{\circ}$ will serve for the determination of a more accurate $\alpha_{\circ}$, which in turn, will lead to the final values of $\tau_{\circ}$ and T. T is required only for the observing list and is not necessary for the reductions.

It is desirable to collect in two separate tables all the data necessary for the observation and reduction of all time stars which are likely to be observed at any given place. The first of these will serve as an observing list, and will contain the number of the star, its name, magnitude, approximate sidereal time of transit, T, and the setting, calculated both for circle west and circle east. The second will be used only in the reductions, and will contain the constants ν, B', C', x, and F. If the observations are to be made in accordance with Method III, it will not be necessary to tabulate C', since the values of this constant will be needed only for the determination of x. Moreover, since the coefficient of C' in x is 0.02 or less, the values of C' need be known only to the first place of decimals and the computation can be greatly abbreviated. Further, the most convenient form for the tabulation of ν will vary somewhat with the instrument used and the method of observation. As already remarked, the second term of ν is negligible for many instruments in Methods I and II, while for the first term

$$i_m I = i_m C$$

will suffice, unless it should happen that some of the threads have been missed. In this case it may be necessary to take into account the factor $\sec y$ in the coefficient. With the careful observer, this will happen so rarely that usually it will be better to calculate directly this factor

when needed than to tabulate the more precise values of ν. In case both terms of ν are required for the reduction, it will be advisable to tabulate them separately in order to reduce more readily observations in which threads have been missed, and to take into account the variations in the coefficient I which may arise when the transit micrometer is used.[1]

An important control formula for the constant B' can be derived as follows: We have

$$B' = AB_0 + B = [\sin(\varphi - \delta)\cot z_0 + \cos(\varphi - \delta)]\sec\delta .$$

But, by (72)

$$\cot z_0 = \tan(\varphi - \varepsilon_0) .$$

Expanding and neglecting squares and higher powers of ε_0,

$$\cot z_0 = (\tan\varphi - \varepsilon_0)(1 - \varepsilon_0\tan\varphi) = \tan\varphi - \varepsilon_0\sec^2\varphi .$$

Substituting this result into the expression for B', we obtain

$$B' = \sec\varphi - \varepsilon_0 A\sec^2\varphi . \qquad (74)$$

For nearly all cases (74) will give B' correct to three places of decimals. This equation is readily used in connection with Table IV of the *American Ephemeris and Nautical Almanac*. The second term is always small and shows that the range of variation in B' is slight. In fact, Döllen neglects the second term entirely and uses the constant value

$$B' = \sec\varphi .$$

In many cases this will be sufficiently close to the true value to bring the final result within the limits of accuracy set for this paper.

The operations involved in the application of the above formulæ to the formation of the observing list and the tables of star constants are fully illustrated by the example in Section 7.

6. Effect upon the Star Constants of Precession, Nutation, Annual Aberration, and Proper Motion

It is of importance to investigate the effect of precession, nutation, annual aberration, and proper motion upon the star constants, and to determine the length of time during which these quantities may be used without modification. Let $\delta\theta$ represent that part of the clock correction which is dependent upon the star constants. We then find for Method II

$$-\delta\theta = \nu + B'b + C'c + x + Ff_0 , \qquad (75)$$

In order to determine the amount of the error introduced into the clock correction as a result of the precessional effect upon the star constants, we must form the derivative of $\delta\theta$ with respect

[1] *Harzer*, p. 40.

to the time and then substitute into the right member of this derivative the values of the differential coefficients expressed in terms of the precession in the coördinates of Polaris and the time star. An examination of the numerical value of the resulting expression will then determine the length of time during which the constants can be used without recalculation.

The required precession in the coördinates of the two stars is given by

$$\frac{da_\circ}{dt} = M + N \sin a_\circ \tan \delta_\circ, \qquad \frac{d\delta_\circ}{dt} = N \cos a_\circ, \qquad \frac{d\delta}{dt} = N \cos a. \qquad (76)$$

M and N are small quantities, at most, only a few minutes of arc, and in the evaluation of all the differential coefficients, excepting $\frac{dx}{dt}$, we may neglect those terms in which M and N appear multiplied into quantities of the order of the azimuth.

We then find with sufficient approximation

$$-\frac{d\delta\theta}{dt} = \frac{d\nu}{dt} + b\frac{dB'}{dt} + c\frac{dC'}{dt} + f_\circ \frac{dF}{dt} + \frac{dx}{dt}$$

$$\frac{d\nu}{dt} = i_m \frac{dI}{dt} + \frac{1}{q}\frac{dI'}{dt}\Sigma i^2$$

$$\frac{dB'}{dt} = A\frac{dB_\circ}{dt} + B_\circ \frac{dA}{dt} + \frac{dB}{dt}$$

$$\frac{dC'}{dt} = A\frac{dC_\circ}{dt} + C_\circ \frac{dA}{dt} + \frac{dC}{dt}$$

$$\frac{dF}{dt} = A\frac{dC_\circ}{dt}\sec^2 a_\circ + C_\circ \frac{dA}{dt}\sec^2 a_\circ + 2AC_\circ \sec^2 a_\circ \tan a_\circ \frac{da_\circ}{dt}$$

$$\frac{dx}{dt} = 3D \tan^2 a_\circ \sec^2 a_\circ \frac{da_\circ}{dt} + (\tfrac{1}{2}A^2 - A\sin\varphi)\tan^3 a_\circ \frac{dA}{dt}$$

$$\frac{dA}{dt} = -NC^2 \cos\varphi \cos a, \qquad \frac{dB}{dt} = NC^2 \sin\varphi \cos a, \qquad \frac{dC}{dt} = NC \tan\delta \cos a,$$

$$\frac{da_\circ}{dt} = -\cos^2 a_\circ \frac{d}{dt}(gG\sin\tau_\circ) = -N\cos(a_\circ - \tau_\circ)\sec\varphi,$$

$$\frac{dC_\circ}{dt} = \frac{d}{dt}(g'G\cos a_\circ) = -N\cos(a_\circ - \tau_\circ)\sec^2\varphi\tan\varphi,$$

$$\frac{dB_\circ}{dt} = \sec z_\circ \frac{dC_\circ}{dt} = \operatorname{cosec}\varphi \frac{dC_\circ}{dt} = -N\cos(a_\circ - \tau_\circ)\sec^2\varphi,$$

$$\frac{dI}{dt} = NC^2 \sin\delta\cos a,$$

$$\frac{dI'}{dt} = \tfrac{1}{2}NC^2 \sin(a_\circ - \tau_\circ)\tan\delta,$$

$$\frac{dB'}{dt} = -NA\cos(a_\circ - \tau_\circ)\sec^2\varphi,$$

$$\frac{dC'}{dt} = -NA\cos(a_\circ - \tau_\circ)\sec^2\varphi\sin\varphi - NC^2\cos a(1 - \sin\delta),$$

$$\frac{dF}{dt} = -NA\cos(a_\circ - \tau_\circ)\sec^2\varphi\sin\varphi - NC^2\cos a,$$

$$\frac{dx}{dt} = -3ND\sin(a_\circ - \tau_\circ)\sec\varphi\tan^2 a_\circ - NC^2\cos a\,(\tfrac{1}{2}A^2 - A\sin\varphi)\cos\varphi\tan^3 a_\circ.$$

It will be observed that in the expression for $\frac{dx}{dt}$ terms of the order of $N\tan^3 a_\circ$, arising from $\frac{da_\circ}{dt}$ have been neglected, whereas, terms of the same order coming from $\frac{dD}{dt}$ have been retained. The term in $\frac{da_\circ}{dt}$ which has been retained is much larger than the neglected terms. If the former does not appreciably affect the clock correction, the latter certainly cannot do so. For those cases in which the principal term becomes of importance, it will be desirable to modify the form of the star constants in such a way as to make the tabular values independent of the precessional effect upon $a_\circ$. In no case, therefore, will it be necessary to investigate the magnitude of the omitted terms. On the other hand, no such condition exists in the case of the terms from $\frac{dD}{dt}$, and they are therefore retained for investigation.

To determine the maximum values of the various derivatives, write

$$\sin(a_\circ - \tau_\circ) = \cos(a_\circ - \tau_\circ) = \sin\tau_\circ = \cos a = 1,$$
$$F_1 = (\tfrac{1}{6}A^3 - \tfrac{1}{2}A\sin\varphi)\sec^3\varphi, \qquad (77)$$
$$F_2 = C^2(\tfrac{1}{2}A^2 - A\sin\varphi)\sec^2\varphi.$$

We then derive

$$\frac{d\nu}{dt} = NC^2\sin\delta\,(i_m + \tfrac{1}{2}\sec\delta\,\frac{1}{q}\Sigma i^2),$$
$$b\frac{dB'}{dt} = -bNA\sec^2\varphi,$$
$$c\frac{dC'}{dt} = -cN[A\sec^2\varphi\sin\varphi + C^2(1-\sin\delta)], \qquad (78)$$
$$f_\circ\frac{dF}{dt} = -f_\circ N(A\sec^2\varphi\sin\varphi + C^2),$$
$$\frac{dx}{dt} = N(F_2\tan^3\pi - 3F_1\tan^2\pi).$$

The absolute magnitude of the error in the clock correction will always be less than the sum of the absolute maximum values of the five expressions in (78), for, in the first place, the trigonometric functions in the first of (77), which are contained in the various expressions of (78) as factors, cannot simultaneously have values as great as unity; second, the combinations of latitude and declination for which these expressions have their maximum values are different for the different expressions, and finally, alternations in algebraic sign will generally occur in such a way as to reduce the actual value below the absolute sums.

The maximum value of the first expression of (78) will occur for extreme northern stars. Assuming 60° as the limit for northern declinations we find

$$\frac{d\nu}{dt} = 3.5\, N\, (i_m + \frac{1}{q}\Sigma i^2)\,. \qquad (79)$$

The values of the factors in the second, third, and fourth expressions, for various combinations of latitude and declination, can be derived from the first of the following tables. The body of this table contains the values of $A \sec^2\varphi$. The last two columns contain those terms which vary with the declination but are independent of the latitude, while the last line gives the values of $\sin\varphi$ in a convenient position for the formation of the first terms of the third and fourth expressions. The table for F_1 and F_2 exhibits the variation of these coefficients with varying declination and latitude.

$A \sec^2\varphi$.

δ \ φ	15°	30°	45°	60°	C^2	$C^2(1-\sin\delta)$
+ 60°	— 1.51	— 1.33	— 1.04	0.00	4.00	+ 0.54
+ 40	— 0.59	— 0.31	+ 0.22	+ 1.80	1.70	0.61
+ 20	— 0.10	+ 0.25	+ 0.90	+ 2.72	1.13	0.74
0	+ 0.28	+ 0.67	+ 1.42	+ 3.48	1.00	1.00
— 20	+ 0.65	+ 1.09	+ 1.92	+ 4.20	1.13	1.51
— 40	+ 1.14	+ 1.64	+ 2.60	——	1.70	+ 2.79
$\sin\varphi$	0.26	0.50	0.71	0.87		

F_1 F_2

δ \ φ	15°	30°	45°	60°	15°	30°	45°	60°
+ 60°	— 0.8	— 0.6	— 0.3	0.0	+ 5.9	+ 5.3	+ 4.1	0.0
+ 40	— 0.1	0.0	0.0	— 0.6	+ 0.5	+ 0.3	— 0.3	— 1.9
+ 20	0.0	0.0	— 0.1	— 1.2	0.0	— 0.1	— 0.5	— 1.6
0	0.0	— 0.1	— 0.3	— 1.8	0.0	— 0.2	— 0.5	— 1.5
— 20	0.0	— 0.1	— 0.5	— 2.3	0.0	— 0.1	— 0.5	— 1.6
— 40	+ 0.1	— 0.1	— 0.7	—	+ 0.5	+ 0.3	— 0.2	—

An examination of the tables yields the following results: The second and third expressions of (78) have their largest values for southern stars observed at northern latitudes. But for latitude 60°, stars south of + 15° will not ordinarily be observed.[1] We therefore obtain

[1] *Harzer*, p. 53.

$$b\frac{dB'}{dt} = -2.9\,bN, \tag{80}$$

$$c\frac{dC'}{dt} = -3.3\,cN. \tag{81}$$

The value of the fourth expression is a maximum for northern stars observed at northern latitudes. For latitude 60°, the variation of this expression with the declination is slight and we assume

$$f_\circ\frac{dF}{dt} = -4.0\,f_\circ N. \tag{82}$$

The coefficient F_1 is greatest for southern stars observed at northern latitudes. For the limits of φ and δ given above, we find

$$|F_1| \lessapprox 1.5.$$

For F_2 we may assume in round numbers

$$|F_2| \lessapprox 2.0,$$

since stars which would give values of F_2 greater than this will not be observed for time. Assuming now $\pi = 1^\circ\,10'$, we find

$$\tan^2\pi = 5^s.70, \qquad \tan^3\pi = 0^s.1,$$

and, finally,

$$\frac{dx}{dt} = N(0^s.2 - 25^s.7). \tag{83}$$

From (79), (80), (81), (82), (83) we then obtain

$$\left|\frac{d\delta\theta}{dt}\right| < |N|\left\{3.5\left|i_m + \frac{1}{q}\Sigma i^2\right| + 2.9\,|b| + 3.3\,|c| + 4.0\,|f_\circ| + 0^s.2 + 25^s.7\right\} \tag{84}$$

For Method II, the quantity $f_\circ$ need never exceed $1^s.5$. In Method III, it may be much larger although a moderate amount of care in the arrangement of the observations will usually result in values of $f_\circ$ not greater than $4^s.0$. In the example given by Harzer, the maximum value is $3^s.8$. For a period of ten years, the value of the precessional coefficient N is very nearly 0.001. Introducing this value of N and the above value of $f_\circ$ and using the limits

$$|i_m| = \left|\frac{1}{q}\Sigma i^2\right| \lessapprox 0^s.2,$$

$$|b| = |c| \lessapprox 1^s.0,$$

we find from (84) for Method II,

$$\left|\frac{d\delta\theta}{dt}\right| < \pm 0^s.0007 + 0^s.0007 + 0^s.0029 \pm 0^s.0033 \pm 0^s.0060 + 0^s.0002 + 0^s.0257, \tag{85}$$

in which the double sign has been prefixed to those terms whose algebraic signs change with a reversal of the instrument. It is assumed that the term in f_0 is of this nature, since it is desirable to observe Polaris on opposite sides of the middle thread in the two positions of the instrument.

The terms arising from I and I', and that due to the precessional effect upon the factor D in the constant x, can be dismissed without further question. The same will generally be true of the term coming from B', since the limit assumed for b is extreme. This removes from the necessity of further consideration the first three and the sixth of the terms in equation (85). The fourth and fifth terms, which are due to C' and F respectively, are somewhat larger, but they cannot seriously affect the final value of the clock correction, since the errors produced by them in the individual results from the time stars of any given set will be of opposite sign and will tend to neutralize each other when these results are combined to form the mean. The resulting compensation of errors will be the more complete according as the declinations of the time stars approach equality. The last term of (85) comes from the factor $\tan^3 a_0$ in the constant x, and requires special attention. In the reduction of observations of the highest precision, it cannot be neglected; and it is necessary, therefore, either to recalculate, at more or less frequent intervals, that part of the constant x which gives rise to this term, or else to arrange the tabulation of the constant in such a way as to cause the large precessional effect to disappear. The value of this term given above is the maximum, reached only for the limits $\varphi = 60°$ and $\delta = +15°$, and even then, only for those stars which transit when Polaris is near elongation, since it contains the factor $\sin^2 \tau_0$, a quantity which, in the above discussion, has been assumed equal to unity. For northern stars observed at northern latitudes, and for all stars observed at intermediate and lower latitudes, this term will be negligible; the constant x can be tabulated as described in the preceding section, and the tabular values can be used for an interval of ten years on either side of the epoch to which they refer. For the exceptional cases mentioned above, in which the last term of (85) can not be neglected, it will be desirable to tabulate the first term of x and the factor D of the second term. The remaining factor can easily be calculated at the time the observations are reduced, since the value of $\tan a_0$ occurs in the regular process of reduction. The formation of this factor and the calculation of the constant x will occupy but an instant of time, nevertheless, the values of x, once obtained, should be preserved, since they can be used unchanged for one or more years according to the position of the star.

In Method I, the conditions are somewhat simpler, since the term in f_0 disappears from the fundamental equation. Further, the precision obtainable in the azimuth through a single setting upon Polaris scarcely warrants the retention of a term of the order of 0.03. In Method I, we may neglect, therefore, the last term of (85). All the constants can be tabulated as previously described and their values can be used without change for the full period of ten years on either side of the epoch for which they are calculated.

In Method III, the term in c disappears, while f_0 is replaced by F_0. For the latter factor there is no compensation as was the case with f_0, and, moreover, as was previously remarked, F_0 will generally be considerably larger than f_0. F must therefore be recalculated every three or four years, and even then, exceptional values of f_0 may require a special calculation of

this constant. On the whole, it will be simplest to calculate the values of F only as they are needed. This can readily be done during the regular process of reduction. The values thus derived will of course be preserved for future use. For the remaining terms of (85), the conclusions are the same as in Method II.

As for the effect of nutation, annual aberration, and proper motion, it will be safe to assume that at no time within the interval during which the star constants are to be used, can the combined effect of these three variations displace a star from its mean position by more than 40″. The component displacements in right ascension and declination will then be given by

$$\Delta\alpha \lessgtr 40'' \sin\sigma \sec\delta, \qquad \Delta\delta \lessgtr 40'' \cos\sigma,$$

where σ is the position angle of the resultant displacement. These variations in the coördinates are but a small fractional part of those due to precession during an interval of ten years, and we may conclude, therefore, that the combined effect of nutation, annual aberration, and proper motion upon the tabular values of the star constants is inappreciable.

7. Examples

The following examples are appended in order to illustrate the application of the process developed in the preceding sections to the reduction of observations made by the vertical circle method. An inspection of the results of this section will facilitate a comparison of the arrangement of formulæ here presented with that proposed by Harzer and will emphasize the advantages claimed for it in Section 1.

The calculation of the auxiliary tables and of the star constants given below is for the latitude of the Laws Observatory, for which $\varphi = 38^\circ\ 56'\ 51''.7$. The first step in the process is the application of equations (71) to the calculation of Table I. This table contains all of the functions of the azimuth and zenith distance of Polaris which are necessary for the calculation of the star constants. The second column gives the azimuth of Polaris to the nearest tenth of a minute of time. The fifth column contains the values of $\tan^3 a_\circ$ expressed in decimals of a second of time, ready for the formation of the term $D\tan^3 a_\circ$, while the sixth column contains the values of $-\frac{1}{2}\cos\varphi\sin a_\circ$ expressed in units of the fourth decimal place. The multiplication of this quantity by C gives $\frac{1}{2}y$. An additional column containing the values of $\sec^2 a_\circ$ to be used in the calculation of F might also have been added; but, for the case under discussion, this is not necessary, since, for the given latitude, the maximum value of $\sec^2 a_\circ$ differs from unity by only a few units of the fourth decimal place. It will be sufficiently accurate, therefore, to assume $\sec^2 a_\circ = 1$ and write $F = AC_\circ$. The values of $B_\circ$ and $C_\circ$ in columns three and four can be checked by (72). This, together with an inspection of the successive differences of the tabular values, affords a sufficient control upon this part of the computation.

Table II is calculated with the aid of a table of squares and cubes from the formula

$$D = \tfrac{1}{6}A^3 - \tfrac{1}{2}A^2\sin\varphi.$$

Table III gives the limiting values of $\frac{1}{2}y$ corresponding to a change of one unit of the fourth decimal place in $\sec y$. This table is of service in the calculation of I and is applicable to all latitudes.

TABLE I

τ_0	a_0	B_0	C_0	$\tan^3 a_0$	$-\frac{1}{2}\cos\varphi\sin a_0$	τ_0
$0^h\ 0^m$	$-\ 0^m.0\ +$	0.843	1.308	$-\ 0^s.000\ +$	$+\ 0\ -$	$24^h\ 0^m$
20	0.5	0.843	1.308	.000	9	40
40	1.1	0.842	1.307	.001	18	20
1 0	1.6	0.842	1.306	.005	27	23 0
20	2.1	0.841	1.306	.011	36	40
40	2.6	0.840	1.305	.020	44	20
2 0	3.1	0.838	1.304	.033	52	22 0
20	3.5	0.837	1.303	.049	60	40
40	3.9	0.835	1.301	.069	67	20
3 0	4.3	0.833	1.301	.092	74	21 0
20	4.7	0.831	1.300	.117	79	40
40	5.0	0.829	1.299	.142	85	20
4 0	5.3	0.826	1.297	.167	89	20 0
20	5.5	0.823	1.295	.191	94	40
40	5.7	0.821	1.294	.212	97	20
5 0	5.9	0.818	1.292	.230	100	19 0
20	6.0	0.815	1.290	.242	101	40
40	6.0	0.812	1.288	.250	102	20
6 0	6.0	0.809	1.286	.252	102	18 0
20	6.0	0.806	1.285	.248	102	40
40	5.9	0.803	1.283	.238	101	20
7 0	5.8	0.801	1.281	.224	98	17 0
20	5.6	0.798	1.279	.205	96	40
40	5.4	0.795	1.277	.183	92	20
8 0	5.2	0.792	1.276	.159	88	16 0
20	4.9	0.790	1.274	.134	83	40
40	4.6	0.787	1.273	.109	78	20
9 0	4.2	0.785	1.271	.086	72	15 0
20	3.7	0.783	1.270	.063	65	40
40	3.4	0 781	1.269	.046	58	20
10 0	3.0	0.779	1.268	.030	51	14 0
20	2.5	0.778	1.267	.018	43	40
40	2.0	0.777	1.266	.010	35	20
11 0	1.5	0.776	1.266	.004	26	13 0
20	1.0	0.775	1.265	.001	18	40
40	0.5	0.775	1.265	.000	9	20
12 0	$-\ 0.0\ +$	0.775	1.265	$-\ 0.000\ +$	$+\ 0\ -$	12 0

TABLE II

δ	D
+ 45°	− 0.008
+ 40	0
+ 35	2
+ 30	9
+ 25	19
+ 20	30
+ 15	43
+ 10	56
+ 5	70
0	83
− 5	96
− 10	− 0.110

TABLE III

$\frac{1}{2}y$	sec y
0.0050	1.0001
87	2
112	3
132	4
150	5
166	6
180	7
193	8
206	9
218	10
229	1.0011
0.0240	

Tables I, II, and III, together with the equations

$$\tau_1 = a_0 + 4^m - a,$$

$$\tau_0 = \tau_1 + a_0 A,$$

$$T = a - a_0 A,$$

$$B' = AB_0 + B = \sec\varphi - \varepsilon_0 A \sec^2\varphi, \qquad C' = AC_0 + C, \tag{86}$$

$$F = AC_0 \sec^2 a_0, \qquad x = -0^s.021\, C' \cos\varphi + D \tan^2 a_0,$$

$$\tfrac{1}{2}y = -\tfrac{1}{2} C \cos\varphi \sin a_0, \qquad I = C \sec y, \qquad I' = \tfrac{1}{2} y\, C \tan\delta,$$

$$\nu = i_m I + \frac{1}{q} I' \Sigma i^2.$$

are to be used for the evaluation of the constants for the individual time stars. The use of these formulæ and the tables is illustrated for the four stars, *A' Tauri*, *γ Tauri*, *ε Tauri*, and *α Tauri*. For completeness, the computation is arranged so as to give the values of both I and I', although the thread intervals of the transit instrument of the Laws Observatory are such that the reduction to middle thread is given with sufficient accuracy by

$$\nu = i_m C,$$

excepting for cases in which threads have been missed. The value of i_m is $\pm 0^s.053$ according as the circle is east or west. The use of the first three of the above equations in connection with Table I has been described on page 57. Otherwise, the arrangement of the computation requires no special explanation. The value of a_0 for 1910.0 is $1^h\ 26^m.9$.

CALCULATION OF STAR CONSTANTS, 1910.0

Time Star	A^1 Tauri	γ Tauri	ε Tauri	α Tauri
δ	+ 21° 50′	+ 15° 25′	+ 18° 59′	+ 16° 20′
α	$3^h\ 59^m.4$	$4^h\ 14^m.7$	$4^m\ 23^s.4$	$4^h\ 30^m.8$
$A\,a_o$	+ 1.1	+ 1 6	+ 1.5	+ 1.7
τ	− 2 28.5	− 2 43.8	− 2 52.5	− 2 59.9
$\tau_o = \tau_1 + Aa_o$	− 2 27.4	− 2 42.2	− 2 51.0	− 2 58.2
$T = \alpha - Aa_o$	3 58.3	4 13.1	4 21.9	4 29.1
B_o	0.836	0.835	0.834	0.833
A	0.317	0.415	0.361	0.401
C_o	1.302	1.301	1.301	1.301
AB_o	0.265	0.347	0.301	0.334
B	1.030	0.951	0.995	0.962
B'	1.295	1 298	1.296	1.296
AC_o	0.413	0.540	0.470	0.522
C	1.077	1.037	1.058	1.042
C'	1.490	1.577	1.528	1.564
$\tan^s a_o$	+ 0^s.056	+ 0^s.071	+ 0^s.081	+ 0^s.090
D	− 0.026	− 0.042	− 0.033	− 0.039
$-0^s.017\ C'$	− 0.025	− 0.027	− 0.026	− 0.027
x	− 0.026	− 0.030	− 0.029	− 0.031
$\frac{1}{2}y$	− 0.0067	− 0.0070	− 0.0075	− 0.0076
$\tan\delta$	0.401	0.276	0.344	0.293
I	1.078	1.038	1.059	1.043
I'	− 0.0029	− 0.0020	− 0.0027	− 0.0023

The numerical values of the constants as thus determined can be used without modification for a period of ten years on either side of the epoch to which they refer.

The results of the calculation are collected in the following tables in a convenient form for use. The exact arrangement of the tables will vary somewhat with special conditions arising for different instruments and for different observatories. For example, the addition of the right ascensions to the Observing List, and of the ordinary transit factors A, B, and C to the list of Star Constants, will make it possible to use the tables either for the vertical circle method or for the meridian method of time determination. In fact this arrangement is to be recommended, for, if adopted, the calculation of the star constants can be very greatly abbreviated. All the given data will appear in the tables, and the use of *Crelle's Rechentafeln* will make it possible to form the various constants without writing down more than three or four numbers for each star other than those which will appear regularly in the Observing List and the table of Star Constants.

The constant B' appears multiplied by the value of a half division of the level, 0^s.060, so that in order to obtain bB', expressed in time, it is only necessary to multiply the tabular value of B' by b, expressed in half divisions of the level.

Observing List, 1910.0

No.	Star	Mag.	T	δ	Setting, C.W.	Setting, C.E.
1	A' Tauri	4 6	$3^h\ 58^m.3$	+ 21° 50′	253° 10′	287° 24′
2	γ Tauri	3.8	4 13.1	+ 15 25	246 45	293 49
3	ϵ Tauri	3.6	4 21.9	+ 18 59	250 19	290 15
4	α Tauri	1.0	4 29.1	+ 16 20	247 40	292 54

Star Constants, 1910.0

No.	ν	B'	C'	F	x
1	± 0.057	0.0777	1.490	0.413	— 0.026
2	55	779	1.577	0.540	30
3	56	778	1.528	0.470	29
4	± 0.055	0.0778	1.564	0.522	— 0.031

The four stars whose constants are given above, were observed with the 2⅛-inch Brunner transit of the Laws Observatory on 1904, Dec. 31. Polaris was brought near the center of the field and its distance from the middle thread measured by means of the micrometer. Three settings of the micrometer were made for each observation. Transits of the time stars were observed across five threads and all the times were recorded on the chronograph. The disposition of the threads and the lack of a reversing apparatus make it impossible to observe by Method III with this instrument. Moreover, the striding level must be removed for each reversal and for the setting upon each star, so that the construction of the instrument is by no means the most favorable for obtaining results of precision. Since the observations are by Method II, the required formulæ are

$$g^s = \tan \pi \sec \varphi \frac{1}{\text{arc } 1^s}, \qquad G = \frac{1}{1 - h \cos \tau_\circ}$$

$$\tau_\circ = a_\circ - \theta_\circ - \Delta\theta_\circ, \qquad h = g \sin \varphi,$$

$$\tan a_\circ = -g^s G \sin \tau_\circ, \qquad \theta = \theta_m + \nu,$$

$$A = \sin(\varphi - \delta) \sec \delta, \qquad f_\circ = R(m - m_\circ),$$

$$\Delta\theta' = a - \theta - A \tan a_\circ - bB' - Ff_\circ - x,$$

$$c = \frac{\Delta\theta'_w - \Delta\theta'_e}{C'_w + C'_e}$$

$$\Delta\theta = \Delta\theta' \pm cC'.$$

The constants g and h are to be calculated but once for any given series of observations, and at certain times their values may be used unchanged for several days in succession.

From the *American Ephemeris* and from the time sheets of the observatory we find

$$\alpha_0 = 1^h\ 25^m\ 20^s.8, \qquad \pi = 1^\circ\ 11'\ 45''.1,$$
$$\Delta\theta_0 = -\ 3^m\ 10^s.0, \qquad \alpha_0 - \Delta\theta_0 = 1^h\ 28^m\ 30^s.8,$$
$$\log h = 8.22718 - 10, \qquad \log g^s = 2.56713.$$

The value of R is $6^s.00$. The correction for clock rate is so small as to be negligible. The reading of the micrometer for coincidence of movable thread and middle thread was 0.426.

For this series of observations, c is so small and the values of C' are so nearly equal that it is sufficient to derive the values of $\Delta\theta$ by forming the means of $\Delta\theta'$ for each set. The data of observation and the reduction are as follows:

LAWS OBSERVATORY, 1904, DECEMBER 31.

Star, Position	A^1 Tauri, C.E.	γ Tauri, C.W.	ε Tauri, C.W.	α Tauri, C.E.
b	+ 1.15	+ 2.05	+ 2.00	+ 0.78
m_0	0.164	0.659	0.679	0.178
f_0	+ 1^s.57	+ 1^s.40	+ 1^s.52	+ 1^s.49
θ_m	$4^h\ 1^m\ 3^s.30$	$4^h\ 15^m\ 52^s.42$	$4^h\ 24^m\ 42^s.06$	$4^h\ 31^m\ 52^s.80$
δ	+ 21° 49′.3	+ 15° 23′.8	+ 18° 58′.1	+ 16° 19′.0
$\varphi - \delta$	+ 17 7.6	+ 23 33.1	+ 19 58.8	+ 22 37.9
θ_0	$3^h\ 56^m\ 42^s.7$	$4^h\ 11^m\ 9^s.3$	$4^h\ 21^m\ 3^s.3$	$4^h\ 27^m\ 39^s.3$
τ_0	− 2 28 11.9	− 2 42 38.5	− 2 52 32.5	− 2 59 8.5
$\cos\tau_0$	9.9021	9.8800	9.8632	9.8511
$h\cos\tau_0$	8.1293	8.1072	8.0904	8.0783
$\log G$	0.00589	0.00559	0.00538	0.00523
$\sin\tau_0$	9.77997_n	9.81396_n	9.83488_n	9.84785_n
$\sin(\varphi - \delta)$	9.46907	9.60160	9.53363	9.58524
$\sec\delta$	0.03229	0.01587	0.02424	0.01785
$\log A\tan a_0$	1.85435	2.00415	1.96526	2.02330
α	$3^h\ 59^m\ 5^s.54$	$4^h\ 14^m\ 24^s.03$	$4^h\ 23^m\ 5^s.03$	$4^h\ 30^m\ 29^s.02$
θ	4 1 3.36	4 15 52.37	4 24 42.00	4 31 52.85
$\alpha - \theta$	− 1 57.82	− 1 28.34	− 1 36.97	− 1 23.83
$-A\tan a_0$	− 1 11.51	− 1 40.96	− 1 32.31	− 1 45.51
$-bB'$	− 0.09	− 0.16	− 0.16	− 0.06
$-\varkappa - Ff_0$	− 0.62	− 0.73	− 0.68	− 0.75
$\Delta\theta'$	− 3 10.04	− 3 10.19	− 3 10.12	− 3 10.15
cC'	− 0.08	+ 0.07	− 0 01	+ 0.01
$\Delta\theta$	− 3 10.12	− 3 10.12	− 3 10.13	− 3 10.14

In order to illustrate the application of the formulæ to Method III, and, at the same time, to afford a more complete comparison with the process of Harzer, the example given in his memoir[1] has been recalculated. The computation is given in detail below. The observations for this example were made by Ristenpart on 1899, June 7, with the transit instrument of the

[1] *Harzer*, pp. 57 and 74.

Kiel Observatory. Four micrometer settings were made upon Polaris for each observation, and forty transits of each time star were observed with a Repsold registering micrometer, the instrument being reversed upon each time star at the middle of the series in accordance with Method IIIb. The following summary of the observed data is formed from the results given by Harzer.

Time Star	ε Herculis	π Herculis	β Draconis
θ_1	$16^h\ 56^m\ 18^s.23$	$17^h\ 16^m\ 18^s.92$	$17^h\ 25^m\ 32^s.52$
θ_2	17 1 14.89	17 11 8.38	17 30 41.63
2γ	4 56.66	5 10.54	5 9.11
θ_0	16 58 46.56	17 13 43.65	17 28 7.08
m_1	$9^s.8870$	$9^s.8235$	$9^s.7635$
m_2	8.9310	10.2825	9.1455
log F_0	0.4490	0.1304n	0.2596
b	$-\ 3^d.3500$	$-\ 2^d.8750$	$-\ 2^d.6125$
θ_m	$16^h\ 59^m\ 14^s.519$	$17^h\ 13^m\ 55^s.659$	$17^h\ 28^m\ 17^s.926$

We have further

$$\varphi = 54^\circ\ 20'\ 28''.5\,, \qquad \alpha_0 = 1^h\ 21^m\ 54^s.81\,, \qquad \pi = 1^\circ\ 13'\ 55''.14\,.$$

Log F_0 is calculated by the formula

$$F_0 = \tfrac{1}{2} R\,(m_1 - m_2) \tag{87}$$

where the logarithm of R expressed in seconds of time is 0.7696. The logarithm of the value of one division of the level in seconds of time is 8.8742-10. The constants for the time stars are contained in the following table.

Time Star	ν	log B'	x	log F
ε Herculis	+ $0^s.042$	9.1042	+ $0^s.001$	9.8909
π Herculis	+ 0.045	9.1054	— 0.004	9.8012
β Draconis	+ 0.060	9.1081	— 0.022	8.9784

The values of ν have been taken directly from the calculation of Harzer where they are designated by the symbol $\mathfrak{C}_1$. The remaining constants have been calculated by equations (5), (19), (23), (31), and (32), with the aid of the apparent positions of the stars for the date of observation. The value of B' is expressed in seconds of time so that in order to form the correction for level, it is only necessary to add log B' to log b and take from the tables the corresponding number.

The formulæ remaining to be calculated are:

$$h = \tan\pi \tan\varphi, \qquad \log \text{arc } 1^s = 5.861666\text{-}10,$$

$$g^s = \tan\pi \sec\varphi \frac{1}{\text{arc } 1^s}, \qquad G = \frac{1}{1 - h\cos\tau_\circ}$$

$$\tau_\circ = a_\circ - \theta_\circ - \Delta\theta_\circ, \qquad A = \sin(\varphi - \delta)\sec\delta, \tag{88}$$

$$\tan a_\circ = -g^s G \sin\tau_\circ, \qquad \theta = \theta_m + \nu,$$

$$\Delta\theta = \alpha - \theta - A\tan a_\circ \cos\gamma - B'b - x - FF_\circ.$$

Table IV gives the limiting values of 2γ corresponding to a change of one unit of the fifth decimal place of $\log\cos\gamma$. The correction which must be applied to $\log A\tan a_\circ$ in order to form $\log A\tan a_\circ\cos\gamma$ is to be taken from this table with the argument 2γ.

TABLE IV

$2\gamma = \theta_2 - \theta_1$	$\log\cos\gamma$
$2^m\ 11^s.97$	0
3 48.57	— 1
4 55.09	— 2
5 49.15	— 3
6 35.90	— 4
7 17.68	— 5
7 55.81	— 6
8 31.10	— 7
9 4.10	— 8
9 35.22	— 9
10 4.73	— 10
10 32.87	— 11
10 59.81	— 12
11 25.69	— 13
11 50.63	— 14

In order to illustrate the procedure when the clock correction is not even approximately known, we shall suppose that for one of the stars $\Delta\theta_\circ = 0$. The value of the clock correction, $\Delta\theta_1$, resulting from this assumption will then be corrected by the differential formula

$$d\Delta\theta = A\tan a_\circ \cot\tau_\circ\, d\Delta\theta_\circ,$$

which will give the final value of the clock correction for that star. This result will also serve as the value of $\Delta\theta_\circ$ for the reduction of the remaining stars. Naturally, one will choose for the preliminary calculation that star which culminates nearest the zenith, in order that the uncertainty in $\Delta\theta_\circ$ may have a minimum effect upon the calculated value of the clock correction. In the present case, the preliminary calculation is made for the third star with the result

$$\Delta\theta_1 = +\ 19^s.192.$$

Owing to the smallness of A for this star, the correction to be applied is only 0ˢ.019, which gives for the final value + 19ˢ.211. A direct six-place calculation with $\Delta\theta_0 = +19^s.2$ gives precisely the same result for the final value of the clock correction as that derived from the differential formula.

The constants required for the determination of the azimuth of Polaris are

$$\log g'' = 2.70525, \qquad \log h = 8.47674 - 10.$$

KIEL OBSERVATORY, 1899, JUNE 7.

Star	ε Herculis	π Herculis	β Draconis
δ	+ 31° 4′.42	+ 36° 55′.30	+ 52° 22′.49
$\varphi - \delta$	+ 23 16.05	+ 17 25.17	+ 1 57.98
τ_0	8ʰ 22ᵐ 49ˢ.0	8ʰ 7ᵐ 52ˢ.0	7ʰ 53ᵐ 47ˢ.7
$\cos\tau_0$	9.7661_n	9.7238_n	9.6779_n
$h\cos\tau_0$	8.2428_n	8.2005_n	8.1546_n
$\log G$	9.99247	9.99316	9.99384
$\sin\tau_0$	9.90958	9.92858	9.94411
$\sin(\varphi - \delta)$	9.59662	9.47620	8.53545
$\sec\delta$	0.06727	0.09720	0.21432
$\log A\tan a_0, \cos\gamma$	$2.27119_n, -3$	$2.20039_n, -3$	$1.39297_n, -3$
a	16ʰ 56ᵐ 28ˢ.829	17ʰ 11ᵐ 35ˢ.018	17ʰ 28ᵐ 12ˢ.280
θ	16 59 14.561	17 13 55.704	17 28 17.986
$a - \theta$	− 2 45.732	− 2 20.686	− 5.706
$-A\tan a_0\cos\gamma$	+ 3 6.707	+ 2 38.621	+ 24.714
$-bB' - x$	+ 0.425	+ 0.370	+ 0.357
$-FF_0$	− 2.188	+ 0.854	− 0.173
			$\Delta\theta_1 = +19.192$
$\Delta\theta$	+ 19.212	+ 19.159	+ 19.211
$\log\Delta\theta_1$			1.2830
$\cot\tau_0$			9.7338_n
$\log d\Delta\theta$			8.2715

Harzer gives two different arrangements for the calculation, the second being somewhat shorter than the first. The two sets of results obtained by him for the above example are

+ 19ˢ.209, + 19ˢ.163, + 19ˢ.211,
+ 19.205, + 19.159, + 19.211.

The results by the formulæ of this paper, taken from the above calculation, are

+ 19ˢ.212, + 19ˢ.159, + 19ˢ.211.

The agreement is therefore satisfactory. With regard to the result for the first star, it may be remarked that a six-place calculation of equation (24) and the rigorous formulæ (1) and (2) gives + 19ˢ.212.

As to the relative amount of labor involved in the method proposed by Harzer and that presented in this paper, the following may be noted. The reductions in both methods depend upon star constants which can be tabulated for each time star and, with the exceptions mentioned for the constants $\varkappa$ and F of this paper, can be used unchanged for a period of twenty years. The time required for the calculation of the star constants is probably not very different for the two methods, although it is difficult to make an accurate estimate without actually performing the computation. The fact that the method of this paper proposes the tabulation of four constants, instead of three, as in the revised scheme of Harzer, is more than offset in the preliminary calculations by the fact that his method of reduction requires the special computation of the $\mathfrak{Q}$ table[1] for each value of the latitude. But slight advantages of one method over the other at this point are not of much consequence since the star constants themselves can be used without modification for so long a period. It is in the reduction of the actual observations, an operation which must be repeated many times each year, that minor points become of importance. The computation given by Harzer for his two sets of formulæ fills thirty-five and thirty lines, respectively. The calculation given above requires nineteen lines on an average for each time star, including the calculation of the constants g and h and the differential correction necessitated by the uncertainty in $\Delta\theta_0$. The addition of the calculation for the constant F would make a total of twenty-one lines. Excluding the preliminary calculations, which are essentially the same in both cases, it would appear that the formulæ here developed permit of a saving of about one-third in the labor of calculation of the clock correction as compared with the revised process of Harzer. At the same time, the precision of the result obtained by the formulæ of this paper is not less than that derived by the method of Harzer, at least for the limits of latitude adhered to in this discussion. Although Harzer assumes a somewhat higher limit for the latitude, namely $\varphi = 66°$, it is not likely that time determinations of extreme precision will be required for points as far north as this, since the northernmost observatory contained in the list of the *American Ephemeris and Nautical Almanac* is that at Åbo, in latitude 60° 26′. Moreover, the formulæ here presented can be used for latitudes somewhat beyond the limit of 60° without any serious effect upon the precision of the results. As for the determination of azimuth, that has been brought into exact agreement with the method most often used for the reduction of such observations, and as a consequence, leaves nothing to be desired from the standpoint of simplicity and rigor. As indicated in the introduction, it is believed that the method here presented will be found advantageous, not only on account of the brevity of the calculations, but also on account of the closeness with which the theoretical developments parallel those of the standard methods for the determination of time and azimuth.

8. Adaptation of the Formulæ to the Southern Hemisphere

The presence of the star σ *Octantis* near the southern pole makes it possible to apply the vertical circle method to the determination of time and azimuth at stations in the southern hemisphere. Although this object is somewhat difficult of observation with small instruments, its

[1] *Harzer*, p. 66.

position renders it even more satisfactory than Polaris for precise observations, since its small polar distance of only 45′ increases to a considerable extent the convergence of the formulæ used in the reductions.

For an instrument in the southern hemisphere, we suppose a to be positive when the western end of the axis of rotation lies *north* of the east and west line, while b and c are to be counted as before. Let A', P', S', $S'_\circ$, be points on the celestial sphere bearing the same relation to the southern hemisphere that the points A, P, S, $S_\circ$, bear to the northern. These points are the vertices of two sets of spherical triangles which are perfectly symmetrical with respect to the celestial equator and which bear the same relation to the direction of the diurnal motion of the stars. If, then, we suppose for a moment that southern declinations and latitudes be counted *positive*, and northern declinations, *negative*, the formulæ of the preceding sections can be applied at once, without modification, to the reduction of observations made in the southern hemisphere. But, in practice, it will be more convenient to count declinations and latitudes in accordance with the usual convention, that is, positive north of the equator and negative to the south, and to modify the formulæ accordingly. The modification is readily made and requires only that the algebraic sign of φ and δ be reversed throughout the entire series of formulæ for the northern hemisphere. With this change, the equations of the preceding sections apply directly to the southern hemisphere. It must be remembered, however, that a is to be measured in the opposite direction from that used for the northern hemisphere.

PART II

ADAPTATION OF THE VERTICAL CIRCLE METHOD TO THE ENGINEER'S TRANSIT

9. Development of Formulæ

The vertical circle method of determining time and azimuth lends itself readily to use with the engineer's transit. For ordinary field operations, an uncertainty in the clock correction of a few tenths, or even a whole second, is of no importance, and, in consequence, many of the terms of higher order appearing in the formulæ developed in Part I can be neglected, with the result that the equations to be calculated assume a very simple form. The simplicity and directness of the calculations, together with the facility with which the observations can be made, render the method one of great value for work in the field.

The method of observing is as follows: The instrument is carefully levelled, and three or four minutes before the transit of a southern star across the vertical circle passing through Polaris, the telescope is turned to the north, and Polaris itself is brought to the intersection of the vertical and horizontal threads. The instrument is clamped in azimuth and the sidereal time of setting, θ_o, is noted. The telescope is then rotated about the horizontal axis until its position is such that the southern or time star will pass through the field of view. The transit of the time star is observed, and the entire process is then repeated for a second time star, with the instrument in the reversed position. The data thus obtained permit of a determination of the error of the timepiece. If the readings of the horizontal circle be taken at the instants of setting on Polaris, the orientation of the circle can be found; and, further, if there be added the circle readings corresponding to settings on a mark, the azimuth of the mark can be determined. The plate levels should be carefully watched, and if there is any evidence of creeping, the instrument should be relevelled immediately before each setting.

The formulæ necessary for the reduction can be written down at once from the corresponding equations of Part I, nevertheless, they are given an independent development in the following paragraphs in order to exhibit more clearly the steps that are essential in the present application.

The right ascension of the time star is numerically equal to the time of its transit across the meridian. The star is observed, not at the instant of meridian passage, but at the instant of transit across the vertical thread of an instrument whose azimuth is a and whose collimation constant is c. But since both a and c are small, the instant of observation will not differ greatly from the time of meridian passage, and it will be possible to determine corrections which, applied to the star's right ascension, will give the true sidereal time of observation. When compared with the clock time of observation, this will determine the error of the clock.

The constant a is to be considered positive when the position of the instrument is such that southern stars are observed before meridian passage; c is positive when the vertical thread lies east of the collimation plane. The zenith distance of the time star at the instant of its observation is sensibly $\varphi - \delta$, and since the angle at the zenith included between the meridian and the vertical circle coinciding with the collimation plane is equal to a, and, further, since the star lies east of this circle by the quantity c, the distance of the star from the meridian is, very approximately,

$$a \sin (\varphi - \delta) + c.$$

The time required for the star to pass from the vertical thread to the meridian will be obtained by multiplying this quantity by sec δ after having expressed a and c in time. The sidereal time of observation is, therefore,

$$\alpha - a \sin (\varphi - \delta) \sec \delta - c \sec \delta.$$

If θ represent the clock time of observation, and $\Delta\theta$ the error of the timepiece, we find

$$\Delta\theta = \alpha - a \sin (\varphi - \delta) \sec \delta - c \sec \delta - \theta. \qquad (89)$$

Writing now

$$A = \sin (\varphi - \delta) \sec \delta, \qquad C = \sec \delta, \qquad (90)$$

the equation for the determination of the clock correction assumes the form

$$\Delta\theta = \alpha - \theta - aA - cC. \qquad (91)$$

The only unknowns in the right member of this equationare the two instrumental constants a and c. It remains to be seen how these can be determined. The former can easily be calculated since the azimuth of the instrument is intimately connected with the azimuth of Polaris at the instant of setting upon that object. The latter can be obtained by a combination of the results from two time stars, one observed circle west, the other circle east.

In the spherical triangle whose vertices are at the zenith, the pole, and Polaris, we have the relations

$$\begin{aligned} \sin z_\circ \sin a_\circ &= - \sin \pi \sin \tau_\circ, \\ \sin z_\circ \cos a_\circ &= \cos \pi \cos \varphi - \sin \pi \sin \varphi \cos \tau_\circ, \end{aligned} \qquad (92)$$

in which $z_\circ$, $a_\circ$, π, and $\tau_\circ$ are the zenith distance, azimuth, north polar distance, and hour angle, respectively, of Polaris. It has been assumed that $a_\circ$ is measured from the north point, positive toward the west, and $\tau_\circ$ from the meridian, positive toward the east. The hour angle, $\tau_\circ$, is given by the relation

$$\tau_\circ = \alpha_\circ - \theta_\circ - \Delta\theta_\circ, \qquad (93)$$

in which a_0 is the right ascension of Polaris; θ_0, the sidereal clock time for which the azimuth a_0 is required; and $\Delta\theta_0$, the error of the clock. Since the azimuth of Polaris varies but slowly with the time, an approximation for the clock correction will answer for the determination of τ_0, a fact which is indicated by the addition of the subscript zero to the symbol $\Delta\theta$ in equation (93). We shall assume for the present that such an approximation is always at hand. We then derive from equations (92)

$$\tan a_0 = -gG\sin\tau_0, \tag{94}$$

in which

$$g = \tan\pi\sec\varphi, \quad h = \tan\pi\tan\varphi,$$
$$G = \frac{1}{1 - h\cos\tau_0}. \tag{95}$$

These equations are entirely rigorous and determine the azimuth of Polaris at the instant its hour angle is τ_0. For the present application they may be abbreviated by writing

$$\tan a_0 = a_0, \qquad \tan\pi = \pi. \tag{96}$$

The resulting uncertainty in the clock correction will amount to only $0^s.3$ or $0^s.4$ even for extreme values of the azimuth, and generally it will be much less. Further, the calculation can be considerably shortened by the tabulation of the values of G for a mean value of the latitude. Expanding G in a series of ascending powers of h, and introducing the values of g and h, we find for the azimuth,

$$a_0 = -\pi\sec\varphi\sin\tau_0\,(1 + \pi\tan\varphi\cos\tau_0 + \cdots).$$

Since $\tan\varphi$ appears with a coefficient of the order of π^3 it is clear that a moderate change in this quantity cannot sensibly affect the value of the azimuth. It is also evident that the precessional change in π as it occurs in G, will produce an appreciable effect in a_0 only after a considerable interval of time. In consequence, the tabular values of G can be used unchanged for a period of six or eight years and throughout a range of two or three degrees in latitude on either side of the mean value for which the table has been constructed. The resulting error will vary with the latitude. For the clock correction, it will not amount to more than $0^s.2$ or $0^s.3$ at a maximum, while for the azimuth it will be quite inappreciable as compared with the uncertainty of the circle readings of the ordinary engineer's transit. If the error of the timepiece is needed only for a proper determination of the azimuth, the range of application of the table for G can be considerably extended without sensible error in the final result. It is to be understood, however, that the values of π and φ occurring in g must refer to the epoch of observation and to the place in which the observations are actually made. The equation for the azimuth of Polaris is, therefore,

$$a_\circ = -\pi G \sec\varphi \sin\tau_\circ . \qquad (97)$$

If many determinations of time and azimuth are to be made at any given place, it will be convenient to combine the factor $\sec\varphi$ with the tabular values of G. The table will then give the values of $G\sec\varphi$ for the argument $\tau_\circ$. This multiplied by $-\pi\sin\tau_\circ$ will give at once the azimuth of Polaris.

Table V contains the values of G calculated for $\varphi = 40°$; and also the values of $G\sec\varphi$, in which the factor $\sec\varphi$ corresponds to the latitude of the Laws Observatory, that is, 38° 57′.

It is assumed throughout that the correction for level is zero. If the collimation were also zero the azimuth of the instrument would be equal to the azimuth of Polaris. But, in general, this will not be the case, and we must have recourse to the relation,

$$a = a_\circ + c\,\mathrm{cosec}\, z_\circ . \qquad (98)$$

Consider the spherical right triangle whose sides are the arc formed by the intersection of the celestial sphere with the collimation plane, the vertical circle through Polaris when that object is on the vertical thread, and the arc which measures the distance of Polaris from the collimation plane. The first and second sides are sensibly equal to $z_\circ$. The third side is equal to c, and the angle opposite this side is $a - a_\circ$. We have, therefore,

$$\sin(a - a_\circ)\sin z_\circ = \sin c . \qquad (99)$$

But since $a - a_\circ$ and c are very small quantities we may write

$$(a - a_\circ)\sin z_\circ = c ,$$

which is equivalent to equation (98). Since $z_\circ$ never differs from the colatitude of the place by more than the north polar distance of Polaris, it will be sufficient to replace $z_\circ$ by $90° - \varphi$, whence we have,

$$a = a_\circ + c\sec\varphi . \qquad (100)$$

This substituted into (91) gives

$$\Delta\theta = a - \theta - a_\circ A - cC' , \qquad (101)$$

where

$$C' = A\sec\varphi + C. \qquad (102)$$

Assuming that (101) refers to circle west, we have for circle east

$$\Delta\theta = a - \theta - a_\circ A + cC' . \qquad (103)$$

For convenience we may write

$$\Delta\theta' = \alpha - \theta - a_0 A, \qquad (104)$$

and adding the subscripts w and e to distinguish the results for the two stars, one observed circle west, the other circle east, we find for the collimation

$$c = \frac{\Delta\theta'_w - \Delta\theta'_e}{C'_w + C'_e}, \qquad (105)$$

and finally for $\Delta\theta$,

$$\Delta\theta = \Delta\theta' \pm cC', \qquad (106)$$

in which the upper sign refers to circle east, the lower, to circle west. Equations (90), (93), (95), (97), (102), (104), (105), and (106) completely determine the error of the clock.

It has been assumed that an approximation sufficiently close to determine a_0 without sensible error is known for the error of the clock. It may happen, however, that the uncertainty in $\Delta\theta_0$ is so great as to seriously affect the final result. In this case, the value of $\Delta\theta'$ resulting from the original assumption will serve as an improved value for $\Delta\theta_0$ with which the equations for $\Delta\theta$ can be recalculated. The convergence of the formulæ is so rapid that a second approximation will always be sufficient, and since many of the quantities in the computation remain unchanged it will require but a moment's time. This will usually be the simplest method of procedure, although the final result can also be obtained from

$$\Delta\theta' = \Delta\theta'_1 + d\Delta\theta, \qquad (107)$$

where $\Delta\theta_1'$ represents the value of $\Delta\theta'$ derived from the first approximation. Differentiating equation (104) with respect to τ_0 and neglecting quantities of the order of π^3, we find

$$d\Delta\theta = -\pi A \sec\varphi \cos\tau_0 (\Delta\theta'_1 - \Delta\theta_0) = a_0 A \cot\tau_0 (\Delta\theta'_1 - \Delta\theta_0), \qquad (108)$$

in which $\Delta\theta'_1 - \Delta\theta_0$ has been written for $-d\tau_0$. An inspection of the difference $\Delta\theta'_1 - \Delta\theta_0$ will show whether the uncertainty in $\Delta\theta_0$ is such as to require the calculation of the differential correction. In case the clock correction is entirely unknown, we may derive a first approximation from the equation

$$\Delta\theta_0 = \alpha - \theta - a_0 A \qquad (109)$$

by simply estimating the magnitude of the last term. If $\Delta\theta$ is known to be small, we may use $\Delta\theta_0 = 0$. In any case, the first approximation will be carried through for a single star, and one will naturally select for this purpose the star which culminates nearest the zenith in order that the uncertainty in a_0 may have a minimum effect upon the calculation. The final value of $\Delta\theta'$ from this star will then serve as a value for $\Delta\theta_0$ in the reduction of the remaining observations.

The constant C' defined by equation (102) may be written in the form

$$C' = \tan\varphi + E, \tag{110}$$

where

$$E = \sec\delta - \tan\delta. \tag{111}$$

Since two places of decimals are sufficient for C' it is possible to tabulate the values of E in a very compact form. Table VI is such a table and gives the values of E for the argument δ. The values of C' are to be obtained for any given latitude by the simple addition of $\tan\varphi$ to the tabular values of E. Table VI also contains the values of C' corresponding to the latitude of the Laws Observatory.

To determine the orientation of the circle and to calculate the azimuth of the mark let

S = the reading of the horizontal circle when the vertical thread is on Polaris.
M = the reading of the horizontal circle when the vertical thread is on the mark.
N = the reading of the horizontal circle when the collimation plane coincides with the meridian.
K = the azimuth of the mark measured from the south, positive toward the west.

Then, for an instrument with a stationary circle whose readings increase in a clockwise direction, we have,

$$N = S + a, \tag{112}$$

and introducing for a its value from (100), we find

$$\begin{aligned} N &= (S + a_\circ)_w + c\sec\varphi, \\ N &= (S + a_\circ)_e - c\sec\varphi, \end{aligned} \tag{113}$$

where the subscripts w and e refer to observations made with the circle west and east, respectively. By addition and subtraction, we find

$$N = \tfrac{1}{2}[(S + a_\circ)_e + (S + a_\circ)_w], \tag{114}$$

$$c = \tfrac{1}{2}[(S + a_\circ)_e - (S + a_\circ)_w]\cos\varphi. \tag{115}$$

Again, let us assume for a moment that M' is the horizontal circle reading when the collimation plane passes through the mark. We then have similarly to (100)

$$M' = M \pm c\sec h,$$

where h is the altitude of the mark. But since

$$K = M' - N - 180^\circ,$$

we find by substituting the values M' and N

$$K = M_w - (S + a_\circ)_w + c\,(\sec h - \sec \varphi) - 180°,$$

$$K = M_e - (S + a_\circ)_e - c\,(\sec h - \sec \varphi) - 180°,$$

and, finally,

$$K = \tfrac{1}{2}\,[M_e - (S + a_\circ)_e + M_w - (S + a_\circ)_w] - 180°, \qquad (116)$$

$$c = \tfrac{1}{2}\,[M_e - (S + a_\circ)_e - M_w + (S + a_\circ)_w]\frac{1}{\sec h - \sec \varphi} \qquad (117)$$

Equation (114) gives the orientation of the circle; (116) the azimuth of the mark. The values of c from (115) and (117) must agree with that from (105) within the errors of the observation, which will serve as a control on the calculation. In general, however, it will not be necessary to calculate equations (115) and (117), as a simple inspection of the results will usually show whether or not the agreement is satisfactory. Collecting the results thus obtained we have the following notation and formulæ:

$a_\circ$, π, and α, δ are the coördinates of the Polaris and the time star, respectively.
$\theta_\circ$ and θ are the sidereal clock times, respectively, of their observation.
S and M are the readings of the horizontal circle corresponding to the settings on Polaris and the mark, respectively.
K is the azimuth of the mark measured from the south, positive toward the west.
$\Delta\theta$ is the error of the timepiece, and $\Delta\theta_\circ$ an approximation for this quantity.

We have then to calculate

$$\tau_\circ = a_\circ - \theta_\circ - \Delta\theta_\circ, \qquad a_\circ = -\pi G \sec\varphi \sin\tau_\circ,$$

$$A = \sin(\varphi - \delta)\sec\delta, \qquad C' = \tan\varphi + E,$$

$$\Delta\theta' = \alpha - \theta - a_\circ A, \qquad (118)$$

$$c = \frac{\Delta\theta'_w - \Delta\theta'_e}{C'_w + C'_e},$$

$$\Delta\theta = \Delta\theta'_w - cC'_w = \Delta\theta'_e + cC'_e.$$

G or $G\sec\varphi$ is to be taken from Table V with the argument $\tau_\circ$; E or C', from Table VI with the argument δ. The subscripts w and e refer to observations made circle west and circle east, respectively. Finally, calculate

$$K = \tfrac{1}{2}\,[M_e - (S + a_\circ)_e + M_w - (S + a_\circ)_w] - 180°, \qquad (119)$$

where the subscripts attached to M refer to settings made with the instrument in such a position that if it were turned toward the north by rotation about the vertical axis, the circle would then be west or east, respectively, according to the subscript.

For the determination of the error of the clock, $a_\circ$ should be expressed in seconds of time; for the determination of the azimuth, in minutes of arc. The values of A are needed to

four places of decimals, and, when once obtained, should be preserved for future use, since for a given latitude, they may be used unchanged for a several months at a time. If the collimation is known to be small, and the declinations of the two time stars do not differ too greatly it will be sufficient to take the mean of the values of $\Delta\theta'$ for circle west and circle east and call this the error of the timepiece.

TABLE V, 1910.0

τ_0	log G	log G sec φ	τ_0
0^h	0.0075	0.1167	24^h
1	0.0073	0.1165	23
2	0.0065	0.1156	22
3	0.0053	0.1145	21
4	0.0037	0.1129	20
5	0.0019	0.1111	19
6	0.0000	0.1092	18
7	9.9981	0.1072	17
8	9.9963	0.1055	16
9	9.9948	0.1039	15
10	9.9936	0.1028	14
11	9.9928	0.1020	13
12	9.9926	0.1018	12

TABLE VI

δ	E	C'
+ 30°	0.58	1.39
+ 25	0.64	1.45
+ 20	0.70	1.51
+ 15	0.77	1.58
+ 10	0.84	1.65
+ 5	0.92	1.72
0	1.00	1.81
− 5	1.09	1.90
− 10	1.19	2.00
− 15	1.30	2.11
− 20	1.43	2.24
− 25	1.57	2.38
− 30	1.73	2.54

10. EXAMPLES

The application of the formulae of the preceding section is illustrated by the following example:

On 1904, Nov. 8, the transits of eight time stars were observed across the single vertical thread of a Buff & Buff engineer's transit. The readings of the horizontal circle were taken for the settings on Polaris. The aperture of the telescope is 1⅛ inches and its focal length 12 inches. The magnifying power is 26.5 and the least reading of the horizontal circle 0.′5. The following table contains the data of observation for four of the stars and shows the arrangement of the calculation for the determination of the clock correction and the orientation of the circle. From the *Ephemeris* we find

$$a_0 = 1^h\ 25^m\ 57^s, \qquad \pi = 1^\circ\ 11'\ 58''.7,$$

whence

$$\log \pi^s = 2.4593, \qquad \log \pi' = 1.8573.$$

Although the clock correction is assumed to be entirely unknown, the first five lines, and also the eleventh, twelfth, thirteenth, fourteenth, and twentieth lines of the calculation can be filled in at once for all of the stars. By estimating the term $a_0 A$ to be -1^m, we derive from the second star as a first approximation $\Delta\theta_0 = +8^m\ 3^s$, which gives for this star

$$\Delta\theta_1' = +\ 8^m\ 43^s.8\,.$$

Using this as a new value for $\Delta\theta_\circ$ we form

$$\alpha_\circ - \Delta\theta_\circ = 1^h\ 17^m\ 13^s\,,$$

and find that $\log \sin \tau_\circ$, and consequently $\log a_\circ{}^s$, must be corrected by the subtraction of twenty-three units of the fourth place of decimals. The final value of $\Delta\theta'$ is therefore $+\ 8^m\ 43^s.3$. The above value of $\alpha_\circ - \Delta\theta_\circ$ is then used for the reduction of the remaining observations. The correction for clock rate is negligible.

Star, Circle	θ Piscium, C. E.	i Piscium, C.W.	i^1 Aquarii, C.W.	δ Sculptoris, C.E.
δ	+ 5° 51′	+ 5° 7′	− 18° 48′	− 28° 39′
$\varphi - \delta$	33 6	33 50	57 45	67 36
$\sin(\varphi - \delta)$	9.7373	9.7457	9.9272	9.9659
$\cos \delta$	9.9977	9.9983	9.9762	9.9433
$\theta_\circ$	$23^h\ 12^m\ 9^s$	$23^h\ 23^m\ 14^s$	$23^h\ 29^m\ 46^s$	$23^h\ 35^m\ 43^s$
$\tau_\circ$	2 5 4	{ 1 54 40 / 1 53 59	1 47 27	1 41 30
$G \sec \varphi$	0.1156	0.1158	0.1159	0.1160
$\sin \tau_\circ$	9.7152	9.6810, −23	9.6550	9.6320
$\log a_\circ'$	1.6881_n	1.6518_n	1.6283_n	9.6052_n
$\log a_\circ{}^s$	2.2901_n	2.2561_n	2.2302_n	2.2073_n
$\log A$	9.7396	9.7474	9.9510	0.0226
α	23 23 8.9	23 35 3.8	23 39 16.3	23 43 58.5
θ	23 16 12.6	23 28 0.8	23 33 4.2	23 38 3.7
$\alpha - \theta$	+ 6 56.3	+ 7 3.0	+ 6 12.1	+ 5 54.8
$A\,a_\circ$	− 1 47.1	− 1 40.8	− 2 31.8	− 2 49.8
$\Delta\theta'$	+ 8 43.4	+ 8 43.3	+ 8 43.9	+ 8 44.6
C'	1.7	1.7	2.2	2.5
cC'				
$\Delta\theta$	+ 8 43.4	+ 8 43.3	+ 8 44.3	+ 8 44.3
S	179° 7′.8	179° 3′.0	179° 0′.3	178° 58′.5
$a_\circ$	− 48.8	− 44.9	− 42.5	− 40.3
$S + a_\circ$	178 18.5	178 18.1	178 17.8	178 18.2
N	178° 18′.3		178° 18′.0	

The values of the collimation from the above stars are —0^s.03 and —0^s.15, while, for the four stars whose reduction is not printed, they are —0^s.05 and —0^s.02, respectively. The agreement of the values for this constant is therefore well within the errors of observation. The four values of N from the eight settings on Polaris are

$$N = 178^\circ\ 18'.3$$
$$18.0$$
$$18.4$$
$$18.8$$

The values of $\Delta\theta$ for the eight time stars observed on this date are given in column I of the following table.

In order to illustrate more clearly the degree of precision attainable by the vertical circle method when applied to small instruments, the values of the clock correction are given in the remaining columns of the table for each star of three other series of observations.

I	II	III	IV
$+ 8^m\ 43^s.4$	$- 3^m\ 8^s.7$	$- 3^m\ 9^s.5$	$- 3^m\ 11^s.9$
43.3	8.8	9.2	11.3
44.3	8.8	9.0	11.6
44.3	8.9	9.0	11.0
44.1	8.6	8.8	10.5
44.1	8.7	9.3	10.9
44.2	8.3	9.1	11.5
44.0	[9.8]	9.3	10.5
			11.8
			11.2
			9.6
			10.6
			11.0
$+ 8\ 43.96$	$- 3\ 8.69$	$- 3\ 9.15$	$- 3\ 11.03$
$\pm\ 0^s.26$	$\pm\ 0^s.13$	$\pm\ 0^s.15$	$\pm\ 0^s.42$

The results in columns II and III were obtained on 1904, Oct. 28 and 29, with a Gregg & Rupp universal instrument of 1⅜ inches aperture and 17 inches focal length; those in IV, on 1904, Nov. 1, with a Brandis engineer's transit of 1¼ inches aperture and 11 inches focal length. The magnifying power of the former instrument is about 30 diameters; that of the latter, 24. The somewhat greater irregularity of the fourth series is due to the fact that the slow motion of the Brandis transit is defective and tends to throw the instrument out of level when adjustments in azimuth are made. The results for all of the stars observed on any given night are printed, but the last value of the second series has been rejected in forming the means, as the instrument was accidentally moved in azimuth during the observation of the last star. In all cases the transits were observed across a single vertical thread. Although the instruments were mounted upon one of the piers of the transit room, it is not likely that this fact has contributed anything to the precision of the results, since, with this method of observing, the stability of the instrument is assumed for only two or three minutes time. The last two lines of the table give the means for the series and the probable error of the clock correction as obtained from a single star.

It would appear, therefore, that a single set of two stars, observed by the vertical circle method with the ordinary engineer's transit, should determine the correction to the clock with an error not exceeding $0^s.3$ or $0^s.4$.

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BULLETIN NO. 6

THE ALGOL VARIABLE 188.1904 DRACONIS

The variability of this star was discovered by Madame Ceraski during an examination of photographic plates made by Blajko at the Observatory of Moscow. Upon five out of six plates the star was of magnitude 9.3, while upon the sixth, taken 1904, Oct. 12, $7^h\ 22^m$–$9^h\ 18^m$ Moscow M. T., the star was not to be found. Upon this date it must have been of the twelfth magnitude or fainter[1]. Other observations were obtained by Blajko as follows:

1904,	Nov.	30,	$7^h\ 30^m$–$9^h\ 24^m$	Moscow	M. T.,	magnitude,	9.4
"	Dec.	2,	9 8	"		"	10.4
"	"	2,	30	"		"	10.2

These results indicate a variation of the Algol type, and such a probability was, indeed, suggested by Professor Ceraski in announcing the discovery.

The star has been on the observing list of the Laws Observatory since June of the current year, and there is at the present moment sufficient data to determine its period with considerable accuracy. The observations have all been made with an equalizing wedge photometer attached to the 7½-inch equatorial. Part of those obtained during June and July were made by Mr. E. S. Haynes, who served as a volunteer assistant in the observatory during the past summer.

Ordinarily, the star is 0.3 of a magnitude fainter than B. D. + 62°.1639 (9.3), but at intervals of about $2^d\ 20^h$ the brightness diminishes somewhat over three magnitudes in about four and one-half hours time. The light curve appears to be nearly if not quite symmetrical, so that approximately nine hours are required for the entire light change. I am unable to state the exact brightness at the time of minimum, since the star disappears from view in the 7½-inch equatorial about an hour before this stage in its variation is reached; but, judging from the rate of light change at the instants of disappearance and reappearance, its minimum brightness is probably in the neighborhood of the thirteenth magnitude. Just before disappearance, and immediately after reappearance, the brightness changes about a magnitude in half an hour. The time corresponding to any given brightness included within this range can therefore be determined with considerable precision.

[1] *Astronomische Nachrichten*, v. 167, p. 41.

Since the observations are not yet fully reduced, the following method has been used for the derivation of the period. The differences in the wedge readings for settings on the variable and comparison star were plotted for the various periods of light change observed. The times corresponding to a difference of twenty wedge divisions were then read from the curves drawn through the plotted observations. The results are as follows:

FROM CURVES OF DIMINISHING LIGHT

Date	G. M. T.	Julian Day	O — C
1905, June 29	15h 0m	2417026.625	— 0.003
July 13	18 24	040.767	— 0.016
Sept. 2	17 48	091.742	— 0.001
Oct. 26	12 48	145.533	0.000

FROM CURVES OF INCREASING LIGHT

Date	G. M. T.	Julian Day	O — C
1905, July 16	18h 6m	2417043.754	+ 0.023
Aug. 19	17 0	077.708	+ 0.004
Oct. 26	15 36	145.650	0.000

The results for July 13 and 16 are uncertain to the extent of twenty or thirty minutes, partly on account of the difficulty with which the observations for these dates were secured, and partly because it was necessary to exterpolate the curves somewhat in order to determine the instant when the difference between the variable and comparison star was equal to the chosen number of wedge divisions. These results are therefore excluded in determining the period. The observations for the other dates agree closely, and it is not likely that the corresponding results given in the tables are in error more than four or five minutes.

The intervals separating the various dates in the two tables must correspond to an exact number of periods. From the first I have obtained as the value of the period, 2.83114 days, while from the second, 2.83092 days, with weights of 3 and 1, respectively. The mean is, therefore,

$$P = 2.83108 \text{ days, weight } 4.$$

Assuming the light curve to be symmetrical, the results for Oct. 26 give for the minimum, Oct. 26, $14^h\ 12^m$ G. M. T. = J. D. 2417145.592. On Aug. 19 a single observation of four comparisons was obtained before the disappearance of the star. This, combined with the results obtained after its reappearance, gives for this minimum Aug. 19, $15^h\ 31^m$ G. M. T. = J. D. 2417077.647. From these two minima there results,

$$P = 2.83104 \text{ days, weight } 1,$$

which is in close agreement with the mean given above.

Since the variable does not appear upon the plate obtained by Blajko on 1904, Oct. 12, it is probable that the middle of the exposure coincided closely with a minimum. Assuming, therefore, a minimum for 1904, Oct. 12, $5^h\ 50^m$ G. M. T. = J. D. 2416766.243, and comparing this with the minimum of 1905, Oct. 26, there is obtained,

$$P = 2.83096 \text{ days.}$$

This last value can be regarded only as a control upon the two given above, since the uncertainty as to the instant of minimum on 1904, Oct. 12 is probably an hour at least. We may therefore adopt provisionally,

$$P = 2.83107 \text{ days} = 2^d\ 19^h\ 56^m\ 44^s,$$

with an uncertainty of perhaps 5^s.

From the value of the period thus derived it appears that the star passed a minimum on 1904, Dec. 2 and was regaining its light when Blajko's observations of that date were made, as, in fact, is indicated by the observations themselves.

The residuals corresponding to the accepted value of the period are printed in the last columns of the tables.

Unless the assumption of a symmetrical light curve is unjustifiable, the time given for the minimum of 1905, Oct. 26 can scarcely be in error more than four or five minutes. Starting with this, the value for the period gives for the minimum of 1904, Oct. 12, J. D. 2416766.229, whence we have the following formula for the recurrence of the minima:

$$m = \text{J. D. } 2416766.229 + 2.83107E.$$

An ephemeris giving the Greenwich Mean Time of minima occurring during the next four months is contained in the following table.

The position of the star is,

1900.0	R. A. $= 18^h\ 40^m\ 48^s.62$	Decl. $= +62^\circ\ 34'\ 32''.2$
1855.0	$= 40\ 32$	$= +62\ 31.1$

EPHEMERIS

E	Julian Day	G. M. T.	E	Julian Day	G. M. T.
141	2417165.409	1905, Nov. 15 9h 50m	166	2417236.186	1906, Jan. 25 4h 28m
142	168.241	18 5 46	167	239.017	28 0 25
143	171.072	21 1 43	168	241.848	30 20 22
144	173.903	23 21 40	169	244.679	Feb. 2 16 18
145	176.734	26 17 37	170	247.511	5 12 15
146	179.565	29 13 33	171	250.342	8 8 12
147	182.396	Dec. 2 9 30	172	253.173	11 4 9
148	185.227	5 5 27	173	256.004	14 0 5
149	188.058	8 1 23	174	258.835	16 20 2
150	190.889	10 21 20	175	261.666	19 15 59
151	193.720	13 17 17	176	264.497	22 11 55
152	196.551	16 13 14	177	267.328	25 7 52
153	199.382	19 9 11	178	270.159	28 3 49
154	202.213	22 5 7	179	272.990	Mar. 2 23 46
155	205.044	25 1 4	180	275.821	5 19 42
156	207.876	27 21 1	181	278.652	8 15 39
157	210.707	30 16 57	182	281.483	11 11 36
158	213.538	1906, Jan. 2 12 54	183	284.314	14 7 33
159	216.369	5 8 51	184	287.146	17 3 29
160	219.200	8 4 48	185	289.977	19 23 26
161	222.031	11 0 44	186	292.808	22 19 23
162	224.862	13 20 41	187	295.639	25 15 20
163	227.693	16 16 38	188	298.470	28 11 16
164	230.524	19 12 35	189	301.301	31 7 13
165	233.355	22 8 31	190	304.132	Apr. 3 3 10

This star is of unusual interest on account of the rapidity and range of its variation, and it is desirable that those possessing instruments of sufficient power should secure observations of minima at as early a date as possible. Among the known Algol variables it is probably second in range of variation only to H1174 = 035727, whose extremes of brightness are separated by about four magnitudes.

Columbia, Missouri, 1905, Nov. 3. *F. H. Seares.*

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BULLETIN NO. 7

PHOTOMETRIC INVESTIGATIONS

During the last three years several pieces of photometric apparatus have been added to the equipment of the Laws Observatory in pursuance of a plan which has for its aim the systematic observation of variable stars. The principal instruments thus acquired are a disc photometer by Brashear and an equalizing wedge photometer constructed by the Alvan Clark & Sons Corporation after designs by Pickering. The former instrument is used for calibration purposes, while the latter, in combination with the 7½-inch equatorial, forms the principal part of the equipment used in the observation of stars. These instruments have been subjected to a careful investigation in order to determine the various instrumental constants, and, at the same time, to gain some idea of the precision which they are capable of affording. The present paper contains an account of the methods used in this investigation and a summary of the results obtained.

The Disc Photometer

A disc photometer is a device by means of which the apparent intensity of a luminous source can be diminished by a definite and known amount. It consists essentially of a rotating disc from which a number of sector shaped areas have been cut away. A source of light viewed through one of the openings when the disc is at rest will be seen with its full intensity; but if the disc be rotated rapidly a part of the rays will be obstructed and the source will appear diminished in brightness by an amount depending upon the relative area of the disc and the openings. Frequently the size of the openings can be varied, which makes it possible to diminish the intensity of the source by any desirable amount within a considerable range of percentage.

The instrument belonging to the Laws Observatory is mounted upon a heavy wooden support at the end of an optical bench, with the disc perpendicular to the axis of the bench, and at such an elevation that the pieces of auxiliary apparatus used in the testing of absorption plates and in the calibration of wedges can be conveniently assembled. The luminous source is contained within a small box situated just beyond the disc and in line with the bench. The side of the box facing the disc and the bench contains a window which can be fitted with a pin hole diaphragm or with a diffusing surface, according as it is desirable to compare the intensities of point

images or of luminous surfaces. The rays of light which pass the window and the openings of the disc are rendered parallel by a collimating telescope mounted on the bench next to the disc. A second telescope receives the rays from the collimator and forms an image of an artificial star or of a luminous surface as may be desired.

The base of the photometer proper is a heavy casting, 36 by 61 cm. in size. From this spring three supports which are in alignment and which carry the moving parts of the instrument. The end supports are higher than that in the middle and hold the bearings for the shaft of the rotating disc. The middle support and one of the others carry the gears which communicate power to the disc and give it the proper speed. Usually the power is supplied by a ¼ H. P. motor bolted to one corner of the base, but, if necessary, the motor can be disconnected by shifting the gears. The disc can then be rotated by hand by means of a crank attached to the end of the gear shaft.

The disc itself consists of four circular plates of brass each 1.4 mm. thick and 51 cm. in diameter, mounted side by side upon the shaft with their surfaces in contact. Each plate has, near its edge, a series of ten openings of equal size distributed uniformly throughout the circumference. The openings, which are approximately 5 by 10 cm., are bounded by radii of the disc and by arcs of circles which are concentric with the disc itself. The angle included between the bounding radii of any opening is 27°. The total area of the openings in any plate is therefore three times the area of that portion of the plate which is included between the bounding concentric circles. The plate nearest the motor is rigidly attached to the supporting shaft, while that farthest away is joined to a sleeve of steel that fits the shaft loosely at this point. The intermediate plates are also free from the shaft, but their motion, relatively to it, is limited. Starting from a position in which the openings of all four plates coincide, the sleeve plate can be rotated toward the left, relatively to the shaft, through an angle of 9° without disturbing any of the others. At this point a pin in the sleeve plate engages in a slot in the second plate so that upon further rotation the latter is also set in motion; after a rotation through a second angle of 9°, the third plate is similarly set in motion; after a third rotation of 9°, the sectors of the sleeve and intermediate plates are spread out like the rays of a fan and completely cover the openings of the remaining plate. No further rotation in this direction is possible and, when in this position, no light from the luminous source can pass the disc. A rotation of the sleeve plate in the opposite direction through an angle of 27° brings the plates into their original position. The intensity of the light transmitted by the rotating disc can therefore be varied at will from 0 to 75 percent of what would be transmitted were the disc at rest with one of the openings in alignment with the source and the collimator.

The adjustment of the size of the openings is accomplished by means of a lever connected by a swivel joint with a rod moving longitudinally within the disc shaft. The rod terminates in a pin whose ends project through slots cut in the shaft parallel to its axis. The ends of the pin also project through other slots cut in the sleeve which encircles the shaft at this point. The sleeve slots are *inclined* to the axis of the shaft so that any longitudinal motion of the rod and the

pin must produce a corresponding relative rotation of the sleeve and the shaft, and, therefore, of the plates composing the disc. This device makes it possible to vary the size of the openings without stopping the disc. An index on the lever and a scale, graduated in 5 percent divisions from 0 to 75, indicate the intensity of the transmitted rays as compared with what the intensity would be were there no obstruction.

Although it is possible to reduce to zero the apparent intensity of the rays, it is not desirable to attempt observations for settings less than 10, since, when the openings are small, a slight error in the scale reading produces a relatively large error in the calculated brightness of the image seen in the view telescope.

In order to determine the graduation errors of the scale the lever was clamped in position, successively, for each of the readings 10, 15, 20, ... 75. For each setting the distance between the bounding radii was measured at the outer edge of each opening by means of a vernier caliper reading to 0.01 mm. The distance between the outer edges of diametrically opposite openings was also measured by means of a beam compass and a millimeter scale. From the data thus obtained the total angular area of the openings was calculated for each of the scale readings. The quotients formed by dividing each of these results by 360° express the ratio of the intensity of the light transmitted by the rotating disc for different settings to the intensity of that which would pass were there no obstruction. The values of the ratios, expressed as percentages, are given in column two of Table I. The difference between any ratio, thus expressed, and the corresponding scale reading is the index correction for that division of the scale. Column three of Table I contains these corrections for each of the divisions examined.

The change in the magnitude of an artificial star corresponding to different settings can be calculated by the formula

$$\Delta m = 2.5\,(\log h_0 - \log h_1),$$

where h_0 and h_1 are the percentages for the settings to be compared. If we assign the value unity, or 100 percent, to the intensity of the artificial star when all of the rays are transmitted, the formula becomes

$$\Delta m = 2.5\,(2.00 - \log h_1).$$

The values of Δm have been calculated for the various scale divisions by substituting into this formula the values of h_1 given in column two of Table I. The results are contained in the last column of this same table. They represent the reduction in brightness of the artificial star, produced by rotating the disc, as compared with its brightness when the disc is at rest.

The uncertainty in the percentages and index corrections given in Table I probably amounts to one or two tenths of one percent. The corresponding uncertainty in Δm varies from 0.01 or 0.02 of a magnitude for division 10 to an amount so small as not to affect the second place of decimals for the higher scale readings.

It is necessary to redetermine the graduation errors from time to time, since a small amount of wear in the slots or in the pin whose motion causes the relative rotation of the disc plates produces an appreciable change in the index corrections. For example, a comparison of the results of a recent determination with those obtained about a year ago shows that during this interval the absolute values of the index corrections have systematically diminished by about 0.5 percent. For division 10 this corresponds to a change of 0.06 of a magnitude in Δm, a quantity which cannot be neglected. Care must be taken always to rotate the disc in the same direction in order to eliminate the effect of lost motion upon the size of the openings.

TABLE I

DISC PHOTOMETER

Division	Percentage	Index Corrn.	Δm
10	9.2	— 0.8	2.59
15	14.1	— 0.9	2.13
20	18.9	— 1.1	1.81
25	23.8	— 1.2	1.56
30	28.7	— 1.3	1.36
35	33.7	— 1.3	1.18
40	38.7	— 1.3	1.03
45	43.7	— 1.3	0.90
50	48.6	— 1.4	0.78
55	53.5	— 1.5	0.68
60	58.6	— 1.4	0.58
65	63.4	— 1.6	0.49
70	68.2	— 1.8	0.42
75	72.4	— 2.6	0.35

THE WEDGE PHOTOMETER

The wedge photometer of the Laws Observatory is of the form designed by Pickering and differs only in minor particulars from instruments already described by Parkhurst and Maddrill[1]. All these instruments are constructed upon the principle involved in the stellar photometer of Zöllner, in which the essential feature consists in the formation of an artificial star in the field of view of the telescope alongside the image of the real star whose brightness is to be determined. The intensity of the former is varied until the two images appear to be of equal brightness. The corresponding scale reading compared with a similar reading for a star of known brightness gives

[1] Parkhurst, J. A., *Astrophysical Journal*, v. 13, p. 249.
Maddrill, J. D., *Pub. Astron. Society Pacific*, v. 17, p. 121.
Astrophysical Journal, v. 22, p. 138.
Lick Observatory Bulletin, No. 83.

the desired information. The principal difference in the two instruments lies in the method of varying the light of the artificial star. In the Zöllner photometer this is accomplished by means of Nicol prisms, while in the Pickering instruments the variation is produced by means of an absorption wedge. The wedges used, however, are not wedges at all in the generally accepted sense of the term, but strips of photographic negative which have been exposed in such a manner that the blackening of the film increases from one end to the other. This form of construction is relatively inexpensive, and possesses the advantage that the source of light for the artificial star need be only one-half the intensity of that required when Nicol prisms are used. In practice, the instrument is very easily handled and the reduction of the observations is very simple, when once the absorption curve of the wedge has been well determined. The most serious disadvantage lies in the amount of labor necessary for the satisfactory determination of this curve, and in the fact that there is no simple method of varying the color of the artificial star. Although this last point is of importance in the observation of highly colored stars, nevertheless, the instrument can be very successfully used as it stands for the majority of objects which come within its range.

It is evident that an ordinary wedge of absorption glass might have been employed in place of the photographic wedge. Such an arrangement has been in use at the Laws Observatory for more than a year, and my experience leads me to believe that the change is a desirable one. The first cost of an instrument fitted in this way is slightly greater than when the photographic wedge is used; but, on the other hand, the time required for the investigation of the absorption curve is less, on account of its greater regularity; and, at the same time, certain other difficulties, to be described later, are entirely avoided. Satisfactory wedges of shade glass can easily be obtained, for the importance of neutral tint is not so great as with the form of the wedge photometer used by Pritchard and others. In the Pickering instruments the rays of the *artificial* star traverse the wedge, and since the quality of the light of this object can be kept the same, it is evident that there may exist, without serious consequence, an amount of selective absorption which, with the older form of instrument, would be entirely inadmissible. It is always necessary to introduce a slip of colored glass in order to give the artificial star the tint of the average real star. If the wedge shows selective absorption it is only necessary to vary the color of the glass so as to meet the peculiarities of the given wedge.

The source of light used for the artificial star is a miniature incandescent lamp of about one candle power, requiring 0.18 amperes at 3.5 volts. When these investigations were begun, it was impracticable to make use of storage cells, and, in order to secure the requisite constancy of current, recourse was had to primary cells of the Edison-Lalande type. Generally, six or eight cells of 300 ampere-hours capacity have been used. The cells are connected in series and the circuit includes sufficient resistance to reduce the current to the required amount. A Weston mil-ammeter, reading directly to 0.002, and by estimation to 0.0002 amperes, is used as a control upon the constancy of the current, and is sufficiently sensitive to reveal any variation capable of affecting the measures. Although the observations are always arranged so as to eliminate any gradual change, experience has shown that, when the cells are in good condition, they will run

for days with a variation of not more than 0.002 or 0.003 amperes. Occasionally, rapid fluctuations have made their appearance, but usually they have been traced to exhausted elements or to defective connections. On the whole, this type of battery has proved very satisfactory. One strong point in its favor is convenience. Cells of the capacity mentioned above will run for months without the slightest attention. On open circuit the difference in potential per cell is 0.95 volt, but when the circuit is closed it drops to the normal of 0.67 in abour half an hour. The best results are obtained by keeping the cells on closed circuit through high resistance when not in active use. The variation in the voltage is thus avoided, with the expenditure of only a small amount of current.

With the normal intensity of the artificial star and the full aperture of the 7½-inch equatorial, stars as bright as the seventh magnitude can be observed. Still brighter objects can be reached by diaphragming or by introducing shade glasses into the path of the rays of the real star, although the latter method is open to some objections and is not used here. The range of the instrument has been increased still further by the use of a lateral ocular in accordance with the suggestion of Ceraski[1]. With this device the artificial star is observed directly, while the rays of the real star are reflected from the plane parallel plate in front of the ocular. The greater part of the disc photometer observations for the determination of the absorption curves were made with this lateral ocular. But, on the other hand, all of the Pleiades observations for the testing of these curves were secured with the direct eye piece. The close agreement of the two sets of observations is evidence as to the satisfactory performance of the device.

A total reflecting prism, attached to the ocular immediately in front of the eye lens, has been used throughout practically all of the observations thus far made. It reduces enormously the fatigue always attendant upon continuous observation, no matter what the position of the object may be, and makes it possible to measure easily objects near the zenith, which otherwise could be observed only with the greatest difficulty. The only objection to the prism appears to be the loss of light, due to the increased absorption, but this is of importance only in the case of faint objects.

Investigation of the Wedges

An attempt to determine the absorption constant of the photographic wedge from observations upon selected pairs of Pleiades stars soon showed that the absorption curve presents marked deviations from a rectilinear form, and that the calibration of the wedge from the Pleiades observations alone would be impracticable. Further, considerable difficulty has been experienced in securing satisfactory images of the artificial star with this wedge. The star image itself is quite as well defined as that of a real star under average atmospheric conditions, but, under certain circumstances, it appears surrounded by a faint luminosity whose intensity gradually diminishes from the center outward. The visibility of the luminosity depends upon the blackness of the sky

[1] *Astronomische Nachrichten*, v. 110, p. 75; v. 120, p. 217.
Annales de l'obs. de Moscou, Série 2, v. 1, livr. 2, p. 13.

background. During moonlight or twilight it disappears entirely, and an ordinary whitish sky is sufficient to reduce greatly its intensity. The phenomenon is most striking when viewed by averted vision; but even when the artificial star is regarded directly, there still remains, upon moonless nights, a distinct trace of the luminosity. An examination of a second photographic wedge, kindly furnished by Professor Parkhurst, gave practically the same results. A special series of experiments indicates that the source of the difficulty lies in the photographic film, and that almost any piece of negative is capable of producing a similar effect. The phenomenon does not appear to have been noted by any of the other observers who are using photographic wedges, and, in consequence, I have been reluctant to believe that the appearance described is due to anything more than some peculiarity in the arrangement of the instrument used here. Nevertheless, repeated experiments all point to the conclusion given above. Without describing these experiments in detail, they may be summarized as follows: The removal of the photographic wedge from its cell causes the luminosity surrounding the artificial star to disappear. When the photographic wedge is replaced by one of shade glass it also disappears. The introduction of strips of ordinary negative in place of the wedge always gives the characteristic appearance to a greater or less degree. Similar experiments with the artificial star of the disc photometer, using an entirely different set of apparatus, give similar results.

In fact, considering the structure of the photographic film, the appearance of diffused light under the conditions described is not surprising; for the reduction in intensity of light traversing such a medium is largely due to an actual obstruction of the rays by the opaque particles of silver suspended in the film. A portion of the rays emerging from the film will be scattered by diffuse reflection from these same particles, and, in the case of a pencil of rays forming a point image, will not be reunited with the main body of the pencil. The result will be an image surrounded by a certain amount of diffused light. As seen here the phenomenon is not very striking when the image is regarded fixedly in such a manner as to form a central retinal image. I am not prepared to assert that it affects the accuracy of the observations, but a prejudicial influence is certainly to be feared. The demonstration of the existence of such an influence would doubtless be a difficult matter, since it would be more or less entangled with the determination of the absorption curve. In as much as the phenomenon varies with the observing conditions, it is probable that any detrimental effect upon the settings would be confused with the accidental errors of observation, and would first reveal itself through an abnormally large probable error.

Under the circumstances it seemed best to discard the photographic wedge in favor of one of shade glass which was furnished by the Zeiss Optical Works. The latter has proved very satisfactory, and, although it too shows some deviation from a rectilinear absorption curve, the precision of the results given below is sufficient evidence that the instrument, thus equipped, is capable of giving valuable results. Moreover, the absorption curve of the shade glass wedge seems to be independent of the arrangement of the diaphragm, which is a distinct advantage not

enjoyed by photographic wedges[1].

A considerable number of variable star observations had been made before it was decided to discard the photographic wedge, and, consequently, it has been necessary to investigate the absorption curves of both wedges. The method employed in this investigation was the following:

The photometer was removed from the telescope and mounted on the bench of the disc photometer where it was fitted with an objective of 2⅛ inches aperture and 22 inches focal length. A telescope of 2 inches aperture served as a collimator, and the window of the lamp box was fitted with a minute diaphragm and with a piece of blue glass to give the rays the proper color. When adjusted for alignment, focus, etc., the rays from the box, traversing the disc and collimator, formed the sharply defined image of an artificial star in the field of view of the telescope. For convenience this will be referred to as the disc star. A second artificial star, the wedge star, formed from rays originating in the lamp of the stellar photometer also appeared in the field of view, alongside the first. The brightness of the former could be varied by rotating the disc for different settings of the controlling lever, while that of the latter could be changed by shifting the wedge. The first observations made were for the determination of the length of the wedge corresponding to a reduction of 1.81 magnitudes in the brightness of the disc star. The order of the settings was as follows: Four settings of the wedge for disc photometer reading 100 percent; eight for 20 percent; four for 100 percent. Before beginning any series the brightness of the disc and wedge stars was always adjusted by varying the resistance of the circuits so that the wedge readings were made to cover a definite section of the wedge. Different sections of the wedge were repeatedly measured in this manner. For the Zeiss wedge the smaller readings were made to coincide as closely as possible with the series 1, 4, 8, 12, 16, ... 40 divisions. For the photographic wedge 10, 15, 20, ... 40 divisions were taken as the beginning of the sections.

The final results for the Zeiss wedge depend upon sixteen observations for each section, each observation consisting of four comparisons between bright and faint disc stars. The observations were distributed over a number of days, and, generally, only four observations of any section were made upon the same day. Moreover, the conditions were purposely varied so as to cover a wide range of intensities for the artificial stars and of apertures for the telescope. None of the various modifications tried produced any appreciable effect upon the measures, and hence they were all included in forming the means for each section. An equal number of observations was also made by Mr. E. S. Haynes for the first five sections of the Zeiss wedge in order to determine whether or not systematic differences depending upon personality are likely to appear in observations made with such an instrument. The results for the Zeiss wedge are contained in Table II. Columns 2-5 show the results of my own observations, while columns 6-9 give those of Mr. Haynes. The second and sixth columns give the range in wedge divisions of the various

[1] *Annals of Harvard College Observatory*, v. XLI, p. 244.
Lick Observatory Bulletin, No. 83, p. 164.

sections measured. Columns three and seven contain the means of the differences of the wedge readings for each section, while columns four and eight give the probable error of each mean, expressed in fractions of a wedge division. The value of Δd corresponds to a variation of 1.81 magnitudes in the disc star; the differences between this quantity and the values of Δm derived from Table V are printed in the columns headed O—C, and represent the deviations of the individual means from the finally accepted absorption curve. The last column contains the

TABLE II

ZEISS WEDGE AND DISC PHOTOMETER

No.	Range	Δd	P. E.	O – C	Range	Δd	P. E.	O – C	S – H
1	1.6 – 15.1	13.47	±0.10	+ 0.21	1.0 – 14.9	13.96	±0.13	+ 0.18	— 0.49
2	4.0 – 18.3	14.36	0.08	+ 0.01	4.0 – 18.6	14.63	0.12	— 0.02	— 0.27
3	8.0 – 22.8	14.87	0.09	— 0.03	7.8 – 22.8	15.01	0.10	— 0.05	— 0.14
4	11.7 – 27.4	15.68	0.12	— 0.03	11.3 – 27.7	16.37	0.04	— 0.12	— 0.69
5	15.6 – 32.4	16.75	0.11	— 0.07	15.9 – 32.7	16.81	±0.03	— 0.06	— 0.06
6	20.1 – 36.8	16.66	0.13	— 0.01					
7	24.4 – 41.0	16.54	0.19	0.00					
8	27.9 – 43.8	15.89	0.12	+ 0.05					
9	32.1 – 48.1	16.00	0.12	+ 0.05					
10	36.5 – 51.9	15.41	0.13	+ 0.05					
11	40.3 – 55.7	15.39	±0.14	— 0.01					

TABLE III

PHOTOGRAPHIC WEDGE AND DISC PHOTOMETER

No.	Range	Δd	P. E.	O – C
1	10.2 – 21.4	11.18	±0.17	+ 0.02
2	15.4 – 26.4	11.05	0.13	+ 0.07
3	20.2 – 34.6	14.40	0.11	+ 0.04
4	25.1 – 41.0	15.88	0.08	0.00
5	30.4 – 44.8	14.42	0.20	+ 0.08
6	35.5 – 49.9	14 35	0.12	+ 0.07
7	40.6 – 56.1	15.55	±0.14	— 0.02

differences between the values of Δd obtained by Mr. Haynes and by myself for sections one to five. An average systematic difference of — 0.33 division, or — 0.04 magnitude, is hereby revealed.

The average probable error for the mean of sixteen observations is ± 0.12 division for myself, and ± 0.08 division for Mr. Haynes. Expressed in magnitudes these quantities become ± 0.013 and ± 0.009, respectively. The corresponding probable errors of a single observation of four comparisons are therefore ± 0.052 and ± 0.036 magnitudes, respectively.

Table III, which depends entirely upon my own observations, gives the results for the photographic wedge. The arrangement is the same as that of Table II. The only point requiring mention is that the values of Δd are the means of eight observations instead of sixteen, and that the probable errors refer to this number of observations. Further, the values of O—C are based upon the absorption curve of Table VI. The average probable error for a single observation of four comparisons derived from these measures is $\pm$ 0.042 magnitude.

In the case of both wedges, the values of Δd correspond to a constant value of Δm. It is evident, therefore, that the absorption curves of both wedges present considerable deviations from a rectilinear form, and that the data in Tables II and III is insufficient for a satisfactory determination of the curves. Further observations were then made with the disc photometer as follows: Four settings were made for each of the disc photometer readings 100, 75, 60, 40, 30, 20, 15, and 10; the series was then repeated in the reverse order. In all, six observations of four

TABLE IV

Disc Photometer Observations for Form of Absorption Curves

D. P. Setting	Δm	Zeiss Wedge		Photographic Wedge			
		d	O – C	d	O – C	d	O – C
100	0.00	1.70	0.00	13.54	0.00	27.35	0.00
75	0.35	5.44	— 0.01				
60	0.58	7.74	— 0.05	16.90	+ 0.01	32.44	+ 0.02
40	1.03	10.83	+ 0.01	19.16	+ 0.02	36.00	+ 0.03
30	1.36	13.34	0.00	20.99	+ 0.01	39.23	0.00
20	1.81	16.08	+ 0.08	24.11	+ 0.04	42.49	+ 0.05
15	2.13	19.88	— 0.06	27.44	— 0.01	45.90	— 0.02
10	2.59	23.50	+ 0.02	31.38	+ 0.04	49.40	0.00

settings each were made with the Zeiss wedge for each reading of the disc photometer, care always being taken to adjust the intensities of the artificial stars so as to make the wedge readings for all of the series as nearly the same as possible.

The means of the twenty-four Zeiss wedge readings for each disc photometer setting are contained in column three of Table IV. The values of Δm in column two are taken from Table I. The results in these two columns, when plotted, represent graphically the form of the absorption curve between wedge readings 1.7 and 23.5. This much of the curve having been determined, the results contained in Table II afford the means for its completion. For example, the point on the curve whose abscissa is 11.7 divisions can be located. The data in Table II under No. 4 then leads to the location of the point whose abscissa is 27.4, and so on, step by step, until the whole curve has been determined. When the final curve had been drawn the ordinates corresponding to integral wedge divisions were tabulated in Table V.

The same method, in general, was employed for the photographic wedge, only, in this case, no settings were made for the disc photometer reading 75. The number of observations for the other readings was eight. Further, the form of the curve was investigated for two different sections of the wedge, namely, from 13.5 to 31.4 divisions, and from 27.4 to 49.4 divisions. The

TABLE V

ABSORPTION CURVE OF ZEISS WEDGE

d	0	10	20	30	40	50
0	0.00	1.06	2.35	3.42	4.51	5.66
1	0.09	1.20	2.45	3.53	4.62	5.78
2	0.17	1.33	2.56	3.65	4.74	5.91
3	0.26	1.46	2.67	3.76	4.86	6.03
4	0.36	1.60	2.77	3.87	4.97	6.16
5	0.46	1.73	2.87	3.98	5.08	6.28
6	0.58	1.87	2.98	4.09	5.19	6.40
7	0.70	2.00	3.09	4.20	5.30	
8	0.81	2.12	3.20	4.30	5.41	
9	0.93	2.24	3.31	4.40	5.53	
10	1.06	2.35	3.42	4.51	5.66	

TABLE VI

ABSORPTION CURVE OF PHOTOGRAPHIC WEDGE

d	10	20	30	40
0	0.00	1.57	2.80	3.98
1	0.08	1.75	2.91	4.10
2	0.18	1.91	3.03	4.23
3	0.32	2.04	3.17	4.35
4	0.47	2.16	3.29	4.48
5	0.62	2.28	3.41	4.59
6	0.79	2.39	3.53	4.69
7	0.98	2.49	3.64	4.81
8	1.18	2.60	3.75	4.93
9	1.38	2.70	3.86	5.07
10	1.57	2.80	3.98	5.23

results are contained in columns 5-8 of Table IV. A graphical process similar to that described above led to the absorption curve whose ordinates are contained in Table VI. Both curves are illustrated by Fig. 1. The probable error of a single observation of four comparisons was also calculated from the observations which served for the construction of Table IV. The result for

the Zeiss wedge is ±0.048 magnitude; for the photographic wedge, ±0.041. Although these values are in close agreement with those derived from the measures entering into Tables II and III, they do not, on account of insufficiency of the data, afford any reliable test of the relative precision to be obtained by the two wedges. It should be remarked that during the disc photometer observations the field of view was of sufficient brightness to render the diffused light about the photographic wedge star invisible. The remark on page 97, concerning the possible

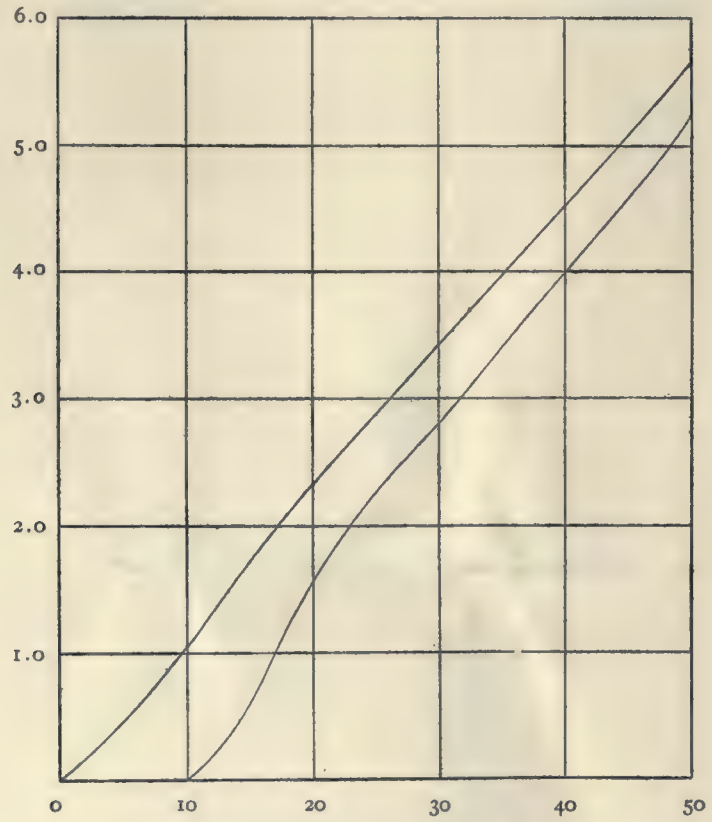

Fig. 1 – Abscissæ, wedge divisions; ordinates, magnitudes.
Upper Curve – Zeiss Wedge.
Lower Curve – Photographic Wedge.

effect of this luminosity upon the probable error, has no bearing upon the present discussion, therefore.

In order to test the agreement of the absorption curves with standards of magnitude which have been introduced into astronomical observations, a number of selected pairs of Peliades stars were measured and the magnitude differences resulting from the curves compared with the differences given by the catalogue of Müller and Kempf. The results, contained in Table VII, include every observation made for this purpose but one, and this is clearly a case of mistake in

identification of one of the stars. The pairs measured differed in brightness by amounts varying from 0.80 to 3.52 magnitudes. Eight observations with the Zeiss wedge were made on each pair. Pairs 17, 18, 19, were measured on three different nights; 7, 8, 20, 21, on five nights; each of the remaining pairs, on four nights. Further, the intensity of the artificial star was varied from time to time so as to bring different parts of the wedge into use for each pair.

TABLE VII

RESULTS OF OBSERVATIONS ON PLEIADES STARS

Pair	M & K Numbers	M & K Magnitudes	Zeiss Wedge			Photographic Wedge					Z – Ph.
			Δm	P. E.	P – Δm	Δm	P. E.	P – Δm	$\Delta m'$	P – $\Delta m'$	
1	54 – 64	8.85 – 10.00	1.02	±0.028	+ 0.13	1.04	±0.099	+ 0.11	1.00	+ 0.15	+ 0.02
2	26 – 61	7.63 – 9.76	2.06	0.058	+ 0.07	1.99	0.062	+ 0.14	1.91	+ 0.22	+ 0.15
3	49 – 77	9.05 – 10.42	1.36	0.056	+ 0.01	1.53	0.091	— 0.16	1.47	— 0.10	— 0.11
4	31 – 49	7.99 – 9.05	1.07	0.077	— 0.01	1.23	0.050	— 0.17	1.18	— 0.12	— 0.11
5	28 – 59	7.82 – 9.67	1.91	0.103	— 0.06	1.97	0.111	— 0,12	1.89	— 0.04	+ 0.02
6	36 – 54	8.31 – 9.44	1.12	0.065	+ 0.01	1.24	0.066	— 0.11	1.19	— 0.06	— 0.07
7	29 – 67	7.84 – 10.08	2.11	0.103	+ 0.13	2.27	0.083	— 0.03	2.18	+ 0.06	— 0.07
8	34 – 76	8.12 – 10.36	2.04	0.091	+ 0.20	2.15	0.096	+ 0.09	2.07	+ 0.17	— 0.03
9	37 – 63	8.33 – 9.88	1.47	0.043	+ 0.08	1.50	0.042	+ 0.05	1.44	+ 0.11	+ 0.03
10	38 – 75	8.40 – 10.35	1.88	0.033	+ 0.07	2.08	0.136	— 0.13	2.00	— 0.05	— 0.12
11	32 – 73	8.03 – 10.29	2.20	0.087	+ 0.06	2.33	0.101	— 0.07	2.24	+ 0.02	— 0.04
12	25 – 66	7.53 – 10.06	2.55	0.102	— 0.02	2.67	0.079	— 0.14	2.57	— 0.04	— 0.02
13	47 – 70	9.00 – 10.20	1.23	0.073	— 0.03	1.30	0.057	— 0.10	1.25	— 0.05	— 0.02
14	33 – 55	8.05 – 9.57	1.54	0.061	— 0.02	1.69	0.091	— 0.17	1.62	— 0.10	— 0.08
15	44 – 58	8.65 – 9.65	0.89	0.030	+ 0.11	0.87	0.059	+ 0.13	0.84	+ 0.16	+ 0.05
16	57 – 78	9.63 – 10.43	0.86	0.043	— 0.06	0.93	0.065	— 0.13	0.89	— 0.09	— 0.03
17	31 – 80	7.99 – 10.53	2.46	0.075	+ 0.08	2.65	0.076	— 0.11	2.55	— 0.01	— 0.09
18	21 – 88	7.24 – 10.76	3.46	0.070	+ 0.06	3.57	0.069	— 0.05	3.43	+ 0.09	+ 0.03
19	23 – 55	7.31 – 9.57	2.41	0.078	— 0.15	2.47	0.100	— 0.21	2.37	— 0.11	+ 0.04
20	28 – 54	7.82 – 9.44	1.64	0.040	— 0.02	1.71	0.057	— 0.09	1.64	— 0.02	0.00
21	36 – 59	8.31 – 9.67	1.42	±0.067	— 0.06	1.62	±0.048	— 0.26	1.56	— 0.20	— 0.14

Column two of Table VII shows the grouping of the stars, the numbers being those of the Müller and Kempf list[1]. Column three contains the Müller and Kempf magnitudes. Column four gives the means of the eight values of the magnitude differences obtained from the absorption curve for the Zeiss wedge, and column six, the correction which must be applied to these means in order to reproduce the Müller and Kempf differences. The fifth column contains the probable error of a single observation of four comparisons as derived from the observations on each pair.

[1] *Astronomische Nachrichten*, v. 150, p. 193.

The results of the observations with the photographic wedge are contained in columns 7-9. In this case each value of Δm is the mean of five observations, each pair being measured on three different nights.

The precision of the observations, in so far as the settings are concerned, is indicated by the probable errors. For the Zeiss wedge, the average probable error for a single observation of four comparisons is $\pm$ 0.066 magnitude; for the photographic wedge, $\pm$ 0.078 magnitude. The agreement of the magnitude scale underlying the absorption curves with that of Müller and Kempf is exhibited by the columns headed P—Δm. For the Zeiss wedge, the average deviation, disregarding algebraic sign, is 0.07 magnitude, and of the twenty-one differences, only four exceed 0.10 magnitude. Twelve of the differences are positive and nine negative. The algebraic sum is +0.58, which would indicate a deviation of +0.028 magnitude between the two magnitude scales for the average range 8.2 – 10.0 magnitudes. The data is insufficient to establish the reality of this deviation, although probably there is a small difference in the direction indicated. But, in any case, the agreement must be considered satisfactory, for the quantity 0.028 magnitude is not greater than differences which may arise from personality alone.

With the photographic wedge the case is different. The large excess of negative signs in the column P—Δm indicates a real difference in magnitude scale amounting to —0.073 magnitude for the above mentioned range. The scale of the photographic wedge can be brought into agreement with that of Müller and Kempf, however, by multiplying each value of Δm by 0.961. The products thus formed are given in the column headed $\Delta m'$, while the following column contains the corrections which must be applied in order to reproduce the Müller and Kempf differences. The algebraic sum of these corrections is —0.01; their average, 0.09 magnitude. This result would indicate that the ordinates of the absorption curve of the photographic wedge should be multiplied by the factor 0.961 in order to obtain a curve agreeing with the standard magnitude scale. But this has not been done for the following reason: It was not possible to make the Pleiades observations under quite the same observing conditions, as regards lamp and diaphragm, as obtained during the disc photometer investigation; the difference in the two sets of results is probably due to this fact. At the same time, the variable star observations secured with the photographic wedge were made with the same arrangement of lamp, diaphragm, etc., as the disc photometer observations. It is anticipated, therefore, that the curve of Table VI will suffice for the reduction of these observations, but further investigation is necessary before this point can be determined.

The last column of Table VI is of interest in that it exhibits the agreement of the magnitude scales of the two wedges. The numbers in this column are the differences between Δm obtained with the Zeiss wedge, and the modified magnitude differences of the photographic wedge. The maximum difference is 0.15 magnitude; the average, 0.06 magnitude.

Columbia, Missouri, 1905, Dec. 14. *F. H. Seares*

UNIVERSITY OF MISSOURI

LAWS OBSERVATORY
UNIVERSITY OF MISSOURI

BULLETIN NO. 8

A GRANT FROM THE GOULD FUND

A grant of five hundred dollars has been made to the Laws Observatory from the Benjamin Apthorp Gould Fund of the National Academy of Sciences to aid in the photometric observation of variable stars. The funds thus acquired will be used for the support of an assistant who will take part in the observations and reductions. Mr. E. S. Haynes, who has served as volunteer assistant in the Observatory from time to time, has been appointed to this position from June 1, 1906. All observations by Mr. Haynes subsequent to this date have been made under the grant in question.

GENERAL REMARKS CONCERNING VARIABLE STAR OBSERVATIONS

The instrumental equipment used in securing the photometric observations contained in this bulletin, and in others to appear in the near future, is the equalizing wedge photometer described in Laws Observatory Bulletin No. 7. No changes of any consequence have been made in the apparatus excepting the substitution of storage cells for the Edison Primary Batteries as a source of power for the miniature lamp. Used in connection with the 7½-inch equatorial the instrument is suitable for the observation of stars between the seventh and eleventh magnitudes, although the greatest precision is obtained only between somewhat narrower limits. The photometer has proved satisfactory although the definition of the artificial star leaves something to be desired. At present it is employed exclusively for the observation of variable stars, particularly those of recent discovery.

Certain points concerning the methods followed in securing these observations may properly find consideration at this place. An observation, in the sense in which the term is used in the following pages, is a group of eight settings of the wedge, four upon the comparison star and four upon the star whose brightness is to be determined. Generally the settings are made alternately upon the two stars, although for widely separated objects this is not done ; but in all cases the observations are so arranged as to eliminate uniform changes in transparency of the atmosphere and in the brightness of the artificial star. At first it was customary to make several

observations in succession, even in the case of the longer period stars ; but experience has shown this to be unnecessary, for the deviation from night to night in the means of several observations appears to be quite as large as the accidental error of a single observation. As a result, the practice at present is to make only two observations per night on all but the very short period stars, although occasionally for some special reason this number is increased. The two observations thus secured are, in general, referred to different comparison stars, and the set of sixteen settings is so arranged as to give not only the magnitude difference between the variable and comparison stars, but also that of the comparison stars themselves. Whenever possible, the observations are made alternately for each star by Mr. Haynes and myself.

In making the settings the real star is always brought between the images of the artificial stars, of which only the right-hand one is used for comparison purposes. In general it is desirable to reverse the images, but in calibrating the wedges by means of the disc photometer it was not convenient to make such a reversal. Any personal equation arising from this source therefore enters into the absorption curves of the wedges and will be nearly if not quite eliminated from observations made upon stars in the same manner. This applies in my own case at least ; for Mr. Haynes, who took but little part in the disc photometer observations, the matter requires further investigation, although a preliminary examination indicates that the systematic difference between the two observers is small.

Both the photographic and Zeiss wedges have been used in the observations ; the former for all observations previous to 1906, Jan. 1, and the latter for all subsequent to this date.

The precision of the observations varies somewhat from star to star and will be discussed in each individual case.

A NEW VARIABLE, 88.1906 LACERTAE

AG. Hels. 13458 1855.0 22^h 43^m 8.89 55° 39′ 45.″0
1900.0 44 58.37 53 58.4

The star BD. +55°.2817, (8.7), used as a comparison object in observations upon V Lacertae (110.1904), is a variable of the continuous variation type with a range of 0.4 magnitude and a period of about 5.4 days. In preparing the V Lacertae observations for publication it was found that the resulting light-curve presented irregularities much greater than the accidental errors of the observations, and of such a character as to suggest a variability of the comparison star. The comparison star was referred to star *b*, which is BD. +56°.2858, (8.0), and a few nights' observation sufficed to establish its variability. These observations, together with others, are given in Table I. The date, Greenwich Mean Time, and Julian Day for each observation, are contained in the first three columns of this table. The observer is indicated by the initial in column five, and column six gives the wedge readings, which in all cases are the means of four settings, the first value referring to the comparison star, the second to the variable. The resulting magnitude differences, derived by means of Table V, Bulletin No. 7, are given in column seven. All of the measures were referred to star *b* excepting the four for which the values of Δm are in

brackets. These were referred to star *c*, which is BD. +56°2872, (8.5). The corresponding magnitudes for the variable in column eight were obtained by adding the values of Δm to the comparison star magnitudes derived below.

Although the observations in Table I are sufficient for the determination of the main features of the light-curve, they do not suffice for an accurate evaluation of the period on account of the shortness of the interval covered. The V Lacertae observations referred to the new variable as comparison object, and extending back over two years, can be used for this purpose, however.

TABLE I

OBSERVATIONS ON BD. +55°2817

Date	G. M. T.	Julian Day	Phase	Obs.	Wedge		Δm	Mag.	O – C
1906, July 16	14h 50m	2417408.62	1.26	S	25.8	27.2	+ 0.15	8.44	— 3
	15 0	.62	1.26	H	21.0	23.5	+ 0.27	8.56	+ 9
17	14 40	7409.61	2.25	S	18.0	17.4	— 0.07	8.22	— 2
	49	.62	2.26	H	14.5	14.1	— 0.05	8.24	0
	20 53	.87	2.51	H	13.6	14.4	+ 0.11	8.40	+ 17
	21 1	.88	2.52	S	11.5	11.1	— 0.05	8.24	+ 1
18	14 35	7410.61	3.25	H	18.5	18.9	+ 0.05	8.34	+ 7
	44	.61	3.25	S	19.7	19.8	+ 0.01	8.30	+ 3
	21 50	.91	3.55	H	12.0	12.4	+ 0.05	8.34	+ 3
	58	.92	3.56	S	13.4	13.4	0.00	8.29	— 2
20	14 47	7412.62	5.26	H	15.0	17.0	+ 0.27	8.56	— 2
	59	.62	5.26	S	17.6	20.0	+ 0.28	8.57	— 1
	19 22	.81	0.01	H	11.2	13.6	+ 0.31	8.60	+ 1
	33	.81	0.01	S	11.9	13.8	+ 0.25	8.54	— 5
21	14 31	7413.60	0.80	S	16.3	17.9	+ 0.20	8.49	— 6
	41	.61	0.81	H	16.3	17.9	+ 0.20	8.49	— 6
	18 54	.79	0.99	S	16.4	18.3	+ 0.24	8.53	— 1
	19 4	.79	0.99	H	15.2	16.7	+ 0.20	8.49	— 5
22	15 32	7414.65	1.85	S	19.1	19.2	+ 0.01	8.30	+ 1
	42	.65	1.85	H	18.6	18.7	+ 0.01	8.30	+ 1
	55	.66	1.86	S	18.8	18.0	— 0.10	8.19	— 10
23	14 44	7415.61	2.81	H	21.6	21.3	— 0.04	8.25	+ 1
	15 0	.62	2.82	S	20.4	19.4	— 0.11	8.18	— 6
24	14 14	7416.59	3.79	H	16.1	16.2	+ 0.02	8.31	— 3
	26	.60	3.80	S	18.9	19.1	+ 0.02	8.31	— 3
	19 18	.80	4.00	S	14.5	15.0	+ 0.07	8.36	— 2
	28	.81	4.01	H	17.9	17.9	0.00	8.29	— 9
25	15 5	7417.63	4.83	S	17.7	19.5	+ 0.22	8.51	— 2
	16	.64	4.84	H	19.0	21.1	+ 0.22	8.51	— 2
26	15 6	7418.63	0.39	S	25.4	27.2	+ 0.20	8.49	— 9

Date	G. M. T.	Julian Day	Phase	Obs.	Wedge		Δm	Mag.	O – C
1906, July 26	16h 24m	2417418.68	0.44	H	19.0	21.2	+ 0.23	8.52	— 6
28	14 24	7420.60	2.36	S	18.1	16.7	— 0.17	8.12	— 11
	32	.61	2.37	H	17.8	17.3	— 0.06	8.23	0
	21 6	.88	2.64	S	15.2	15.0	— 0.03	8.26	+ 3
	15	.89	2.65	H	15.9	14.8	— 0.16	8.13	— 10
29	15 48	7421.66	3.42	H	15.9	16.0	+ 0.01	8.30	+ 1
30	14 17	7422.60	4.36	S	23.2	25.1	+ 0.19	8.48	+ 3
	19 13	.80	4.56	H	18.3	20.6	+ 0.25	8.54	+ 4
Aug. 1	14 59	7424.62	0.94	H	19.7	23.2	+ 0.37	8.66	+ 11
	16 2	.67	0.99	S	18.1	20.5	+ 0.27	8.56	+ 2
	19 44	.82	1.14	S	17.0	19.4	+ 0.28	8.57	+ 6
	52	.83	1.15	H	17.0	19.8	+ 0.33	8.62	+ 11
2	14 23	7425.60	1.92	S	23.3	23.6	+ 0.03	8.32	+ 4
	23	.60	1.92	S	28.3	24.1	[— 0.45]	8.24	— 4
3	15 28	7426.64	2.96	H	20.8	22.1	+ 0.14	8.43	+ 19
	28	.64	2.96	H	23.4	21.0	[— 0.26]	8.43	+ 19
	42	.65	2.97	S	19.6	19.2	— 0.05	8.24	0
	42	.65	2.97	S	22.8	20.0	[— 0.30]	8.39	+ 15
7	14 16	7430.59	1.46	S	17.4	18.0	+ 0.07	8.36	— 5
	16	.59	1.46	S	21.2	17.5	[— 0.41]	8.28	— 13

These observations give a curve which represents the combined variation of the two variables. The problem was to determine the dates of minima for BD. +55°.2817 from the resultant curve, the character of the variation of V Lacertae itself being unknown. The resultant curve showed a period of 4.98 days with maxima and minima of varying heights. The variation of BD. +55°.2817 appeared to be small as compared with that of V Lacertae, which made it possible to determine an approximate light-curve for the latter by neglecting the variation of the former. The V Lacertae observations were therefore combined in the usual manner for the formation of a mean light-curve, as though BD. +55°.2817 were of constant brilliancy. The comparison of the means of the observations by nights with this approximate light-curve gave a series of residuals which oscillated from positive to negative values with a period of about 5.4 days. The minima of the curve traced with these residuals correspond to the instants of minimum brightness of the new variable. Two of these minima near the beginning of the series are

J. D. 2416772.9
678.3.

These are well determined. The following, from recent observations in Table I, are also well determined:

J. D. 2417412.8
418.2.

Other minima, intermediate to these, are shown by the curve of residuals, although less distinctly than those given above; but, owing to the peculiar distribution of the observations, the total number of periods between the extreme dates is somewhat uncertain. Values of the period corresponding to 137, 139, and 141 cycles all satisfy the observations within the limits of error, with perhaps a slight preponderance of evidence in favor of 137. This value is therefore adopted provisionally. The resulting elements are

$$\text{Min.} = \text{J. D. } 2417412.8 + 5^{d}.440E, \qquad \text{G. M. T.}$$

The alternative values of the period are 5.36 and 5.28 days, respectively.

TABLE II

NORMAL PLACES FOR BD. +55°2817

No.	Phase	Δm	Mag.	No. Obs.	O − C
1	$0^{d}.08$	+ 0.26	8.55	6	— 0.04
2	0.92	+ 0.25	8.54	6	— 0.01
3	1.28	+ 0.20	8.49	4	+ 0.02
4	1.87	— 0.01	8.28	3	— 0.01
5	2.38	— 0.05	8.24	6	+ 0.01
6	2.81	— 0.05	8.24	6	0.00
7	3.41	+ 0.02	8.31	5	+ 0.02
8	3.90	+ 0.03	8.32	4	— 0.04
9	4.65	+ 0.22	8.51	4	+ 0.01

The observations of Table I were combined into nine normal places for the determination of the mean light-curve of the new variable. The results are shown in Table II. The values of Δm in the third column are the magnitude differences of the variable and star *b*. The corresponding magnitudes for the variable are in column four. The ordinates of the resulting light-curve are given for every 0.5 day in Table III, the phase being counted from minimum. The curve itself is shown in Fig. 1.

The residuals in the last column of Table II show the deviation of the normal places from the mean light-curve, and the last column of Table I gives a similar result, expressed in hundredths of a magnitude, for the individual observations. The phase value necessary for this comparison is found in the fourth column of Table I.

The magnitude of *b* was determined by a reference to four PD. stars. The results of the comparisons are shown in Table IV. The first three columns give the designation, PD. number and magnitudes of the reference stars. The fourth and fifth columns give the mean difference in brightness of the PD. stars and star *b* as determined by each of the observers. The mean Δm and the resulting magnitudes for *b* are in the sixth and seventh columns. The last column gives the number of observations upon which the results of the fourth and fifth columns are based. The mean of the four values for *b* is 8.29, which is the magnitude adopted for this star. The mean of four determinations of the difference between *b* and *c* gave $c - b = +0^m.40$, whence the magnitude of *c* is 8.69.

TABLE III

LIGHT-CURVE FOR BD. +55°.2817

Phase	Δm	Mag.	Phase	Δm	Mag.
0.0	+ 0.30	8.59	3.0	— 0.05	8.24
0.5	+ 0.29	8.58	3.5	+ 0.01	8.30
1.0	+ 0.25	8.54	4.0	+ 0.09	8.38
1.5	+ 0.10	8.39	4.5	+ 0.20	8.49
2.0	— 0.03	8.26	5.0	+ 0.26	8.55
2.5	— 0.06	8.23	5.5	+ 0.30	8.59
3.0	— 0.05	8.24	6.0	+ 0.28	8.57

TABLE IV

MAGNITUDE OF COMPARISON STAR *b*

Star	PD. III	PD. Mag.	Δm S	Δm H	Mean Δm	Mag. *b*	No. Obs.
p	3845	7.60	0.62	0.65	0.64	8.24	3, 3
q	3807	7.05	1.15	1.12	1.14	8.19	3, 3
r	3849	7.04	1.23	1.34	1.28	8.32	3, 2
s	3858	7.78	0.57	0.72	0.64	8.42	3, 2
					Mean	8.29	

The probable errors for a single observation derived from the values of O − C in Table I are $\pm 0^m.041$ for Seares and $\pm 0^m.055$ for Haynes, or for the whole series, $\pm 0^m.050$.

To determine the systematic difference between the two observers, the differences between the magnitudes derived by each were formed in the order S − H. The average for the fourteen nights upon which the variable was observed by both observers is $S - H = -0^m.045$. A similar

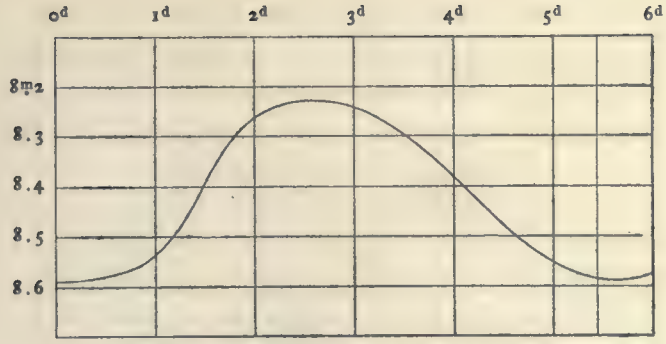

Fig. 1 — Light-curve of BD. + 55°2817

result follows from a consideration of the average of the residuals O – C for each observer, attention being paid to the algebraic sign. These averages are S – C = — 0m.014, and H – C = + 0m.027, whence S – H = — 0m.041, which is in close agreement with the value previously obtained. The systematic difference is small but appreciable ; the actual probable errors are therefore somewhat smaller than those given above, since the systematic difference enters into the data upon which the errors are based.

PRELIMINARY RESULTS FOR V LACERTAE, 110.1904

BD. $+55°.2815$, (8.7) 1855.0 $22^h\ 42^m\ 43^s.85$ $+55°\ 33'\ 21''.4$[1]
1900.0 44 33.29 47 34.3

The variable V Lacertae was discovered by Madame Ceraski from plates made at Moscow by Blajko, and was announced in A. N. No. 3953. Five hundred and thirty-nine photometric observations of the star have been made at this observatory since June, 1904. Most of these were referred to BD. $+55°.2817$ as comparison object, but as has been shown in the preceding section, this star is itself a variable. As a result, the reduction of the V Lacertae observations has been somewhat complicated, and indeed, a final reduction cannot be made until the uncertainty in the period of BD. $+55°.2817$ mentioned above is removed. The following elements derived from a preliminary reduction cannot be far from the truth, however.

$$\text{Max.} = \text{J. D. } 2416666.74 + 4^d.9835E, \qquad \text{G. M. T.}$$

TABLE V

LIGHT-CURVE FOR V LACERTAE

Phase	Mag.	Phase	Mag.
0.0	8.48	2.5	9.22
0.5	8.66	3.0	9.37
1.0	8.79	3.5	9.37
1.5	8.93	4.0	9.18
2.0	9.08	4.5	8.85
2.5	9.22	5.0	8.48

Table V gives the ordinates of the light curve for every 0.5 day, the phase being counted from maximum. The range is from 8.48 to 9.42 magnitudes. The minima precede the maxima by only 1.7 days, so that the ascending branch of the curve is steeply inclined. The descending branch appears to be nearly rectilinear throughout an interval of about three days. The maxima are sharply defined. The observations will be printed in detail in a future bulletin.

[1] Position from A. N. No. 3953.

THE VARIABLE V VULPECULAE, 4.1904

BD. $+26°.3937$, (8.1) 1855.0 $20^h30^m22^s$ $+26°\ 6'.2$
1900.0 32 17 15.4

A light-curve of the Beta Lyrae type with a period of 75.3 days was derived for this star by Williams and published in A. N. No. 3929. An examination of the data upon which these results are based indicates that they are open to considerable doubt. Fifty-eight observations were used by Williams, thirteen of which are photographic. Six of the photographic observations are near secondary minima, and two principal minima are well determined by the visual observations; but neither of these series cover both principal and secondary minima, although one of the visual observations precedes a secondary minimum by only seven or eight days. The reduction of the photographic observations to the visual light-scale as carried out by Williams indicates that the photographic minima differ from those determined visually by some 0.5 or 0.6 magnitude. But the assumptions involved in this reduction are, as Williams himself states, of doubtful validity; and it does not appear, therefore, that the two sets of minima are necessarily different in brightness. The question naturally arises as to whether the light-curve is not of the ordinary continuous variation type with a period one-half of that assigned by Williams. Both of his series of observations, considered separately, are satisfied within the limits of error by such a curve; and, since there is no conclusive evidence that the minima of the one series differ from those of the other, a variation of this kind is probable in view of the comparative rarity of variables of the Beta Lyrae type.

That the light-curve is really one of equal minima seems to be proved by observations made by Mr. Haynes and myself during the past year. Five minima, two principal and three secondary as determined by the elements of Williams, have been observed, and in all of these minima the variable was sensibly of the same brightness. One hundred and eighty-four observations were secured upon seventy-two nights. Generally the variable was referred to two comparison stars upon each night in the manner described upon page 108. The comparison stars were

$$a = \text{BD.} + 25°.4301, (8.8), \qquad b = \text{BD.} + 25°.4304, (8.9).$$

The observations are printed in detail in Table VI, in which the first three columns give the date, Greenwich Mean Time, and Julian Day for each observation. The fifth and sixth columns indicate the observer and comparison star; the seventh contains the wedge readings, each of which is the mean of four settings. The eighth column contains the magnitude difference of the comparison stars resulting from each night's observation. This difference is entered only when the sixteen settings on the two comparison stars and the variable were made in a single set as described on page 108. The ninth column gives the magnitude differences for the variable and comparison stars. The results in columns eight and nine were derived from the wedge readings by means of Table V in Bulletin No. 7. The magnitudes for the variable in column ten were obtained by adding the numbers in column nine to the comparison star magnitudes derived below.

TABLE VI

OBSERVATIONS ON V VULPECULAE

Date	G. M. T.	Julian Day	Phase	Obs.	Star	Wedge		$b-a$	Δm	Mag.	O − C
1905, July 16	17h	2417043.7	22.4	S	*a*	11.6	13.0		+ 0.18	8.73	+ 2
				S	*a*	10.6	12.0		+ 0.19	8.74	+ 3
				H	*a*	5.0	7.4		+ 0.28	8.83	+ 12
				H	*a*	5.4	7.5		+ 0.25	8.80	+ 9
17	17	7044.7	23.4	S	*a*	11.8	13.8		+ 0.27	8.82	+ 6
18	16	7045.7	24.4	S	*a*	12.5	13.0		+ 0.06	8.61	— 18
				S	*a*	11.0	12.3		+ 0.17	8.72	— 7
				H	*a*	7.9	8.6		+ 0.08	8.63	— 16
				H	*a*	6.0	7.7		+ 0.20	8.75	— 4
23	18	7050.7	29.4	S	*a*	10.7	12.6		+ 0.25	8 80	— 6
				S	*a*	11.2	12.4		+ 0.15	8.70	— 16
28	15	7055.6	34.3	S	*a*	14.3	16.4		+ 0.28	8.83	— 2
				S	*a*	13.7	15.4		+ 0.23	8.78	— 7
29	16	7056.7	35.4	S	*a*	12.7	15.3		+ 0.35	8.90	+ 11
				S	*a*	12.0	13.8		+ 0.24	8.79	0
31	16	7058.7	37.4	S	*a*	10.4	9.6		— 0.11	8.44	— 14
				S	*a*	10.0	8.6		— 0.18	8.37	— 21
Aug. 1	16	7059.7	0.6	S	*a*	16.3	15.5		— 0.11	8.44	+ 2
				S	*a*	16.0	14.8		— 0.17	8.38	— 4
4	16	7062.7	3.6	S	*a*	11.9	8.8		— 0.41	8.14	— 5
				S	*a*	12.2	9.1		— 0.42	8.13	— 6
7	16	7065.7	6.6	S	*a*	14.6	12.9		— 0.23	8.32	+ 10
				S	*a*	14.6	12.3		— 0.31	8.24	+ 2
11	19	7069.8	10.7	S	*a*	14.0	11.3		— 0.36	8.19	— 7
				S	*a*	13.0	10.0		— 0.40	8.15	— 11
17	15	7075.6	16.5	S	*a*	12.0	10.0		— 0.27	8.28	— 14
				S	*a*	11.8	10.0		— 0 24	8.31	— 11
19	22	7077.9	18.8	S	*a*	21.5	21.6		+ 0.01	8.56	+ 3
				S	*a*	20.4	20.8		+ 0.04	8.59	+ 6
21	16	7079.7	20.5	S	*a*	15.7	17.0		+ 0.17	8.72	+ 11
				S	*a*	16.4	17.2		+ 0.10	8.65	+ 4
23	16	7081.7	22.5	S	*a*	15.4	16.4		+ 0.13	8.68	— 3
				S	*a*	14.2	15.1		+ 0.11	8.66	— 5
25	15	7083.6	24.5	S	*a*	15.2	17.4		+ 0.29	8.84	+ 4
				S	*a*	15.3	17.2		+ 0.25	8.80	0
26	15	7084.6	25.5	S	*a*	19.3	22.2	+ 0.78	+ 0.31	8.86	+ 4
				S	*b*	26.6	22.9		— 0.39	8.85	+ 3
30	15	7088.6	29.5	S	*a*	8.2	10.4	+ 0.86	+ 0.29	8.84	— 2
				S	*b*	14.7	11.8		— 0.39	8.85	— 1

Date	G. M. T.	Julian Day	Phase	Obs.	Star	Wedge		$b-a$	Δm	Mag.	O − C
1905, Aug. 31	14^h	2417089.6	30.5	S	*a*	11.5	14.8	+ 0.91	+ 0.44	8.99	+ 11
				S	*b*	18.4	14.7		− 0.48	8.76	− 12
Sept. 1	18	7090.8	31.7	S	*a*	6.1	9.5		+ 0.41	8.96	+ 7
				S	*a*	6.2	8.6		+ 0.28	8.83	− 6
2	15	7091.6	32.5	S	*a*	11.1	13.5		+ 0.32	8.87	− 2
				S	*a*	10.6	12.7		+ 0.28	8.83	− 6
4	18	7093.7	34.6	S	*a*	19.3	19.4	+ 0.79	+ 0.01	8.56	− 28
				S	*b*	26.7	19.5		− 0.76	8.48	− 36
Oct. 7	16	7126.7	29.7	S	*a*	14.1	17.3		+ 0.43	8.98	+ 11
				S	*a*	13.4	16.3		+ 0.39	8.94	+ 7
10	16	7129.7	32.8	S	*a*	10.0	13.0		+ 0.40	8.95	+ 6
				S	*a*	11.0	13.7		+ 0.36	8.91	+ 2
11	16	7130.7	33.7	S	*a*	14.6	17.1		+ 0.33	8.88	0
				S	*a*	13.2	15.9		+ 0.37	8.92	+ 4
				S	*b*	19.1	14.9		− 0.53	8.71	− 17
12	16	7131.7	34.8	S	*a*	8.7	12.0	+ 0.74	+ 0.44	8.99	+ 16
				S	*b*	14.2	11.3		− 0.39	8.85	+ 2
25	18	7144.7	10.0	S	*a*	18.6	17.4	+ 0.70	− 0.14	8.41	+ 16
				S	*b*	25.2	16.8		− 0.92	8.32	+ 7
1906, June 12	17	7374.7	13.3	S	*a*	25.2	22.2	+ 0.60	− 0.31	8.24	− 7
				S	*b*	30.6	23.1		− 0.81	8.43	+ 12
				H	*a*	23.8	20.4	+ 0.54	− 0.36	8.19	− 12
				H	*b*	28.8	20.6		− 0.88	8.36	+ 5
17	18	7379.7	18.3	S	*a*	16.7	16.0	+ 0.71	− 0.09	8.46	− 4
				S	*b*	23.0	16.6		− 0.72	8.52	+ 2
				H	*a*	17.2	18.1	+ 1.00	+ 0.11	8.66	+ 16
				H	*b*	26.4	18.4		− 0.85	8.39	− 11
19	17	7381.7	20.3	S	*a*	9.8	10.8	+ 0.94	+ 0.14	8.69	+ 8
				S	*b*	16.8	11.6		− 0.69	8.55	− 6
				H	*a*	11.4	13.0	+ 0.87	+ 0.21	8.76	+ 15
				H	*b*	18.0	12.8		− 0.69	8.55	− 6
21	18	7383.7	22.3	S	*a*	16.2	16.9	+ 0.63	+ 0.09	8.64	− 7
				S	*b*	21.7	16.7		− 0.57	8.67	− 4
				H	*a*	14.5	15.4	+ 0.69	+ 0.13	8.68	− 3
				H	*b*	20.0	14.8		− 0.65	8.59	− 12
23	20	7385.8	24.4	S	*a*	15.8	18.7	+ 0.92	+ 0.36	8.91	+ 12
				S	*b*	23.9	19.3		− 0.49	8.75	− 4
				H	*a*	15.3	18.1	+ 0.85	+ 0.36	8.91	+ 12
				H	*b*	22.5	17.6		− 0.55	8.69	− 10
25	18	7387.8	26.4	S	*a*	17.6	19.6	+ 0.69	+ 0.24	8.79	− 4
				S	*b*	23.9	18.9		− 0.53	8.71	− 12

Date	G. M. T.	Julian Day	Phase	Obs.	Star	Wedge		$b-a$	Δm	Mag.	O − C
1906, June 25	18h	2417387.8	26.4	H	*a*	16.9	19.6	+ 0.84	+ 0.32	8.87	+ 4
				H	*b*	24.6	20.2		− 0.46	8.78	− 5
27	17	7389.7	28.3	S	*a*	13.1	15.4	+ 0.92	+ 0.32	8.87	+ 2
				S	*b*	20.4	16.3		− 0.48	8.76	− 9
				H	*a*	8.3	10.2	+ 0.61	+ 0.24	8.79	− 6
				H	*b*	13.0	10.0		− 0.40	8.84	− 1
July 1	17	7393.7	32.3	S	*a*	12.2	14.9	+ 0.74	+ 0.36	8.91	+ 2
				S	*b*	17.8	14.7		− 0.41	8.83	− 6
				H	*a*	12.2	15.6	+ 0.72	+ 0.45	9.00	+ 11
				H	*b*	17.7	15.7		− 0.25	8.99	+ 10
3	16	7395.7	34.3	S	*a*	13.0	15.6	+ 0.89	+ 0.35	8.90	+ 5
				S	*b*	20.0	15.5		− 0.55	8.69	− 16
				H	*a*	14.1	16.1	+ 0.93	+ 0.27	8.82	− 3
				H	*b*	21.8	16.3		− 0.63	8.61	− 24
8	15	7400.7	1.5	H	*a*	13.2	10.6	+ 0.73	− 0.35	8.20	− 8
				H	*b*	18.8	10.3		− 1.12	8.12	− 16
				S	*a*	16.2	13.1	+ 0.68	− 0.43	8.12	− 16
				S	*b*	22.2	12.7		− 1.16	8.08	− 20
10	16	7402.7	3.5	H	*a*	13.2	9.4	+ 0.68	− 0.51	8.04	− 14
				H	*b*	18.4	9.5		− 1.17	8.07	− 11
12	16	7404.7	5.5	S	*a*	18.8	14.7	+ 0.68	− 0.53	8.02	− 18
				S	*b*	25.3	14.2		− 1.27	7.97	− 23
14	15	7406.6	7.4	H	*a*	12.6	10.6	+ 0.86	− 0.27	8.28	+ 5
				H	*b*	19.3	10.3		− 1.17	8.07	− 16
16	15	7408.6	9.4	S	*a*	24.9	20.6	+ 0.45	− 0.45	8.10	− 15
				S	*b*	29.0	19.8		− 0.98	8.26	+ 1
18	15	7410.6	11.4	H	*a*	18.5	17.0	+ 0.51	− 0.18	8.37	+ 10
				H	*b*	23.2	16.0		− 0.82	8.42	+ 15
20	16	7412.7	13.5	S	*a*	17.1	14.8	+ 0.66	− 0.31	8.24	− 7
				S	*b*	23.0	15.6		− 0.86	8.38	+ 7
23	17	7415.7	16.5	H	*a*	20.6	19.7	+ 0.54	− 0.09	8.46	+ 4
				H	*b*	25.7	18.5		− 0.77	8.47	+ 5
24	15	7416.6	17.4	S	*a*	20.7	19.3	+ 0.68	− 0.15	8.40	− 6
				S	*b*	27.1	18.6		− 0.91	8.33	− 13
26	15	7418.6	19.4	H	*a*	28.0	27.3		− 0.08	8.47	− 9
				H	*b*	31.9	24.4		− 0.83	8.41	− 15
28	15	7420.6	21.4	S	*a*	17.9	18.1	+ 0.59	+ 0.02	8.57	− 9
				S	*b*	23.3	19.1		− 0.45	8.79	+ 13
30	16	7422.7	23.5	H	*a*	23.2	25.0	+ 0.41	+ 0.18	8.73	− 3
				H	*b*	27.1	23.2		− 0.41	8.83	+ 7
31	15	7423.6	24.4	H	*a*	17.4	19.0	+ 0.52	+ 0.19	8.74	− 5
				H	*b*	22.1	19.0		− 0.30	8.94	+ 15

Date	G. M. T.	Julian Day	Phase	Obs.	Star	Wedge		$b-a$	Δm	Mag.	O − C
1906, Aug. 1	15h	2417424.6	25.4	S	*a*	21.9	24.3	+ 0.61	+ 0.25	8.80	− 2
				S	*b*	27.6	24.1		− 0.38	8.86	+ 4
	20	7424.8	25.6	H	*a*	18.5	22.2	+ 0.77	+ 0.40	8.95	+ 13
				H	*b*	25.7	21.7		− 0.42	8.82	0
2	15	7425.6	26.4	S	*a*	25.8	28.7		+ 0.32	8.87	+ 4
				S	*b*	33.2	28.2		− 0.56	8.68	− 15
3	15	7426.6	27.4	H	*a*	21.8	25.2	+ 0.67	+ 0.35	8.90	+ 6
				H	*b*	28.1	25.3		− 0.31	8.93	+ 9
5	15	7428.6	29.4	S	*a*	23.4	26.1		+ 0.28	8.83	− 3
				S	*b*	28.4	26.0		− 0.26	8.98	+ 12
7	14	7430.6	31.4	H	*a*	19.2	21.0	+ 0.66	+ 0.19	8.74	− 14
				H	*b*	25.5	20.8		− 0.49	8.75	− 13
				S	*a*	19.5	21.8	+ 0.53	+ 0.24	8.79	− 9
				S	*b*	24.6	22.3		− 0.24	9.00	+ 12
8	14	7431.6	32.4	S	*a*	21.3	23.7	+ 0.56	+ 0.26	8.81	− 8
				S	*b*	26.5	24.0		− 0.27	8.97	+ 8
9	15	7432.6	33.4	H	*a*	19.0	22.6	+ 0.58	+ 0.39	8.94	+ 6
				H	*b*	24.5	22.6		− 0.19	9.05	+ 17
				S	*a*	20.4	22.8	+ 0.63	+ 0.26	8.81	− 7
				S	*b*	26.4	24.0		− 0.25	8.99	+ 11
10	14	7433.6	34.4	S	*a*	25.4	28.6	+ 0.72	+ 0.36	8.91	+ 6
				S	*b*	31.8	28.9		− 0.33	8.91	+ 6
11	16	7434.7	35.5	H	*a*	23.4	24.8	+ 0.59	+ 0.14	8.69	− 9
				H	*b*	28.9	24.9		− 0.44	8.80	+ 2
				S	*a*	24.4	27.1	+ 0.64	+ 0.29	8.84	+ 6
				S	*b*	30.3	26.1		− 0.46	8.78	0
12	14	7435.6	36.4	H	*a*	27.6	29.3	+ 0.85	+ 0.18	8.73	+ 2
				H	*b*	35.3	29.4		− 0.66	8.58	− 13
	15	.6	36.4	H	*a*	27.5	29.3	+ 0.69	+ 0.20	8.75	+ 4
				H	*b*	33.6	29.6		− 0.45	8.79	+ 8
13	14	7436.6	37.4	H	*a*	22.8	23.1	+ 0.68	+ 0.03	8.58	0
				H	*b*	29.2	23.7		− 0.59	8.65	+ 7
14	14	7437.6	0.6	S	*a*	29.7	27.4	+ 0.37	− 0.26	8.29	− 13
				S	*b*	33.0	28.4		− 0.52	8.72	+ 30
15	14	7438.6	1.6	H	*a*	20.6	18.7	+ 0.46	− 0.21	8.34	+ 7
				H	*b*	25.0	18.6		− 0.68	8.56	+ 29
16	15	7439.6	2.6	S	*a*	31.8	29.4	+ 0.62	− 0.28	8.27	+ 7
				S	*b*	37.5	29.5		− 0.89	8.35	+ 15
				H	*a*	28.8	26.0	+ 0.62	− 0.31	8.24	+ 4
				H	*b*	34.4	25.6		− 0.97	8.27	+ 7
17	16	7440.7	3.6	S	*a*	15.8	13.2	+ 0.76	− 0.35	8.20	+ 1
				S	*b*	22.4	13.6		− 1.06	8.18	− 1

Date	G. M. T.	Julian Day	Phase	Obs.	Star	Wedge		$b-a$	Δm	Mag.	O − C
1906, Aug. 18	14	2417441.6	4.6	H	*a*	20.6	18.3	+ 0.47	— 0.25	8.30	+ 10
				H	*b*	25.1	18.6		— 0.69	8.55	+ 35
20	15	7443.6	6.6	S	*a*	18.5	15.5	+ 0.48	— 0.38	8.17	— 5
				S	*b*	22.9	16.1		— 0.78	8.46	+ 24
	18	7443.7	6.7	H	*a*	16.4	15.4	+ 0.73	— 0.13	8.42	+ 20
				H	*b*	22.8	15.7		— 0.82	8.42	+ 20
21	15	7444.6	7.6	S	*a*	15.5	13.1	+ 0.69	— 0.33	8.22	— 2
				S	*b*	21.4	12.5		— 1.09	8.15	— 9
22	14	7445.6	8.6	H	*a*	15.4	13.1	+ 0.73	— 0.32	8.23	— 2
				H	*b*	21.6	13.4		— 1.00	8.24	— 1
23	19	7446.8	9.8	H	*a*	12.8	10.7	+ 0.90	— 0.27	8.28	+ 3
				H	*b*	19.8	10.9		— 1.14	8.10	— 15
26	14	7449.6	12.6	S	*a*	19.6	15.9	+ 0.60	— 0.45	8.10	— 19
				S	*b*	25.4	16.1		— 1.03	8.21	— 8
27	14	7450.6	13.6	H	*a*	26.9	24.6	+ 0.74	— 0.25	8.30	— 2
				H	*b*	33.5	25.0		— 0.95	8.29	— 3
29	14	7452.6	15.6	H	*a*	18.3	17.1	+ 0.67	— 0.15	8.40	+ 2
				H	*b*	24.6	17.5		— 0.77	8.47	+ 9
	15		15.6	S	*a*	26.0	24.0	+ 0.50	— 0.21	8.34	— 4
				S	*b*	30.5	23.2		— 0.79	8.45	+ 7

The period was determined by a comparison of the instants for which the light of the variable, when increasing, was equal to *a*. The variation at this time is rapid, and the instant mentioned can be determined with an uncertainty of not much more than half a day. From the observations of Table VI we find J. D. 2417437.0. Other values, less accurately determined, are also given by these observations, but they were not used in the evaluation of the period. The curve by Williams in A. N. No. 3929 gives J. D. 2416416.7 and 2416492.3. The resulting period is 37.79 days, with an uncertainty of 0.05 or 0.06 of a day.

To determine the mean light-curve, the observations were combined into eleven normal places, the results for which are given in Table VII. The phase value required for the combination, counted from the point on the ascending branch of the curve for which $v = a$, is found in the fourth column of Table VI. The ordinates of the light-curve are given in Table VIII, and the curve itself is shown in Fig. 2. The deviation of the normal places from the mean light-curve is given in the last column of Table VII, and the residuals for the individual observations, in hundredths of a magnitude, in the last column of Table VI.

From the mean light-curve it is found that the minima precede the zero points adopted for the phase values by 5.3 days, whence we find for the first minimum observed visually by Williams: J. D. 2416411.4. The elements of the variable are therefore

$$\text{Min.} = \text{J. D. } 2416411.4 + 37^{d}.79E, \qquad \text{G. M. T.}$$

Table IX contains the results of a comparison of these elements with the minima observed by Williams. The brightness of star *a* was determined by a reference to four PD. stars. The results of this comparison are shown in Table X, which is similar in arrangement to Table IV.

TABLE VII

NORMAL PLACES FOR V VULPECULAE

No.	Phase	Mag.	Nights	Obs.	O – C
	d				
1	0.3	8.46	7	18	0.00
2	3.9	8.19	6	14	0.00
3	7.7	8.24	6	14	0.00
4	11.9	8.28	8	18	0.00
5	17.2	8.44	7	16	— 0.01
6	21.3	8.65	7	20	0.00
7	24.2	8.81	8	21	+ 0.02
8	27.6	8.84	5	14	— 0.01
9	30.3	8.86	6	14	— 0.01
10	32.9	8.90	6	17	+ 0.01
11	34.8	8.84	6	16	+ 0.01

TABLE VIII

MEAN LIGHT-CURVE FOR V VULPECULAE

Phase	Mag.	Phase	Mag.
0d	8.55	19d	8.54
1	8.35	22	8.69
2	8.23	25	8.81
3	8.18	28	8.85
4	8.19	31	8.88
7	8.23	34	8.87
10	8.25	35	8.82
13	8.30	36	8.75
16	8.40	37	8.65
19	8.54	38	8.48

Sixty-six values of $b-a$ are available for the determination of the magnitude difference of the comparison stars, 34 by Seares and 32 by Haynes. The mean for the former is $+ 0^{m}.684 \pm 0^{m}.016$; for the latter, $+ 0^{m}.691 \pm 0^{m}.017$; and for the entire series, $+ 0^{m}.69 \pm 0^{m}.012$, where the appended quantities are the probable errors. The combination of the final mean for $b - a$ with the magnitude of *a* gives for the brightness of *b* 9.24 magnitudes.

Table XI shows the mean deviation from the light-curve when the observations are grouped chronologically. Column one gives the Julian Dates which limit the observations combined; columns three and four, the number of nights and observations in each group; and column five,

TABLE IX

REPRESENTATION OF WILLIAMS' MINIMA

E	Observed	Computed	O – C
— 39	J. D. 2414937	4937.6	— 0.6
— 28	5355	5353.3	+ 1.7
— 20	5659	5655.6	+ 3.4
— 18	5734	5731.2	+ 2.8
0	6410	6411.4	— 1.4
+ 2	6487	6487.0	0.0

TABLE X

BRIGHTNESS OF COMPARISON STAR *a*

Star	PD. II.	PD. Mag.	Δm		Mean Δm	Mag. *a*	No. Obs.
			S	H			
p	3716	7.36	1.36	1.21	1.28	8.64	1, 1
q	3689	7.17	1.59	1.39	1.49	8.66	1, 1
r	3699	6.68	1.82	1.81	1.82	8.50	3, 3
s	3715	7.23	1.27	1.31	1.29	8.52	3, 3
					Mean	8.55	

TABLE XI

SYSTEMATIC DEVIATION OF OBSERVATIONS

No.	Julian Dates	Nights	Obs.	O – C
1	2417043 – 060	8	19	— 0.04
2	061 – 080	6	12	— 0.02
3	081 – 093	8	16	— 0.04
4	126 – 132	4	9	+ 0.03
5	379 – 395	8	32	— 0.02
6	400 – 418	10	22	— 0.07
7	420 – 434	12	32	+ 0.02
8	435 – 453	14	36	+ 0.04

the systematic deviation of each group from the curve. The numbers in this column would have been appreciably smaller had three or four of the most discordant observations been rejected.

Groups 1, 3, 4, 5, and 7 cover minima, of which Nos. 3 and 5 are principal, and Nos. 1, 4, and 7 secondary minima, on the basis of the elements of Williams. It is evident that there is no sensible difference between the minima observed.

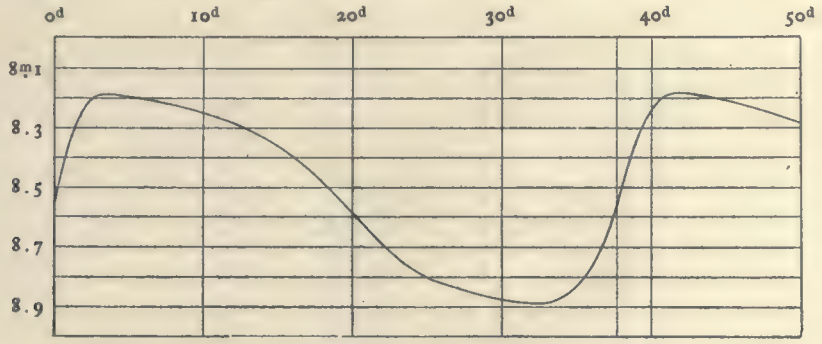

Fig. 2 — Light-curve of V Vulpeculae.

The precision of the results is indicated by Table XII which gives for each observer the average residual for a single observation and the average systematic deviation of the observations referred to each of the comparison stars. The last line shows the number of observations involved in the determination of the various residuals.

TABLE XII

AVERAGE RESIDUALS AND SYSTEMATIC DEVIATIONS

	Seares		Haynes	
	a	*b*	*a*	*b*
Average Residual	0m.075	0m.105	0m.077	0m.113
Average Systematic Deviations . .	— 0.020	— 0.010	+ 0.018	+ 0.008
Number of Observations	77	37	37	33

That the residuals for the observations referred to *b* are larger than those referred to *a* is probably due to the proximity of a 6th-magnitude star which rendered the settings on *b* somewhat difficult, and to the fact that the average magnitude difference measured is greater for *b* than for *a*. The average systematic difference for the two observers is

S – H = — 0m.038, from observations referred to *a*,
S – H = — 0.018, from observations referred to *b*.

For star *a* the observing conditions are very similar to those existing in the case of the variable 88.1906; and the systematic differences are sensibly the same for the two cases. Finally, the difference for the two observers, derived from the values of $b-a$, is

$$S - H = -0^{m}.007.$$

The probable error for a single observation derived from the entire series is in the neighborhood of ± 0.07 magnitude, but as in the case of BD. $+55^{\circ}.2817$, this result is not free from systematic errors.

PRELIMINARY NOTE ON VARIABLE 108.1905 CAPRICORNI

1855.0	$20^{h}53^{m}27^{s}$	$-15^{\circ}46'.0$
1900.0	55 55.14	37 6".1[1]

Astronomische Nachrichten No. 4053 contains an article by Götz concerning this star, in which it is stated that the variation is probably of the Algol type with intermediate minima and a period of 3.25 days. Observations made by Mr. Haynes and myself show that it is not of the Algol type, but has a light curve very similar to that given by Enebo in A. N. No. 4094 for 76.1905 Cygni, and that the period is approximately $10^{h}44^{m}.6$. Maxima have been observed on the following approximate geocentric dates:

G. M. T.	Julian Day
1906, Aug. 13 $20^{h}.9$	2417436.87
14 18.2	437.76
15 15.8	438.66
27 17.8	450.74
28 15.4	451.64

The rise to maximum is rapid, an increase of 1½ magnitudes occupying only 1¾ hours. The fall from maximum through the same amount requires about five hours. For the remainder of the cycle the light is nearly if not quite constant, so that the question of antalgol variation is raised here as in the case of 76.1905. At maximum the variable is about 0.1 magnitude fainter than BD. $-15^{\circ}.5863$, (9.1). The following are approximate elements:

$$\text{Max.} = \text{J. D. } 2417436.87 + 0^{d}.4476E, \qquad \text{G. M. T.}$$

An ephemeris, giving the Greenwich Mean Time of every tenth maximum, is appended in Table XIII.

[1] Position from A. N. No. 4053.

TABLE XIII

EPHEMERIS OF MAXIMA FOR 108.1905 CAPRICORNI

E	G. M. T.	J. D.	E	G. M. T.	J. D.
72	1906, Sept. 15^{d} 2^{h}.4	2417469.10	202	1906, Nov. 12^{d} 6^{h}.7	2417527.28
82	19 13.7	473.57	212	16 18.2	531.76
92	24 1.2	478.05	222	21 5.8	536.24
102	28 12.5	482 52	232	25 17.0	540.71
112	Oct. 3 0.0	487.00	242	30 4.6	545.19
122	7 11.5	491.48	252	Dec. 4 15.8	549.66
132	11 22.8	495.95	262	9 3.4	554.14
142	16 10.3	500.43	272	13 14.9	558.62
152	20 21.6	504.90	282	18 2.2	563.09
162	25 9.1	509.38	292	22 13.7	567.57
172	29 20.6	513.86	302	27 1.0	572.04
182	Nov. 3 7.9	518.33	312	31 12.5	576.52
192	7 19.4	522.81	322	1907, Jan. 5 0.0	581.00

Columbia, Missouri, 1906, Sept. 5. *F. H. Seares.*

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BULLETIN NO. 9

THE ALGOL VARIABLE RR DRACONIS (188.1904)[1]

1900.0 R.A. $18^h40^m48^s.62$ Dec. $+ 62°34'32''.2$[2]
1855.0 40 32 62 31.1

A preliminary announcement concerning the character of variation and period of the variable RR Draconis was made in Laws Observatory Bulletin No. 6. Since then further observations have been secured, and it is now possible to state the period with some precision and to exhibit the characteristic features of the greater part of the light-curve. That the complete light-curve cannot be given is due to the fact that the variable is invisible in the 7½-inch equatorial for a period of about two hours in the neighborhood of the minimum. Enough has been learned, however, to show that the star is one of unusual interest, for the variation is apparently more than three magnitudes, and the rate of change, at the time of disappearance, a magnitude in half an hour. These figures mark the object as an extreme example of the class to which it belongs, and distinguish it, therefore, as one worthy of special attention. During the greater part of the interval of variation the star is too difficult an object for precise observation with the equipment of the Laws Observatory. When it was placed upon the observing list, this could not be foreseen, as the character of the variation was then unknown. When the nature of the light change became evident, it seemed desirable to continue the observations for a time in order to secure data for the proper reduction of the measures already obtained. Since the majority of the observations were made when the variable was fainter than magnitude ten and one-half, the errors are considerably larger than those affecting measures secured under more favorable conditions.

The material available for the present investigation is distributed over thirty-three nights, and consists of two hundred and sixty-five observations, ninety-five of which fall within the in-

[1] Through the courtesy of the Trustees of the Benjamin Apthorp Gould Fund, a sum of five hundred dollars has been placed at the disposal of the Laws Observatory to be used in securing photometric observations of variable stars. This grant has made possible those observations in the following pages which were obtained by Mr. E. S. Haynes subsequently to June 1, 1906. Mr. Haynes has also rendered assistance, under the same grant, in the preparation of this bulletin for publication.

[2] The position is by Ebell, *Astronomische Nachrichten*, v. 167, p. 41.

tervals of constant light. All of the observations were made with the equalizing wedge photometer in connection with the 7½-inch equatorial. The star BD. +62°1639, 9.3 mag., was used as comparison object throughout. Its brightness was determined by a reference to two Potsdam DM. stars. The details of the comparison are shown in Table I, which is similar in arrangement to tables in former bulletins. The resulting magnitude for the comparison star is 9.64.

The observations during constant light are summarized in Table II. Column one gives the date; column two, the limiting G.M. Times within which the measures are included; column three, the Julian Date, with the limiting times expressed in decimals of a day of G.M. Time; column four, the number of observations; and column five, the mean magnitude difference between the variable and the comparison star for each night. The weighted mean magnitude dif-

TABLE I

MAGNITUDE OF COMPARISON STAR *a*

Star	BD. No.	PD. Mag.	Δm		Mean Δm	Mag. *a*	No. Obs.
			S	H			
p	+62°1629	7.53	2.15	2.06	2.10	9.63	2, 3
q	+62°1674	7.10	2.59	2.53	2.55	9.65	2, 3
					Mean	9.64	

ference for all the nights is 0.34, whence we have for the normal brightness of the variable, 9.98 mags. The deviations of the means by nights from the final mean magnitude difference are given in column six, and the average residual for each night, referred to the mean of the night, in the last column. The average residual for a single observation referred to the mean of the series is 0.092 mag., while that referred to the means of the nights is only 0.057 mag. The difference between these residuals is surprisingly large. Two explanations are possible: Either the measures of the different nights are affected by large systematic errors of observation, or the star undergoes a minute variation during what has been assumed to be a period of constant light. Small errors peculiar to a night are undoubtedly present, but it is doubtful if they are capable of explaining the discrepancy in question, for a comparison with the residuals of observations made during light change renders the former alternative improbable. Furthermore, the evidence resulting from an arrangement of the observations in order of phase is distinctly in favor of the latter. But the variation indicated is so slight, not exceeding a tenth of a magnitude, that its reality cannot at present be asserted.

The one hundred and seventy observations during light change are printed in detail in Table III. Fourteen observations on 1905, July 16, made immediately preceding those in the table under this date, have been rejected on account of unusual irregularities; and, as will be seen later, there is reason for believing that the balance of the measures for this date is affected by a considerable systematic error. The star was faint during the earlier observations, and the

atmospheric conditions were bad throughout the entire series. For the most part, the arrangement of Table III is obvious. The phase values in column four are counted from the nearest geocentric minimum given by the elements on page 135; they are positive for increasing light, and negative for diminishing. Each of the values in the column of wedge readings is the mean of four settings, and in all cases the first number refers to the comparison star, the second to the

TABLE II

OBSERVATIONS DURING CONSTANT LIGHT

Date	G. M. T.	Julian Date	No.Obs.	Δm	v_1	v_2
1905, June 24	$15^h.0 - 15^h.4$	2417021.63 – .64	4	+ 0.15	+ 0.19	4
July 3	15.3 – 15.7	030.64 – .65	3	0.26	+ 0.08	15
12	17.5 – 17.8	039.73 – .74	3	0.24	+ 0.10	2
14	14.5 – 21.4	041.61 – .89	24	0.37	— 0.03	8
15	14.5 – 19.0	042.61 – .79	12	0.35	— 0.01	7
17	14.5 – 14.7	044.61 – .61	2	0.40	— 0.06	0
18	15.0 – 15.4	045.63 – .64	4	0.42	— 0.08	10
22	14.6 – 14.9	049.61 – .62	3	0.29	+ 0.05	1
23	15.7 – 18.6	050.66 – 77	8	0.48	— 0.14	6
25	14.7 – 15.3	052.61 – .64	4	0.33	+ 0.01	4
28	17.2 – 17.4	055.72 – .72	2	0.46	— 0.12	5
29	14.5 – 14.6	056.60 – .61	2	0.26	+ 0.08	0
31	14.5 – 15.2	058.60 – .63	4	0.32	+ 0.02	6
Aug. 1	15.2 – 15.3	059.63 – .64	2	0.32	+ 0.02	0
3	14.5 – 14.6	061.60 – .61	2	0.30	+ 0 04	2
5	18.3 – 18.4	063.76 – .77	2	0.23	+ 0.11	2
7	15.2 – 19.6	065.63 – .82	4	0.20	+ 0.14	3
11	17.4 – 17.5	069.72 – .73	2	0.27	+ 0.07	3
14	17.8 – 17.9	074.74 – .74	2	0.28	+ 0.06	0
25	18.5 – 18.6	083.77 – .78	2	0.29	+ 0.05	7
26	17.8 – 17.9	084.74 – .74	2	0.35	— 0.01	7
1906, Apr. 17	16.6 – 16.7	318.69 – .69	2	+ 0.24	+ 0.10	2
		Means	4 +	+ 0.34	0.062	0.057

variable. The magnitude differences were derived by means of Table V, Bulletin No. 7; the magnitudes in column eight, by adding the values of Δm to the magnitude of the comparison star given above. The calculation of the residuals in columns nine and ten will be explained later.

The observations made during light change are distributed over eleven different nights. Upon five of these both branches of the light-curve were more or less completely observed. The results for 1905, June 29, July 16, July 19, and Aug. 5, are fragmentary and of no value for the determination of the period. A direct determination of the instants of minima is impossible on

account of the faintness of the star; but even were this possible, a comparison of the abscissæ of selected points on the steeply inclined branches of the curve would doubtless give more precise results for the period. Such, in fact, was the method used. The observations of each night were plotted. An inspection of the resulting curves showed that in nearly every case they are well determined for the ordinate Δm=2.00 mags. Points having these ordinates were accordingly used for the comparison. The corresponding abscissæ read from the curves are given in columns two and three of Table IV, expressed in Julian days and decimals of G. M.

TABLE III

OBSERVATIONS DURING LIGHT CHANGE

Date	G. M. T.	Julian Day	Phase	Obs.	Wedge		Δm	Mag.	v_1	v_2
1905, June 29	14h 51m	2417026.619	— 0.066	S	17.3	34.6	+ 1.90	11.88	— 10	— 10
	59	.624	— 0.061	S	17.0	37.3	2.23	12.21	+ 3	+ 3
July 13	14 51	040.619	— 0.221	S	9.0	12.0	0.40	10.38	+ 5	+ 6
	57	.623	— 0.217	S	8.2	12.2	0.53	10.51	+ 17	+ 18
	15 7	.630	— 0.210	S	8.7	11.3	0.35	10.33	— 2	— 2
	16 34	.690	— 0.150	S	26.3	30.3	0.44	10.42	— 26	— 27
	40	.694	— 0.146	H	18.2	23.8	0.61	10.59	— 12	— 15
	43	.696	— 0.144	H	17.3	24.5	0.78	10.76	+ 4	□
	50	.701	— 0.139	S	22.8	31.0	0.88	10.86	+ 9	+ 7
	56	.705	— 0.135	S	23.6	31.3	0.84	10.82	+ 2	— 3
	17 3	.710	— 0.130	H	15.6	23.7	0.93	10.91	+ 6	0
	6	.712	— 0.128	H	16.5	25.8	1.02	11.00	+ 13	+ 6
	16	.719	— 0.121	S	20.7	31.0	1.11	11.09	+ 15	+ 6
	21	.723	— 0.117	S	21.3	31.6	1.12	11.10	+ 11	+ 1
	30	.729	— 0.111	H	16.4	26.2	1.08	11.06	0	— 13
	34	.732	— 0.108	H	16.5	29.1	1.39	11.37	+ 27	+ 11
	47	.741	— 0.099	S	27.2	38.5	1.24	11.22	0	— 23
	54	.746	— 0.094	S	26.3	40.3	1.53	11.51	+ 20	+ 9
	18 6	.754	— 0.086	H	24.5	41.6	1.87	11.85	+ 40	+ 3
	12	.758	— 0.082	H	26.4	45.2	2.08	12.06	+ 53	+ 11
16	19 34	043.815	+ 0.144	S	15.2	22.7	0.88	10.86	+ 14	— 6
	42	.821	+ 0.150	S	15.4	22.2	0.79	10.77	+ 9	— 7
	50	.826	+ 0.155	H	9.6	16.7	0.95	10.93	+ 29	+ 14
	54	.829	+ 0.158	H	9.1	16.1	0.94	10.92	+ 30	+ 15
	20 2	.835	+ 0.164	S	15.5	20.0	0.55	10.53	— 5	— 18
	7	.838	+ 0.167	S	15.1	20.7	0.68	10.66	+ 10	— 1
	16	.844	+ 0.173	H	9.8	14.9	0.69	10.67	+ 15	+ 5
	20	.847	+ 0.176	H	9.2	14.0	0.64	10.62	+ 12	+ 2
	32	.856	+ 0.185	S	14.8	19.3	0.57	10.55	+ 10	+ 2
	41	.862	+ 0.191	S	15.0	18.3	+ 0.43	10.41	— 1	— 8

Date	G. M. T.	Julian Day	Phase	Obs.	Wedge		Δm	Mag.	v_1	v_2
1905, July 16	20 50	2417043.868	+ 0.197	S	15.2	19.5	+ 0.52	10.50	+ 11	+ 5
	21 5	.878	+ 0.207	S	16.9	20.1	0.37	10.35	— 1	— 5
19	14 47	046.616	+ 0.114	S	27.1	34.8	0.86	10.84	— 18	— 8
Aug. 5	14 40	063.611	+ 0.122	S	22.8	28.2	0.57	10.55	— 38	— 27
	54	.621	+ 0.132	S	23.6	29.1	0.59	10.57	— 26	— 16
	15 6	.629	+ 0.140	S	22.6	28.6	0.64	10.62	— 14	— 4
	14	.635	+ 0.146	S	24.6	30.8	0.68	10.66	— 5	+ 6
19	14 8	077.589	— 0.055	S	23.6	46.0	2.46	12.44	+ 2	+ 19
	17 4	.711	+ 0.067	S	17.8	39.4	2.34	12.32	+ 38	+ 21
	11	.716	+ 0.072	S	19.3	36.8	1.91	11.89	+ 11	— 3
	18	.721	+ 0.077	S	19.2	35.8	1.81	11.79	+ 14	+ 3
	28	.728	+ 0.084	S	20.4	34.3	1.51	11.49	+ 1	— 9
	38	.735	+ 0.091	S	19.6	32.8	1.43	11.41	+ 5	+ 3
	46	.740	+ 0.096	S	18.9	31.3	1.34	11.32	+ 4	+ 4
	52	.744	+ 0.100	S	18.7	29.9	1.21	11.19	— 1	— 2
	18 0	.750	+ 0.106	S	18.4	28.6	1.10	11.08	— 4	— 2
	8	.756	+ 0.112	S	17.7	26.7	0.98	10.96	— 9	— 4
	17	.762	+ 0.118	S	19.3	27.5	0.87	10.85	— 13	— 7
	18 28	.769	+ 0.125	S	18.3	26.4	0.86	10.84	— 6	0
	36	.775	+ 0.131	S	18.9	26.2	0.77	10.75	— 9	— 2
	46	.782	+ 0.138	S	17.0	23.0	0.67	10.65	— 13	— 6
	59	.791	+ 0.147	S	19.0	25.4	0.67	10.65	— 5	+ 3
	19 10	.798	+ 0.154	S	19.6	25.5	0.61	10.59	— 5	+ 2
	20	.805	+ 0.161	S	19.7	25.5	0.60	10.58	— 1	+ 6
	28	.811	+ 0.167	S	18.8	24.4	0.59	10.57	+ 2	+ 9
	37	.817	+ 0.173	S	19.4	24.1	0.50	10.48	— 4	+ 4
	50	.826	+ 0.182	S	17.4	20.9	0.39	10.37	— 9	— 3
	56	.830	+ 0.186	S	17.5	20.4	0.33	10.31	— 13	— 7
	20 3	.835	+ 0.191	S	17.4	20.4	0.34	10.32	— 10	— 5
	17	.845	+ 0.201	S	17.2	20.7	0.40	10.38	0	+ 4
Sept. 2	16 32	091.689	— 0.110	S	15.7	23.3	0.87	10.85	— 22	— 10
	39	.694	— 0.105	S	14.0	23.6	1.13	11.11	— 2	+ 9
	48	.700	— 0.099	S	12.9	23.2	1.24	11.22	0	+ 11
	17 3	.710	— 0.089	S	15.4	27.5	1.35	11.33	— 7	+ 3
	7	.713	— 0.086	S	14.4	26.4	1.37	11.35	— 10	— 1
	14	.718	— 0.081	S	13.8	29.4	1.78	11.76	+ 21	+ 28
	20	.722	— 0.077	S	13.4	29.6	1.86	11.84	+ 19	+ 24
	34	.732	— 0.067	S	13.4	31.4	2.06	12.04	+ 10	+ 16
	43	.738	— 0.061	S	18.0	37.9	2.17	12.15	— 3	+ 4
	52	.744	— 0.055	S	18.2	38.6	2.22	12.20	— 22	— 12
	18 0	.750	— 0.049	S	18.5	40.3	2.36	12.34	— 34	— 25
	12	.758	— 0.041	S	18.4	43.2	+ 2.71	12.69	— 43	— 32

Date	G. M. T.	Julian Day	Phase	Obs.	Wedge		Δm	Mag.	v_1	v_2
1905, Oct. 26	12 12	2417145.508	— 0.082	S	27.4	42.2	+ 1.63	11.61	+ 9	+ 21
	18	.512	— 0.078	S	25.9	41.8	1.75	11.73	+ 11	+ 22
	22	.515	— 0.075	S	24.4	40.8	1.79	11.77	+ 6	+ 16
	27	.519	— 0.071	S	25.5	40.5	1.64	11.62	— 20	— 8
	32	.522	— 0.068	S	25.7	42.4	1.84	11.82	— 9	+ 3
	36	.525	— 0.065	S	27.6	46.8	2.12	12.10	+ 8	+ 22
	41	.528	— 0.062	S	28.2	45.9	1.96	11.94	— 20	— 4
	45	.531	— 0.059	S	27.6	46.1	2.04	12.02	— 24	— 8
	51	.535	— 0.055	S	29.0	48.9	2.21	12.19	— 24	— 5
	13 0	.542	— 0.048	S	11.8	35.3	2.71	12.69	— 5	+ 13
	5	.545	— 0.045	S	13.8	36.2	2.54	12.52	— 40	— 16
	12	.550	— 0.040	S	14.3	41.4	3.03	13.01	— 17	+ 4
	15 26	.643	+ 0.053	S	7.0	30.8	2.81	12.79	+ 28	+ 11
	32	.647	+ 0.057	S	7.0	28.2	2.52	12.50	+ 19	0
	38	.651	+ 0.061	S	8.7	28.2	2.33	12.31	+ 13	— 1
	43	.655	+ 0.065	S	8.2	26.4	2.19	12.17	+ 15	— 1
1906, Aug. 22	14 34	445.607	— 0.074	S	24.4	42 2	1.95	11.93	+ 21	+ 17
	40	.611	— 0.070	H	23.6	43.3	2.16	12.14	+ 29	+ 26
	45	.615	— 0.066	S	25.3	44.4	2.11	12.09	+ 11	+ 6
	50	618	— 0.063	H	23.5	44.0	2.25	12.23	+ 14	+ 9
	56	.622	— 0.059	S	24.5	45.8	2.35	12.33	+ 7	+ 5
	15 1	.626	— 0.055	H	24.8	47.2	2.47	12.45	+ 3	— 1
	5	.628	— 0.053	S	23.6	47.1	2.58	12.56	+ 6	+ 2
	10	.632	— 0.049	H	22.5	48.2	2.81	12.79	+ 11	+ 5
	16	.636	— 0.045	S	23.5	50.3	2.98	12.96	+ 4	— 2
	17 46	.740	+ 0.059	S	21.6	42.5	2.28	12.26	0	+ 4
	50	.743	+ 0.062	H	20.4	41.0	2.23	12.21	+ 7	+ 10
	55	.746	+ 0.065	S	22.3	38.8	1.79	11.77	— 25	— 21
	18 0	.750	+ 0.069	H	20.7	39.1	1.99	11.97	+ 9	+ 12
	5	.753	+ 0.072	S	23.1	38.2	1.64	11.62	— 16	— 14
	10	.757	+ 0.076	H	22.8	39.6	1.82	11.80	+ 12	+ 15
	14	.760	+ 0.079	S	22.7	36.5	1.51	11.49	— 11	— 9
	20	.764	+ 0.083	H	22.9	36.2	1.45	11.43	— 7	— 1
Sept. 8	14 6	462.587	— 0.081	H	27.7	40.0	1.34	11.32	— 23	— 22
	14	.593	— 0.075	S	24.4	39.4	1.63	11.61	— 9	— 9
	20	.597	— 0.071	H	23.0	41.9	2.06	12.04	+ 22	+ 22
	23	.599	— 0.069	S	25.0	41.6	1.82	11.80	— 8	— 8
	26	.601	— 0.067	H	24.3	42.2	1.96	11.94	0	— 2
	30	.604	— 0.064	S	23.8	42.6	2.06	12.04	— 2	— 1
	34	.607	— 0.061	H	24.1	43.1	2.09	12.07	— 11	— 11
	38	.610	— 0.058	S	23.4	44.7	2.34	12.32	+ 4	+ 4
	42	.612	— 0.056	H	24.2	44.2	+ 2.20	12.18	— 20	— 20

Date	G. M. T.	Julian Day	Phase	Obs.	Wedge	Δm	Mag.	v_1	v_2
1906, Sept. 8	14 46	2417462.615	— 0.053	S	24.6 47.3	+ 2.50	12.48	— 2	— 2
	50	.618	— 0.050	H	23.4 46.4	2.52	12.50	— 14	— 14
	54	.621	— 0.047	S	24.3 49.7	2.82	12.80	+ 1	0
	58	.624	— 0.044	H	24.2 50.5	2.93	12.91	— 7	— 7
	15 3	.627	— 0.041	S	24.4 53.4	3.27	13.25	+ 15	+ 14
	7	.630	— 0.038	H	23.2 53.5	3.41	13.39	— 3	— 3
	17 8	.714	+ 0.046	S	24.5 51.9	3.08	13.06	+ 20	+ 20
	17 14	.718	+ 0.050	H	23.4 48.5	2.76	12.74	+ 10	+ 1
	20	.722	+ 0.054	S	24.0 45.7	2.39	12.37	— 11	— 11
	24	.725	+ 0.057	H	23.4 44.0	2.26	12.24	— 9	— 9
	30	.829	+ 0.061	S	24.7 43.2	2.04	12.02	— 16	— 16
	34	.732	+ 0.064	H	25.7 45.0	2.13	12.11	+ 3	+ 3
	38	.735	+ 0.067	S	24.0 40.4	1.78	11.76	— 20	— 20
	42	.737	+ 0.069	H	24.4 41.7	1.89	11.87	— 1	— 1
	45	.740	+ 0.072	S	24.2 38.6	1.57	11.55	— 23	— 23
	48	.742	+ 0.074	H	23.8 39.7	1.73	11.71	— 1	— 1
	52	.744	+ 0.076	S	25.1 37.6	1.38	11.36	— 32	— 32
	56	.747	+ 0.079	H	25.0 39.3	1.56	11.54	— 6	— 6
	18 1	.751	+ 0.083	S	23.0 35.9	1.41	11.39	— 11	— 8
	6	.754	+ 0.086	H	25.9 37.9	1.32	11.30	— 15	— 10
	9	.756	+ 0.088	S	24.4 34.9	1.16	11.14	— 27	— 22
	12	.758	+ 0.090	H	25.9 37.8	1.31	11.29	— 9	— 3
25	13 28	479.561	— 0.093	H	23.5 35.0	1.26	11.24	— 8	+ 2
	33	.564	— 0.090	H	24.2 35.8	1.28	11.26	— 12	— 2
	40	.569	— 0.085	H	23.7 37.2	1.48	11.46	0	+ 8
	44	.572	— 0.082	H	23.2 36.1	1.41	11.39	— 14	— 6
	50	.576	— 0 078	H	24.0 37.6	1.49	11.47	— 15	— 11
	54	.579	— 0.075	H	23.8 38.6	1.61	11.59	— 11	— 6
	14 2	.585	— 0.069	H	23.5 40.6	1.86	11.84	— 4	+ 3
	7	.588	— 0.066	H	24.2 41.5	1.89	11.87	— 11	— 5
	14	.593	— 0.061	H	23.6 43.4	2.17	12.15	— 3	+ 5
	14 18	.596	— 0.058	H	23.4 46.0	2.48	12.46	+ 18	+ 25
	25	.601	— 0.053	H	23.5 48.9	2.80	12.78	+ 28	+ 36
	30	.604	— 0.050	H	23.0 50.8	3.09	13.07	+ 43	+ 53
	17 4	.711	+ 0.057	S	24.4 46.5	2.44	12.42	+ 9	0
	9	.715	+ 0.061	H	24.1 45.7	2.38	12.36	+ 18	+ 11
	14	.718	+ 0.064	S	23.0 41.7	2.03	12.01	— 5	— 13
	18	.721	+ 0.067	H	24.3 44.0	2.17	12.15	+ 21	+ 13
	42	.737	+ 0.083	S	25.5 37.4	1.32	11.30	— 20	— 23
	46	.740	+ 0.086	H	24.8 40.4	1.70	11.68	+ 24	+ 23
	51	.744	+ 0.090	S	24.3 38.8	1.58	11.56	+ 18	+ 20
	56	.747	+ 0.093	H	25.3 38.7	+ 1.47	11.45	+ 13	+ 15

Date	G. M. T.	Julian Day	Phase	Obs.	Wedge		Δm	Mag.	v_1	v_2
1906, Sept. 25	18 22	2417479.765	+ 0.111	S	27.2	36.9	+ 1.08	11.06	□	+ 7
	26	.768	+ 0.114	H	25.4	36.2	1.20	11.18	+ 16	+ 23
	29	.770	+ 0.116	S	23.8	35.0	1.23	11.21	+ 21	+ 29
	34	.774	+ 0.120	H	26.0	35.9	1.10	11.08	+ 13	+ 20
	19 0	.792	+ 0.138	S	26.6	31.8	0.58	10.56	— 22	— 13
	4	.794	+ 0.140	H	25.6	32.6	0.78	10.76	0	+ 9
	8	797	+ 0.143	S	25.3	31.0	0.63	10.61	— 12	— 3
	12	.800	+ 0.146	H	25.7	31.2	0.60	10.58	— 13	— 3
	38	.818	+ 0.164	S	26.0	29.6	0.40	10.38	— 19	— 10
	42	.821	+ 0.167	H	24.8	29.2	0.48	10.46	— 9	□
	46	.824	+ 0.170	S	26.2	29.8	0.42	10.40	— 13	— 5
	50	.826	+ 0.172	H	24.5	28.5	0.44	10.42	— 10	— 2
	20 30	.855	+ 0.201	S	27.4	31.3	0.44	10.42	+ 4	+ 8
	34	.857	+ 0 203	H	25.2	28.7	+ 0.39	10.37	□	+ 3

Time. The fourth column of this table contains the differences of the numbers in columns two and three. The mean of these differences is 0.134 day. The light curve seems to be symmetrical, whence it appears that the geocentric minima follow the dates in column two by 0.067 day, and precede those in column three by the same amount. The resulting epochs of minima are in

TABLE IV

DETERMINATION OF MINIMA

Date	Δm=2m00 Desc. Br.	Δm=2m00 Asc. Br.	Diff.	Geocentric Minima	Red. to Sun	O_1= Obs. Hel. Minima	Wt.	E	C_1= Comp. Hel. Minima	O_1-C_1
	d			d	d	d			d	d
1905, June 29	026.618			026.689	+ 0.0004	026.689	□	0	026.686	+ 0.003
July 13	040.766			040.833	5	040.834	1	5	040.841	— 0.007
July 16		43.760		043.693	5	043.694	0	6	043.672	+ 0.022
Aug. 19	077.580	77.714	0.134	077.647	6	077.648	1	18	077.645	+ 0.003
Sept. 2	091.733			091.800	6	091.801	1	23	091.800	+ 0.001
Oct. 26	145.525	145.660	0.135	145.593	2	145.593	2	42	145.590	+ 0.003
1906, Aug. 22	445.611	445.745	0.134	445.678	6	445.679	2	148	445.682	— 0.003
Sept. 8	462.601	462.732	0.131	462.666	6	462.667	2	154	462.668	— 0.001
Sept. 25	479.590	479.724	0.134	479.657	+ 0.0005	479.658	2	160	479.654	+ 0.004

column five. The reduction to the sun is practically negligible on account of the high latitude of the star. Its values in column six, expressed in units of the fourth decimal of a day, were taken from Table V. The heliocentric minima and the weights used in deriving the period are given in columns seven and eight of Table IV. The first and third minima are given zero weight; the former, because it is based upon only two observations; the latter, because an extrapolation

of the observed curve was required for the determination of the epoch. Minima derived from observations upon a single branch of the curve were given weight one ; where both branches of the curve were observed the weight is two, except for Aug. 19, where but a single observation was secured for the descending branch.

The heliocentric minima lead to fifteen equations of condition whose solution gave for the elements :

$$\text{Min.} = \text{J. D. } 2417026.686 + 2^{d}.831056\ E, \qquad \text{G. M. T.}$$

The heliocentric minima calculated with these elements are in column ten of Table IV, and the differences between observation and calculation, in column eleven.

TABLE V

REDUCTION TO SUN

Date	Red.	Date	Red.
	d		d
Jan. 1	— 0.0004	July 1	+ 0.0004
Feb. 1	— 6	Aug. 1	+ 6
Mar. 1	— 6	Sept. 1	+ 6
Apr. 1	— 4	Oct. 1	+ 5
May 1	— 2	Nov. 1	+ 2
June 1	+ 0.0001	Dec. 1	— 0.0001

To determine the mean light-curve the observations were arranged in order of phase, and the overlapping means formed for each five consecutive observations. The means were plotted and smooth curves were drawn for both branches of the light-curve. Since the deviations from a symmetrical curve appeared to be no greater than the errors of observation, the two branches were combined to form a mean light-curve whose ordinates are given in columns two and four of Table VI. The comparison of this curve with the individual observations gave the residuals v_1 in Table III.

In order to exhibit more clearly the relation of these residuals to the light-curve, the mean deviations of the observations included within certain limiting phase values were calculated for each night, regard being paid to the algebraic sign. The results, expressed in hundredths of a magnitude, are in Table VII. The limiting phase values, expressed in hundredths of a day, appear at the head of the table ; the Julian Day and the branch of the curve, ascending or descending, in the first two columns ; and the mean deviations, in the columns headed d_1. The columns headed n show the number of observations used for the formation of each mean deviation. The second and third lines from the last give similar results for all observations on descending and ascending branches. Generally speaking, these quantities are small, from which it would appear that the assumption of a symmetrical light-curve is justified. The last line gives

the mean deviation for the entire series of observations, irrespective of the algebraic sign of the phase values. These numbers are small, as they should be, and indicate that the mean light-curve has been properly drawn, at least in so far as the phase values of Table III are assumed to be correct. Nevertheless, a casual inspection of the results in Tables III and VII shows that in many instances the residuals are abnormal and systematic in character. For example, the mean deviations within the phase interval 0.14 – 0.18 are negative for all the nights excepting J. D. 2417043, which is 1905, July 16. In view of the unfavorable atmospheric conditions already mentioned as existing upon this night, it is probable that the light-curve requires modification at this point. Again, the means for the descending branch on J. D. 2417145 are both negative

TABLE VI

MEAN LIGHT-CURVES REFERRED TO STAR *a*.

Phase from Min.	Δm_1	Δm_2	Phase from Min.	Δm_1	Δm_2
d			d		
0.035	$3^m.60$	$3^m.60$	0.100	$1^m.23$	$1^m.15$
0.040	3.22	3.22	0.110	1.10	0.99
0.045	2.93	2.93	0.120	0.98	0.87
0.050	2.66	2.66	0.130	0.87	0.77
0.055	2.43	2.43	0.140	0.78	0.67
0.060	2.23	2.23	0.150	0.70	0.59
0.065	2.04	2.04	0.160	0.62	0.52
0.070	1.87	1.87	0.170	0.55	0.46
0.075	1.72	1.72	0.180	0.49	0.41
0.080	1.60	1.58	0.190	0.44	0.38
0.085	1.49	1.45	0.200	0.40	0.36
0.090	1.40	1.34	0.210	0.37	0.35
0.095	1.31	1.24	0.220	0.35	0.34
0.100	1.23	1.15	0.230	0.34	0.34

while for the ascending branch they are positive, indicating that the minimum used for the calculation of the phase values for this date differs appreciably from the true value. Considerations of this sort led to a revision of the mean light-curve and a redistribution of the epochs of minima. This was carried out in such a manner as to reduce the average residual for each night to a minimum. The corrections applied to the original phase values in order to accomplish the readjustment are given in column two of Table VIII, and the ordinates of the revised light-curve in columns three and six of Table VI. The curve is illustrated by Fig. 1. The corresponding residuals for the individual observations are in the last column of Table III.

For purposes of comparison these residuals were summarized in precisely the same way as the original series. The mean deviations appear in Table VII under the heading d_2. An improvement in the representation of the observations is evident. The systematic deviations

have been reduced, and the agreement of the two branches of the light-curve noticeably improved. But the effect of the revision is even more clearly shown by the results in Table IX. The body of this table, which is similar in arrangement to Table VII, shows the average residuals for the various groups of observations, derived without regard to algebraic sign. In addition, the last three columns give the average residuals and the total number of observations for each night. The residuals under the heading v_1 refer to the original reduction ; those under v_2, to the revision. As before, the unit is one hundredth of a magnitude. The reduction in the average residuals

TABLE VII

MEAN DEVIATIONS FROM LIGHT-CURVES

J. D.	Br.	< 0.06			0.06 – 0.10			0.10 – 0.14			0.14 – 0.18			0.18 – 0.22		
		d_1	d_2	n	d_1	d_2	n	d_1	d_2	n	d_1	d_2	n	d_1	d_2	n
026	D				− 4	− 4	2									
040	D				+ 28	0	4	+ 10	+ 2	8	− 11	− 14	3	+ 7	+ 7	3
043	A										+ 14	0	8	+ 5	− 2	4
046	A							− 18	− 8	1						
063	A							− 30	− 22	2	− 10	+ 1	2			
077	D	+ 2	+ 19	1												
077	A				+ 12	+ 3	6	− 8	− 3	7	− 3	+ 5	5	− 8	− 3	4
091	D	− 33	− 23	3	+ 4	+ 12	7	− 12	− 1	2						
145	D	− 22	− 2	5	− 2	+ 10	7									
145	A	+ 24	+ 6	2	+ 14	− 1	2									
445	D	+ 6	+ 2	5	+ 19	+ 14	4									
445	A	0	+ 4	1	− 4	− 1	7									
462	D	− 3	− 3	8	− 5	− 4	7									
462	A	+ 2	+ 2	4	− 13	− 12	12									
479	D	+ 18	+ 25	1	− 9	− 1	9									
479	A	+ 9	0	1	+ 10	+ 7	7	+ 6	+ 13	5	− 11	− 2	7	+ 2	+ 5	2
	D	− 8	− 2	23	+ 2	+ 4	40	+ 6	+ 1	10	− 11	− 14	3	+ 7	+ 7	3
Means	A	+ 8	+ 4	8	− 1	− 3	34	− 7	− 1	15	0	+ 1	22	− 1	− 1	10
	All	− 4	− 01	31	+ 1	+ 1	74	− 2	0	25	− 1	− 1	25	+ 1	+ 1	13

resulting from the revision is marked. It will be observed that the discussion is based upon 168 of the total of 170 observations. The two abnormal residuals for 1906, Sept. 25, G. M. T. $14^h\ 25^m$ and $14^h\ 30^m$ have been excluded.

It remains to be seen how much of this improvement in the representation of the observations is real. That some of it may be only apparent is evident from the following : The times of minima derived from observations upon both branches of the light-curve are not likely to be affected by systematic errors, at least not, unless these errors change appreciably during the observations of a night. But for minima derived from observations upon a single branch, one has

no such assurance, and the removal of a systematic difference from the residuals by the application of an arbitrary correction to the phase values may merely throw the error into the epoch of the minimum. If such has been the case, it will be found impossible to represent accurately the corrected minima by any revision of the elements. In the present case, the corrected minima are in column three of Table VIII. They were obtained by subtracting the phase values in the preceding column from the quantities in column ten of Table IV. The solution of the corresponding equations of condition, with the same weights as before, gave for the revised elements :

$$\text{Min.} = \text{J. D. } 2417026.682 + 2^{d}.831079\ E, \qquad \text{G. M. T.}$$

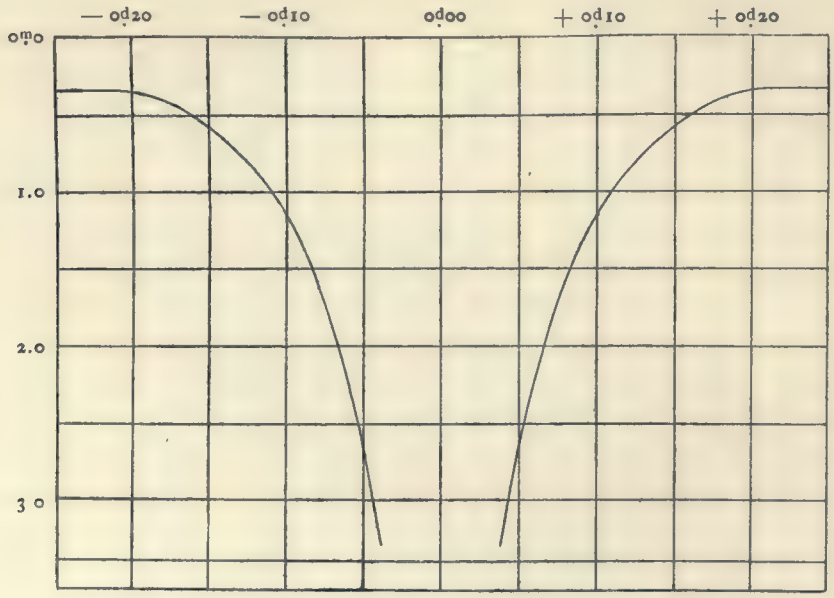

Fig. 1—Light-curve of RR Draconis referred to star $a = 9^{m}.64$

where the uncertainty in the period is probably less than one second. The calculated minima based upon these elements are given in column six of Table VIII, and the outstanding residuals in the minima, under the heading $O_2 - C_2$. A comparison with the residuals $O_1 - C_1$ in Table IV shows that the representation of the well determined minima has been improved. None of these show differences between observation and computation which exceed $0^{d}.001$. On the other hand, the representation of the remaining minima is less satisfactory than before. Doubtless we have here an illustration of the influence of systematic errors mentioned above, for in view of all the evidence, it is probable that the revised elements are nearer the truth than the original set. Some of the residuals suggest the possibility of a variation in the period, but for the present such a possibility can be only conjectural.

Although it is difficult to estimate the part played by external conditions in the introduction of systematic errors into the measures, the influence due to personality of the observers, or at least, the differential effect, is readily determined. To this end the mean deviations from the light-curve were formed for each observer. In order to reveal any progressive change, the means

TABLE VIII

DETERMINATION OF MINIMA

Date	ΔP	O_2 = Obs. Hel. Minima	Wt.	E	C_2 = Comp. Hel. Minima	$O_2 - C_2$
	d	d			d	d
1905, June 29	0.000	026.686	0	0	026.682	+ 0.004
July 13	+ 0.015	040.826	1	5	040.837	— 0.011
16	— 0.030	043.702	0	6	043.668	+ 0.034
Aug. 19	— 0.004	077.649	1	18	077.641	+ 0.008
Sept. 2	— 0.002	091.802	1	23	091.797	+ 0.005
Oct. 26	+ 0.004	145.586	2	42	145.587	— 0.001
1906, Aug. 22	+ 0.001	445.681	2	148	445.682	— 0.001
Sept. 8	0.000	462.668	2	154	462.668	0.000
25	— 0.002	479.656	2	160	479.655	+ 0.001

TABLE IX

AVERAGE RESIDUALS

J. D.	< 0.06			0.06 – 0.10			0.10 – 0.14			0.14 – 0.18			0.18 – 0.22			Means		n
	v_1	v_2	n	v_1	v_2	n	v_1	v_2	n	v_1	v_2	n	v_1	v_2	n	v_1	v_2	
026				6	6	2										6	6	2
040				28	12	4	10	6	8	14	14	3	8	9	3	15	9	18
043										16	8	8	6	5	4	12	7	12
046							18	8	1							18	8	1
063							30	22	2	10	5	2				20	13	4
077	2	19	1	12	7	6	8	3	7	3	5	5	8	5	4	8	6	23
091	33	23	3	10	12	7	12	10	2							16	15	12
145	22	8	7	12	11	9										17	10	16
445	5	3	6	15	13	11										11	9	17
462	10	9	12	13	12	19										12	11	31
479	14	12	2	12	10	16	14	18	5	11	5	7	11	6	11	12	10	32
Means	14	10	31	13	11	74	12	9	25	11	7	25	7	6	13	13	10	168

were first calculated for the phase intervals shown in column one of Table X. The deviations themselves, in the sense Seares minus curve, and Haynes minus curve, are in columns two and five. The number of observations used in forming each mean is given in columns four and seven. The systematic difference, Seares minus Haynes, is shown in column eight. The second line

from the last gives similar results for all the observations during light change ; the last line, for all observations during constant light. It appears that the average difference, S – H, is practically the same for both series, a result of some interest in view of the wide difference in observing conditions existing for the two cases. As for the separate phase intervals, the observations are too few in number for all cases excepting the second to permit any certain conclusion, but, so far as the evidence goes, it seems to indicate that the systematic difference between the observers is nearly independent of the brightness of the variable and the magnitude of the interval measured.

Table X also shows, in columns three and six, the average residuals for each observer ; the last column contains the means given in the last line of Table IX. An intimate relation between the observing conditions and the precision of the results is hereby revealed. With diminishing phase the residuals increase from 0.06 to 0.11 of a magnitude, showing clearly the prejudicial

TABLE X

Systematic Differences and Average Residuals

Phase limits	S – C	v_s	n_s	H – C	v_H	n_H	S – H	v
< 0.06	— 0m.008	0.10	22	— 0m.016	0.10	9	+ 0m.008	0m.098
0.06 – 0.10	— 0.005	0.12	43	+ 0.034	0.09	31	— 0.039	0.110
0.10 – 0.14	— 0.022	0.08	19	+ 0.078	0.12	6	— 0.100	0.093
0.14 – 0.18	— 0.034	0.07	15	+ 0.025	0.07	10	— 0.059	0.070
0.18 – 0.22	+ 0.007	0.06	12	+ 0.080	0.08	1	— 0.073	0.058
All obs. during lt. change	— 0.011	0.097	111	+ 0.030	0.091	57	— 0.041	0.095
" " const. lt.	— 0.015	0.091	73	+ 0.028	0.097	22	— 0.043	0.092

influence of the decreasing brightness of the variable and the increasing value of the magnitude difference measured. The average residual for the series is 0.095 mag., while that for the constant light observations is 0.092 mag.

This brings us back to the anomalous result stated on page 128, which can now be exhibited in another light : Evidence has been presented to the effect that the conditions of observation exert a direct influence upon the precision of the results. Here are two series of observations, one made under much less favorable observing conditions, on the average, than the other. Yet the two series show sensibly the same average residual. The matter can also be stated as follows : Observations near the beginning and end of light change, when observing conditions are similar to those during constant light, show a residual of 0.06 mag. while the average for the constant light observations themselves is 0.092 mag. It may be argued that this is due to a lack of homogeneity in the residuals compared ; that, owing to the method of reduction employed, the residuals during light variation do not contain the full effect of systematic errors. In a measure, this is true ; but the result remains much the same when the comparison is made with

the residuals for the last four nights only. For these there has been no elimination of errors peculiar to a night. Moreover, the discrepancy is not due to personality, for the observations of both Mr. Haynes and myself show the anomaly in question. In view of the general consistency of the measures in other respects one is led to the explanation already suggested, namely, a minute intermediate variation ; but as stated before, the available evidence is insufficient to admit of any definite conclusion.

PRELIMINARY ANNOUNCEMENT CONCERNING THE ALGOL VARIABLE 121.1906 DRACONIS

1900.0 R. A. $19^h\ 1^m\ 8^s$ Dec. $+\ 58°35'.2$
1855.0 0 25 31.3

The Variable 121.1906 Draconis, announced by Ceraski, A. N. No. 4128, has been observed by Mr. Haynes on twelve nights since Nov. 26. Minima were partially observed by him on Dec. 6 and 8, and by myself on Dec. 25. The estimated times of minima, including that given by Ceraski, are contained in column two of the following table :

TABLE XI

MINIMA OF 121.1906 DRACONIS

Date	J. D. Obsn.	E	O − C	Observer
			d	
1906, Oct. 18	2417502.40	0	0.00	Blajko
Dec. 6	551.63	26	— 0.01	Haynes
8	553.55	27	+ 0.01	Haynes
25	570.58	36	0.00	Seares

These observations are satisfied by the period $1^d.894$ which is probably near the true value. Values equal to one-third and one-half this amount are excluded by observations on J. D. 2417563.64 and J. D. 2417552.59, respectively, which showed the variable to be of normal brightness. Ordinarily, the star is 0.40 mag. fainter than BD. + 58°1858, 9.5 mag. The range of variation is 0.8 or 0.9 mag. The interval of light change appears to be about five hours.

Columbia, Missouri, 1907, Jan. 2. *F. H. Seares.*

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BULLETIN NO. 10

ANNOUNCEMENT OF PRELIMINARY RESULTS FOR VARIABLE STARS

I. General Remarks and Star List

The working program of the Laws Observatory contains at present some sixty variables which are under systematic observation with the equalizing wedge photometer used in connection with the 7½-inch equatorial. These stars are all objects whose light-curves and periods are more or less imperfectly known. One star has been under observation since June, 1904, and several others, since the summer of 1905; but most of the list have been systematically observed only since the grant from the Gould Fund[1] became available in June, 1906. In all some 6000 observations, involving about 48,000 individual settings of the wedge, have been secured thus far.

Detailed investigations have already been published in Bulletins 8 and 9 for the following stars: 88.1906 Lacertae, 50 observations; V Vulpeculae (4.1904), 184 observations; and RR Draconis (188.1904), 265 observations. Preliminary announcements have likewise been made in the same place for the following: V Lacertae (110.1904), 539 observations; RV. Capricorni (108.-1905), 260 observations; and 121.1906 Draconis, 45 observations. The accumulation of data is now sufficient to justify preliminary announcements, more or less definite, for 32 other stars. Some of these might well be discussed in detail at once; but, as such discussions must necessarily proceed somewhat slowly, it has seemed preferable to collect the results in a form suitable for immediate publication, and to present them in the following pages, in the hope that they may supplement, in certain instances at least, the conclusions of other observers. In addition, further information has been secured for one of the stars for which preliminary announcement has already been made. The 33 stars in question are collected in the following table in order of right ascension. In general, the positions in the third and fourth columns have been copied directly from the *Ephemeriden* of Hartwig for 1907.[2] The numbers in the last column of the table refer to the pages of the present bulletin upon which will be found the results for the individual stars. These results are based upon 3290 individual observations, slightly more than

[1] *Laws Observatory Bulletin*, No. 8, p. 107.

[2] Ephemeriden veränderlicher Sterne für 1907, *Vierteljahrsschrift der Astronomischen Gesellschaft*, v. 41, p. 304.

one-half of which have been made by Mr. Haynes under the grant from the Gould Fund mentioned above. The remaining observations have been made by myself.

STAR LIST

No.	Name	R. A. 1855	Dec. 1855	Page
1	64.1905 Cassiopeiae	$0^h\ 2^m\ 0^s$	$+50°\ 45'$	145
2	65.1905 Cassiopeiae	0 9 52	+49 29.1	145
3	190.1904 Cassiopeiae	0 15 36	+54 59.6	146
4	RW Cassiopeiae (46.1905)	1 27 49	+57 0.9	146
5	RV Andromedae (17.1904)	1 1 41	+48 14.7	146
6	RV Persei (61.1905)	4 1 15	+33 52.0	147
7	RV Tauri (45.1905)	4 38 12	+25 54.7	147
8	RR Camelopardalis (40.1905)	5 17 59	+72 20.2	147
9	Y Ursae Majoris (134.1904)	12 33 44	+56 38.3	147
10	RY Ophiuchi (77.1905)	18 9 24	+3 38.8	148
11	RZ Ophiuchi (103.1905)	18 38 45	+7 4.3	148
12	RS Draconis (39.1905)	18 41 25	+74 11.4	149
13	X Lyrae	19 7 10	+26 32.0	149
14	UV Cygni	19 26 38	+43 19.9	149
15	U Vulpeculae	19 30 17	+20 0.8	150
16	X Vulpeculae (161.1904)	19 51 27	+26 10.2	153
17	W Vulpeculae (5.1904)	20 3 59	+25 51.6	154
18	RR Delphini (79.1906)	20 36 45	+13 25.5	154
19	VX Cygni (58.1903)	20 51 52	+39 37.2	154
20	TX Cygni (22.1900)	20 54 47	+42 2.0	155
21	VY Cygni (61.1903)	20 58 43	+39 23.7	155
22	YY Cygni (94.1901)	21 16 56	+41 46.7	156
23	SS Cygni	21 37 1	+42 55.4	156
24	VZ Cygni (7.1904)	21 45 53	+42 27.3	157
25	70.1905 Pegasi	21 49 39	+22 10.8	160
26	88.1906 Lacertae	22 43 9	+55 39.8	161
27	RZ Andromedae (166.1904)	23 3 4	+52 16.2	161
28	SS Andromedae (167.1904)	23 5 0	+52 6.4	161
29	RU Aquarii (66.1901)	23 16 48	−18 6.9	162
30	RS Cassiopeiae (108.1904)	23 30 29	+61 37.7	162
31	52.1906 Andromedae	23 31 33	+34 57.6	163
32	RS Andromedae	23 48 4	+47 49.5	163
33	53.1906 Andromedae	23 57 9	+42 45.2	164

II. Preliminary Results

In the following pages the first three figures of the Julian Dates, viz., 241, have been dropped throughout, except in the case of initial dates in systems of elements. The magnitude following a BD. number is in all cases the BD. magnitude. It has been necessary to introduce these magnitudes since the comparison stars, usually fainter than magnitude 7.5, have not yet been connected with the standard magnitude system.

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1. 64.1905 Cassiopeiae—Variability discovered by Mrs. Fleming from H. C. O. plates. Announced in H. C. O. Cir. 98. Photometric observations have been secured as follows :

a) 1905, Aug. 7 - Sept. 19, 25 observations.

b) 1906, June 11 - 1907, March 1, 108 observations.

When near minimum the star is too faint to be seen with the 7½-inch equatorial. The following data represents, in a general way, the location of minima: 1905, Oct. 7, *v* at the limit of visibility, moonlight ; Oct. 10, *v* invisible, moonlight ; 1906, July 8, *v* invisible, sky white and full moon rising ; July 17, 23, 28, Aug. 11, 22, Sept. 5, *v* invisible, moonlight on Aug. 11 and Sept. 5 ; 1906, Dec. 3, 1907, Jan. 25, 27, *v* invisible, moonlight.

The observations show well-defined maxima for J. D. 7082 and J. D. 7504, the latter of which agrees exactly with the date given by Hartwig, Eph., 1907, p. 310. Further, the variable was near a maximum, and diminishing on J. D. 7374. At present it is approaching a maximum. The following will serve as approximate elements :

$$\text{Max.} = \text{J. D. } 2417082 + 141^{d}\text{E.}$$

At the time of the 1905 maximum, viz. J. D. 7082, the variable was $1^{m}.3$ brighter than BD. + 50°11, $9^{m}.3$. At the 1906 maximum on J. D. 7504 it was $0^{m}.6$ brighter than this same star, and sensibly equal to BD. + 50°9, $8^{m}.8$, which is in agreement with the result found by Hartwig. The observations show a range of about three magnitudes. The maxima are sharp, and that part of the light-curve revealed by the observations appears to be nearly symmetrical with respect to the maxima. Maxima for 1907 should occur on J. D. 7645 = March 10, and on J. D. 7786 = July 29.

---

2. 65.1905 Cassiopeiae—Variability announced in H. C. O. Cir. 98 as a result of the examination of 16 plates made during the years 1890-1902. The estimated limits of variation were $7^{m}.5$ - $9^{m}.0$. The Laws Observatory measures are as follows :

a) 1905, Aug. 7 - Oct. 10, 18 observations.

b) 1906, June 11 - 1907, Feb. 8, 50 observations.

The 1905 observations show a gradual increase in brightness amounting to $0^{m}.36$. At the beginning of the series the variable was $0^{m}.22$ fainter than star *a* = BD. + 49°38, $8^{m}.8$. At the end it was $0^{m}.16$ brighter than *a*. The 1906 observations, covering 242 days, give no certain indi-

cation of variation. The average brightness for the 22 nights of observation included within this interval is $0^m.19$ greater than star *a*. The extreme range shown by the means by nights is $0^m.37$, with an average deviation per night from the mean of the series amounting to $0^m.08$.

---

3. 190.1904 CASSIOPEIAE—Twenty-five observations, secured on fifteen nights between 1905, Aug. 7 and Oct. 11, show no appreciable variation. The variable was referred to star No. 10, $8^m.5$, of Hagen's chart for T Cassiopeiae. The maximum difference in brightness of variable and comparison star was $0^m.81$; the minimum, $0^m.57$. The mean of the series was $0^m.67$, with an average residual of $0^m.06$. The variable was fainter than the comparison star.

---

4. RW CASSIOPEIAE (46.1905)—Discovered by Madame Ceraski from plates by Blajko, A. N. 4010. One hundred and sixty-nine observations were secured during the interval 1906, July 18 - 1907, Feb. 16. The range of variation is about one magnitude. Four maxima and four minima are well determined. Blajko has given the following elements in A. N. 4108 :

$$\text{Max.} = \text{J. D. } 2417062.5 + 14^d.80E, \qquad \text{G. M. T.,} \qquad m - M = 9^d.0.$$

The representation of the observed maxima and minima by these elements is as follows :

Obs. Max.	Comp. Max.	O – C	Obs. Min.	Comp. Min.	O – C
J. D. 7433.0	J. D. 7432.5	+ $0^d.5$	J. D. 7426.0	J. D. 7426.7	— $0^d.7$
7447.0	7447.3	— 0.3	7441.0	7441.5	— 0.5
7462.0	7462.1	— 0.1	7470.0	7471.1	— 1.1
7491.0	7491.7	— 0.7	7515.5	7515.5	0.0

The average of the residuals O – C is — $0^d.36$.

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5. RV ANDROMEDAE (17.1904)—Variability was discovered by Williams, A. N. 3944, who derived from three minima a period of 182 days. The epochs of two maxima and two minima, observed by Nijland, are given in A. N. 4116. The Laws Observatory observations are 62 in number and extend from 1906, July 18, to 1907, Feb. 16. They determine two maxima and one minimum. A comparison of the four maxima gives the following provisional elements :

$$\text{Max.} = \text{J. D. } 2416934 + 169^d E.$$

Nijland's observations give 111 and 69 days, respectively, for the intervals $m - M$. The corresponding value from Mr. Haynes's observations is 66 days. The value 72 days is accepted for the present, whence,

$$\text{Min.} = \text{J. D. } 2417006 + 169^d E.$$

Only one of the minima by Williams is given below. The other two were observed photographically, and are uncertain. The representation of the various observations is as follows :

Obs. Max.	O – C	Observer	Obs. Min.	O – C	Observer
J. D. 6934	0d	Nijland	J. D. 6509	+ 10d	Williams
7105	+ 2	Nijland	7045	+ 39	Nijland
7445	+ 4	Haynes	7174	— 1	Nijland
7609	— 1	Haynes	7514	+ 1	Haynes

Mr. Haynes's observations show a range of $2^{m}.8$, with a maximum $1^{m}.0$ brighter than BD. + 48°612, $9^{m}.3$. A maximum should occur on 1907, July 22, and minima on April 16 and Oct. 2.

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6. RV Persei (61.1905)—Discovered by Madame Ceraski from plates by Blajko, A. N. 4016. Fifty-one observations during the interval 1906, July 18 - 1907, March 5, show a well determined maximum for J. D. 7624, and suggest a period of 36 days. The result is uncertain, inasmuch as the earlier observations of the series are scattered ; but the measures made here are all represented by the elements

$$\text{Max.} = \text{J. D. } 2417624 + 36^{d}\text{E.}$$

The maximum is $1^{m}.0$ fainter than BD. + 34°828, $8^{m}.9$.

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7. RV Tauri (45.1905)—Variability discovered by Madame Ceraski from plates by Blajko, A. N. 4010. Mr. Haynes secured 38 observations during the interval 1906, Nov. 27, - 1907, March 5. The observations are scattered, but maxima seem to have occurred about J. D. 7545, J. D. 7585, and J. D. 7626; and minima, about J. D. 7572 and J. D. 7610. The rise to maximum is relatively rapid, occupying not more than 15 or 16 days. When brightest the star is equal to BD. + 26°746, $9^{m}.1$. The range is approximately two magnitudes. Approximate elements are :

$$\text{Max.} = \text{J. D. } 2417545 + 40^{d}\text{E.}$$

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8. RR Camelopardalis (40.1905)—Variability discovered by Madame Ceraski from plates by Blajko, A. N. 4000. Mr. Haynes has secured 20 observations during the interval 1906, Oct. 31-1907, Feb. 21. On J. D. 7515 and J. D. 7625 the variable was sensibly equal to BD. + 72°280, $9^{m}.5$, which is probably its maximum brightness. A minimum, about one magnitude fainter, was passed on or about J. D. 7575. The period is probably near 110 days.

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9. Y Ursae Majoris (134.1904)—Discovered by Mrs. Fleming and announced in H. C.

O. Cir. 80. Although the star was suspected of variability as early as 1890, its period is still unknown. The Laws Observatory observations are as follows :

a) 1905, July 14 - Aug. 30, 36 observations.

b) 1906, April 16 - 1907, Feb. 21, 62 observations.

Series a) indicates a minimum for J. D. 7065. Series b) shows a maximum for J. D. 7465. Minima must have occurred about J. D. 7360 and J. D. 7575. The 1906-7 observations therefore suggest a period of 215 days or thereabout, but this is not in agreement with the results obtained in 1905. The range is about one magnitude with a maximum $0^m.1$ brighter than BD. $+ 56°1612$, $7^m.5$.

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10. RY Ophiuchi (77.1905)—Discovered by Anderson, A. N. 4031. One hundred and forty-four observations were secured upon 51 nights during the interval 1906, June 8 - Nov. 26. The general results are as follows :

1.	1906, June	8 =	J. D. 7370,	$v - a = + 0^m.30$
2.	July	20 =	7412,	$= + 2.50$
3.	Sept.	17 =	7471	$= + 0.78$
4.	Oct.	19 =	7503,	$= - 1.58$
5.	Nov.	26 =	7541,	$= + 0.61$

Comparison star *a* is BD.$+3°3640$, $8^m.9$. Although no observations were secured between Oct. 19 and 26, the curve indicates a maximum for Oct. 21 or 22. With this exception the decrease in brightness between the dates 4) and 5) was continuous. The decrease was also continuous between the dates 1) and 2). From 3) to 4) the increase was continuous. The variable was either invisible, or too faint to measure, in the 7½-inch equatorial on the following dates between 2) and 3), viz., Julian Days 7420, 7432, 7446, 7459, and 7464. The maximum defined by the curve is exactly represented by Graff's elements :

$$\text{Max.} = \text{J. D. } 2417042 + 154^d.5\text{E},$$

which are given by Hartwig in the *Ephemeriden* for 1907, p. 314.

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11. RZ Ophiuchi (103.1905)—Variability discovered by Madame Ceraski from plates by Blajko, A. N. 4046. The star is apparently an Algol variable of long period. Blajko, A. N. 4108, gives the period as 130.9 or 261.8 days. If the former value is correct, there should have been a minimum between the dates 1906, Aug. 15 and Sept. 4. Eighty-two observations on 69 nights between 1906, July 24 and Dec. 7, show no appreciable variation. The star was observed on 12 nights between the above mentioned limits for the minimum. These results are in agreement with those of Jordan, A. N. 4128, and Graff, Hartwig, Eph., 1907, p. 306.

12. RS Draconis (39.1905)—Variability discovered by Madame Ceraski from plates by Blajko, A. N. 4000. No elements appear to have been published, although Hartwig indicates in the *Ephemeriden* for 1907 a maximum for 1907, May 26. The Laws Observatory observations are 116 in number, and are distributed as follows :

a) 1905, June 24 - Oct. 25, 50 observations.
b) 1906, April 16, 2 observations.
c) 1906, June 8 - 1907, Feb. 26, 66 observations.

At the beginning of series a), the variable was $2^{m}.1$ fainter than star c = BD. + 74°790, $9^{m}.0$. It gradually increased in brightness, and at the end of this series was equal to c. On 1906, April 16, it was $1^{m}.5$ fainter than c, and apparently increasing. A secondary maximum was reached on J. D. 7390 with a brightness $0^{m}.3$ greater than c. A principal maximum occurred on J. D. 7450. The star was then $0^{m}.67$ brighter than c. A decrease then began and continued until the principal minimum on J. D. 7580, upon which date it was $2^{m}.83$ fainter than c. The range is therefore about $3^{m}.5$. It is impossible at present to estimate the period with precision, although these observations suggest a value of 270 days. This would require a maximum for 1907, May 24, which is in substantial agreement with the date given by Hartwig. Ebell, A. N. 4000, estimates from Harvard Plate No. 1 that the star was certainly fainter than magnitude 10.5 on 1902, Jan. 4 = J. D. 5754. The above value for the period gives a minimum for J. D. 5690. The results are not necessarily contradictory, for the variable remains fainter than magnitude 10.5 for something like three months. We may therefore assume provisionally

$$\text{Max.} = \text{J. D. } 2417450 + 270^{d}\text{E}, \qquad m - \text{M} = 130^{d}.$$

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13. X Lyrae—Variability discovered by Espin in 1897, A. N. 3447. Sixty-seven observations were secured during the interval 1906, July 24 - Dec. 11. These indicate a range from $1^{m}.5$ fainter than a = BD. + 26°3485, $8^{m}.3$, to $0^{m}.9$ fainter than a, with a period which is not far from 50 days. The following will serve as a rough approximation for the elements :

$$\text{Min.} = \text{J. D. } 2417420 + 50^{d}\text{E}.$$

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14. UV Cygni—Variability discovered by Deichmüller in 1899, A. N. 3544. The star is given by Hartwig in the *Ephemeriden* as only slightly variable. Thirty-four observations were secured during the interval 1906, July 28 - 1907, Feb. 8. At the beginning of the series the variable was equal in brightness to star b = BD. + 43°.3264, $9^{m}.2$. During the 100 days immediately following it increased in brightness $0^{m}.6$. It was then $0^{m}.2$ fainter than star a = BD. + 43°3267, $7^{m}.9$, and has remained of this brilliancy ever since.

15. U VULPECULAE—Variability discovered by Müller and Kempf in 1897. The results thus far published are in part contradictory. The most important papers are: Müller and Kempf, A. N. 3483; Pickering, H. C. O. Cir. 41, A. N. 3561; Luizet, A. N. 3570; Yendell, A. J. 513, 563; and Wilkins, A. N. 4125.

The first approximation for the period was $8^d.00$ which was derived by Müller and Kempf. Luizet, with the aid of additional observations, found $8^d.003$. The light-curves derived by these observers are practically identical, and present as their characteristic feature an almost perfect symmetry with respect to the maxima. Pickering, on the other hand, finds from observations by Wendell that the ascending branch is more steeply inclined than the descending, and derives for the period $7^d.98$. The observations of Wendell have not been published in detail so far as I am aware. The results announced by Pickering are confirmed in the main by Yendell who derived the elements.

$$\text{Max.} = \text{J. D. } 2414200.311 + 7^d.97997E, \qquad \text{G. M. T.}$$

Although these elements are based on data extending over the years 1897-1901, they do not represent the observations of Luizet. In fact, no single value of the period can be found which will represent satisfactorily the maxima determined by the observations of Müller and Kempf, Luizet, and Yendell. The dissymmetry of the light-curve found by Yendell is even more striking than that in the curve of Pickering, and is in sharp contrast with the sinusoidal curves of Müller and Kempf, and Luizet. For example, Yendell finds $M - m = 1^d.98$, while the corresponding value for the symmetrical curves is $4^d.00$. In other words, there is a difference in the position of the maximum relative to the minimum in the two types of curve amounting to two days, or twenty-five per cent of the whole period. It is difficult to understand how a difference of this magnitude could have arisen as a result of errors of observation alone.

The observations of Wilkins are photographic and date from the year 1905. They, too, show the increase in light to have taken place more rapidly than the decrease, although some of the minor peculiarities exhibited by the curve of Wilkins are possibly due to the relatively small number of observations upon which the light-curve is based.

The star has been observed here upon 45 nights between the dates 1906, July 28 - Oct. 31. In all, 94 observations have been secured. These determine a number of maxima and minima, and make it possible to remove the uncertainty in the period.

The difficulty which presents itself in attempting to find a value of the period which shall represent the maxima defined by the older series of observations, seems to be a direct consequence of the diversity of form in the light-curves derived from these observations. Moreover, a comparison of the maxima observed by Luizet with those computed from the elements of Yendell indicates that the maxima alone are affected by irregularities. In other words, if the discussion of the period had been based upon minima instead of maxima, no difficulty would have been encountered. For this reason minima alone have been used in the present investigation. In order

to abbreviate the discussion, the numerous minima available have been combined into means. The epochs of these means are given in column three of Table I, expressed in Julian Dates and decimals of Greenwich Mean Time. The last column gives the source from which the data has been drawn. In addition a few words of explanation are necessary. For example, No. 1 was obtained by adding $4^d.00 = m - M$ to the epoch of the isolated observation in 1895, which was assumed to coincide with a maximum. No. 2 was obtained in a similar manner from the initial epoch given by Müller and Kempf. Nos. 3 and 4 were derived from the original observations of Luizet after they had been plotted in the usual manner. The epochs assigned to Yendell were derived from his published values by a process of averaging. No. 11 was derived from the plot of the original observations by Wilkins. Nos. 12 and 13 result from the Laws Observatory series mentioned above.

TABLE I

EPOCHS OF MINIMA

No.	E	J. D.	v_1	v_2	Source
1	— 85	3524.22	— $1^d.12$	— $1^d.50$	Müller & Kempf, A. N. 3483
2	0	4204.47	— 0.04	— 0.33	" " " " "
3	38	4508.35	+ 0.20	— 0.04	Luizet, A. N. 3570
4	55	4644.30	+ 0.32	+ 0.09	" " "
5	85	4884.00	+ 0.32	+ 0.12	Yendell, A. J. 513
6	126	5211.86	+ 0.57	+ 0.42	" " "
7	136	5291.78	+ 0.59	+ 0.45	" " "
8	184	5674.75	+ 0.02	— 0.06	" " "
9	212	5898.27	— 0.19	— 0.24	Yendell, A. J. 563
10	231	6049.89	— 0.38	— 0.41	" " "
11	357	7056.80	— 0.25	— 0.14	Wilkins, A. N. 4125
12	404	7432.50	— 0.09	+ 0.07	Seares & Haynes
13	414	7512.50	+ 0.01	+ 0.18	" " "

The equations of condition for the initial epoch and the period were assigned equal weight. A least-square solution gave the following elements:

$$\text{Min.} = \text{J. D. } 2413525.34 + 7^d.99029E, \qquad \text{G. M. T.} \qquad \text{(A)}$$

The representation of the observations by these elements is shown by the residuals O – C in column four under the heading v_1. The first minimum is very uncertain, however, and its retention is a questionable proceeding. A second solution, from which this minimum was excluded, gave

$$\text{Min.} = \text{J. D. } 2414204.80 + 7^d.98919E, \qquad \text{G. M. T.} \qquad \text{(B)}$$

The residuals corresponding to this system are given in column five of Table I. With the exception of the first they are satisfactory. The isolated observation upon which the first position depends is probably affected by an abnormal error. The residuals give no evidence of any irregularity in the distribution of the minima similar to that affecting the maxima. Both systems of elements represent, although not very satisfactorily, the early observation by Müller and Kempf in 1890. The residuals in the observed magnitude, on the basis of the Müller and Kempf light-curve, are $+ 0^{m}.13$ and $+ 0^{m}.27$ respectively.

The question as to the true form of the light-curve cannot be settled so conclusively at present. Owing to the near approach to commensurability between the period of the variable and the mean solar day, the phase values for the Laws Observatory series all cluster about a few mean values, so that only a small number of points on the curve is determined. The course of the curve between these points is as yet uncertain; and, as this is particularly true of those sections in the neighborhood of maximum and minimum, it will be necessary to continue the observations for some time in order to arrive at any final conclusion. The results for the present series are collected in Table II. The mean Δm was formed for each night and the results arranged in order of phase. The series was then subdivided into groups and the means formed for each group. The resulting normal places are in Table II, the mean phase value from minimum being in column two and the mean Δm in column three. Column four contains the average deviation of the result for a single night, usually the mean of two observations, from the normal place itself.

TABLE II

NORMAL PLACES

No.	Phase	Δm	Av. Dev.	No. Nights
1	$0^{d}.26$	$+ 0^{m}.33$	$\pm 0^{m}.056$	5
2	1.32	+ 0.32	0.050	3
3	2.32	+ 0 08	0.108	6
4	3.29	— 0.10	0.036	5
5	4.29	— 0.15	0.055	6
6	5.31	— 0.02	0.070	5
7	6.31	+ 0 16	0.096	7
8	7.30	+ 0.29	± 0.052	8

When plotted the normal places reveal the curve shown in Fig. 1, which is nearly if not quite symmetrical. Possibly the ascending branch is slightly steeper than the descending; but, on the whole, the curve seems to be more nearly related to those of Müller and Kempf, and Luizet, than to the dissymmetrical curves of the other observers. But, as already remarked, further observations are necessary for the determination of the exact location of the maxima and minima.

The values of Δm in Table II are referred to the mean brightness of the comparison stars, which are a = BD. + 20°4218, $6^m.7$, and b = BD. + 20°4179, $7^m.8$. Star a is P.DM. II 3411, $6^m.90$. From the observations themselves, $a - b = -0^m.95$. Approximate magnitudes corresponding to the normal places will therefore be found by adding the values of Δm in Table II to 7.38. The amplitude appears to be somewhat less than that found by other observers.

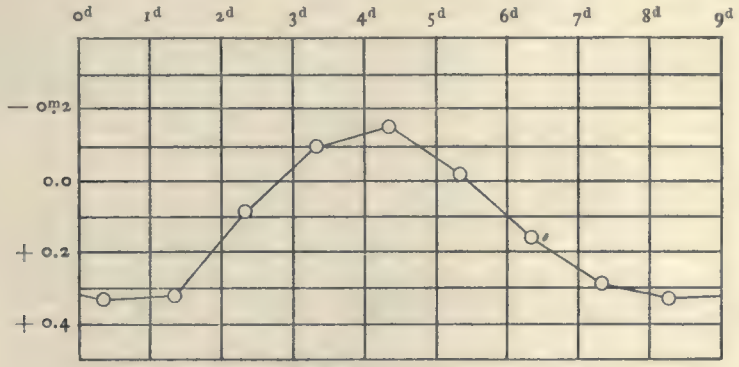

Fig. 1 — Light-curve of U Vulpeculae.

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16. X VULPECULAE (161.1904)—Discovered by Madame Ceraski from plates by Blajko, A. N. 3971. The Laws Observatory measures are in two groups:

a) 1905, July 14 - Aug. 26, 53 observations.

b) 1906, April 17 - Dec. 18, 220 observations.

The observations of 1905 determine 6 maxima; those of 1906, 14 maxima. From these the following elements have been derived:

$$\text{Max.} = \text{J. D. } 2417040.85 + 6^d.317E, \qquad \text{G. M. T.,} \qquad M - m = 2^d.$$

The representation of the maxima is as follows:

Obs. Max.	Comp. Max.	O – C	Obs. Max.	Comp. Max.	O – C
J. D. 7040.8	J. D. 7040.8	$0^d.0$	J. D. 7400.6	J. D. 7400.9	$-0^d.3$
7053.4	7053 4	0.0	7407.2	7407.2	0.0
7059.7	7059.8	— 0.1	7413.8	7413.5	+ 0.3
7066.0	7066.1	— 0.1	7420.0	7419.8	+ 0.2
7078.8	7078.7	+ 0.1	7426.4	7426.2	+ 0.2
7085.2	7085.0	+ 0 2	7432.5	7432.5	0.0
7375.4	7375.6	— 0 2	7439.0	7438.8	+ 0.2
7381.6	7381.9	— 0.3	7445.6	7445.1	+ 0.5
7388.2	7388.3	— 0.1	7451 7	7451.4	+ 0.3
7394.4	7394.6	— 0.2	7477.0	7176.7	+ 0.3

The range is about one magnitude with maxima $0^{m}.5$ brighter than BD. + 26°3739, $8^{m}.9$.

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17. W Vulpeculae (5.1904)—Discovered by Madame Ceraski from plates by Blajko, A. N. 3926. The Laws Observatory measures are in two groups:

a) 1905, July 18 - Oct. 26, 34 observations.

b) 1906, June 8 - Dec. 18, 72 observations.

The 1906 observations show a maximum for J. D. 7515. The 1905 observations are in agreement with the measures of Van Biesbroeck, A. N. 4092, which give a maximum for J. D. 7017. The curve indicates the existence of one intermediate maximum. The following elements may therefore be adopted provisionally:

$$\text{Max.} = \text{J. D. } 2417017 + 249^{d}\text{E.}$$

At maximum the variable is of the ninth magnitude approximately. The range is about one magnitude. The variation seems to be somewhat irregular.

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18. RR Delphini (79.1906)—Algol type. Variability discovered by Madame Ceraski from plates by Blajko, A. N. 4106. Nijland has derived the following elements, A. N. 4120:

$$\text{Min.} = \text{J. D. } 2417424.53 + 4^{d}.601\text{E}, \qquad \text{G. M. T.}$$

Ninety observations were secured during the interval 1906, Aug. 18 - Oct. 9. Upon 31 nights the variable was of normal brightness, which is $0^{m}.17$ fainter than star a = BD. + 13°4504, $9^{m}.4$. Partial minima were observed on Sept. 25 and Oct. 9. The observations of the former date indicate a minimum for J. D. 7479.695, which, compared with the above elements, gives a residual O − C = − $0^{d}.047$. At minimum the variable was $2^{m}.0$ fainter than star a. It was followed for four hours, during which the brightness increased $1^{m}.6$. The observations of Oct. 9 extend over three hours near the end of light change. That part of the curve corresponding to light change appears to be V-shaped, with a slight rounding of the vertices where it joins the line of normal brightness. The total period of light change seems to be 13 or 13½ hours.

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19. VX Cygni (58.1903)—Discovered by Williams, A. N. 3907, who has derived the following elements, A. N. 4010:

$$\text{Max.} = \text{J. D. } 2414934.97 + 20^{d}.125\text{E}, \qquad \text{G. M. T.}$$

One hundred and eighty observations were secured during the interval 1906, July 11 - 1907, Jan. 25. Six maxima are well determined, whose representation by the elements of Williams is as follows:

Obs. Max.	Comp. Max.	O – C
J. D. 7411.0	J. D. 7410.4	+ 0d6
7431.5	7430.5	+ 1.0
7453.0	7450.6	+ 1.4
7472.0	7470.7	+ 1.3
7491.5	7490.8	+ 0.7
7551.8	7551.2	+ 0.6

The general character of the light-curve indicated by the Laws Observatory measures is in agreement with that derived by Williams.

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20. TX Cygni (22.1900)—Variability discovered by Williams, A. N. 3678. Hartwig, Eph., 1907, p. 315, gives the following revised elements :

$$\text{Max.} = \text{J. D. } 2417010.5 + 14^{d}.71E, \qquad \text{G. M. T.}$$

One hundred and twenty-two observations were secured during the interval 1906, July 28 - 1907, Jan. 25. Eight maxima are well determined. Their representation by the above elements is as follows :

Obs. Max.	Comp. Max.	O – C
J. D. 7438.0	J. D. 7437.1	+ 0d9
7452.7	7451.8	+ 0.9
7466.5	7466.5	0.0
7496.5	7495.9	+ 0.6
7511.0	7510.6	+ 0.4
7527.0	7525.4	+ 1.6
7541.5	7540.1	+ 1.4
7557.5	7554.8	+ 2.7

The range of variation is about $1^{m}.2$. At maximum the variable is $0^{m}.5$ brighter than BD. + 42°3937, $9^{m}.2$.

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21. VY Cygni (61.1903)—Variability discovered by Williams, A. N. 3915, who derived the following elements from observations extending over the years 1899 - 1903 :

$$\text{Max.} = \text{J. D. } 2416370.88 + 7^{d}.857E, \qquad \text{G. M. T.}, \qquad m - M = 5^{d}.76, \qquad 8^{m}.79 - 9^{m}.50.$$

The Laws Observatory observations are in two groups :

a) 1905, July 29 - Oct. 26, 24 observations.

b) 1906, June 11 - Sept. 30, 168 observations.

The 1905 observations are scattered; but one minimum is fairly well determined. Those of 1906 show 12 maxima and a number of minima. The representation by the elements of Williams is as follows:

Obs. Min.	Comp. Min.	O – C
J. D. 7083.6	J. D. 7083.8	$-0^d.2$

Obs. Max.	Comp. Max.	O – C	Obs. Max.	Comp. Max.	O – C
J. D. 7384.6	J. D. 7384.4	$+0^d.2$	J. D. 7432.0	J. D. 7431.6	$+0^d.4$
7393.0	7392.3	+ 0.7	7440.0	7439.4	+ 0.6
7400.6	7400.1	+ 0.5	7447.8	7447.3	+ 0.5
7409.2	7408.0	+ 1.2	7463.6	7463.0	+ 0.6
7415.6	7415.9	+ 0 7	7470.6	7470.9	— 0.3
7424.0	7423.7	+ 0.3	7478.6	7478.7	— 0.1

The average O – C is $+0^d.44$. The general character of the light-curve shown by the observations is in agreement with the results of Williams.

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22. YY Cygni (94.1901)—Variability discovered by Deichmüller, A. N. 3751. Graff has derived the following elements, Hartwig, Eph., 1907, p. 309:

$$\text{Max.} = \text{J. D } 2416638 + 378^d\text{E.}$$

The Laws Observatory observations are 38 in number and extend from 1906, June 16 to 1907, Jan. 20. They indicate a maximum for J. D. 7390 which is in substantial agreement with the date J. D. 7394 given by the elements.

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23. SS Cygni—Eighty observations were secured during the interval 1906, July 21 - Dec. 18. At the beginning of the series the star was rapidly diminishing in brightness and it is probable that a maximum occurred about J. D. 7410. Maxima also occurred on or about Julian Days 7455, 7505, and 7548. The first of these seems to have been similar to the maximum illustrated by Parkhurst, A. P. J., v. 12, p. 268. The second was too imperfectly observed to determine its character, while the third appears to have been sharply defined. In each of these cases the rise to maximum was much more gradual than in the outbursts of a few years ago. The distribution of the fluctuations is best represented by the following series of Julian Dates which are the epochs of equality of brightness of the variable and the star *b* =BD. + 42°4186, $9^m.2$: 7417D, 7447A, 7464D, 7513D, 7544A, 7553D, in which the letters A and D indicate the branch, ascending or descending, to which the epochs correspond. At maximum, the variable was, on the average, one magnitude brighter than *b*. The amplitude shown by the observations is $3^m.6$.

24. VZ Cygni (7.1904)—Variability was discovered by Madame Ceraski from plates by Blajko, A. N. 3932. In A. N. 4108 Blajko states that the light-curve has two minima of $9^m.1$, and two maxima of $8^m.3$. He finds the minima to be sharper than the maxima and to have a distribution such that Min. II – Min. I = $5^d.0$, while Min. I – Min. II = $4^d.7$. From visual observations during 1904-5, and photographs extending over the years 1898-1900, Blajko derives the elements:

$$\text{Min. I} = \text{J. D. } 2417060.1 + 9^d.727E, \qquad \text{G. M. T.}$$

Hartwig, Eph., 1907, p. 306, states, "Meine Beobachtungen bestätigen bis jetzt im grossen und ganzen die merkwürdige Lichtkurve." The Laws Observatory measures are in two groups :

a) 1905, Aug. 1 - 26, 28 observations.

b) 1906, June 10 - 1907, Jan. 27, 228 observations.

In view of the interest attaching to a star of the character indicated, it seemed advisable to carry through a preliminary reduction for the formation of the light-curve at once.

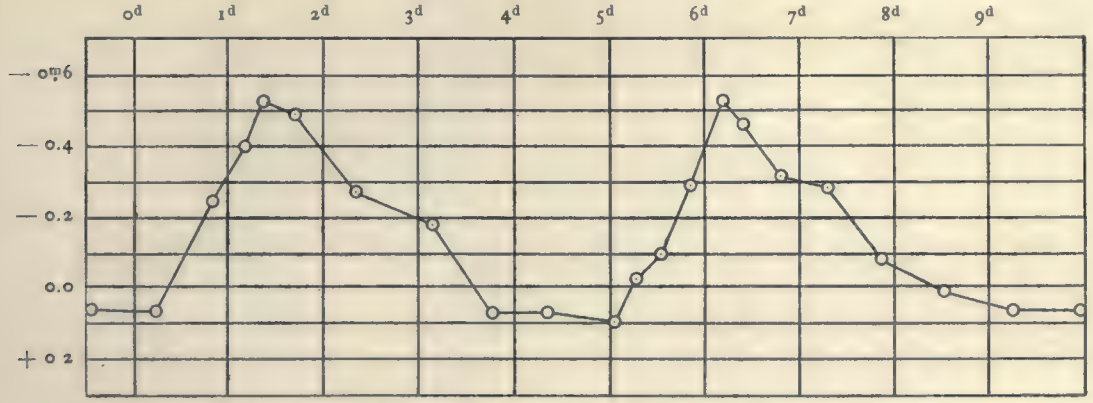

Fig. 2 — Normal Places for VZ Cygni. Referred to BD. + 42°4246, $8^m.0$.

The observations were referred to the comparison stars *a* = BD. + 42°4246, $8^m.0$, and *b* = BD. + 42°4247, $8^m.3$. Usually the measures of each night included settings on both *a* and *b*. From these the magnitude difference *a* – *b* was found to be + $0^m.72$, which value, it will be observed, is opposite in sign to that given by the BD. magnitudes. With this difference the observations referred to *b* were reduced to *a*, and the mean magnitude difference formed for each night. Blajko's elements were then used to compute the phase, referred to Min. I, for each mean result. The means by nights were arranged in order of phase and combined into normal places in the usual manner. These normals are shown in Table III. Column two contains the mean phase, and column four, the mean magnitude difference. The number of nights included in each normal is given in the last column. The results are also shown graphically in Fig. 2. Two facts are at once evident : First, the curve given by observations made here is quite different in character from that derived by Blajko. The maxima are sharp and well-defined, while the minima

are flat and uncertain. Just the opposite conditions prevail for the curve of Blajko. Second, the two sections of the curve, Min. I to Min. II, and Min. II to Min. I, are so nearly identical in form that there is no certain evidence of the inequality in the distribution of the maxima and minima found by Blajko from his observations. It was seen quite early in the reduction that such might be the result, and particular care was taken in forming the normals not to displace the points of maxima through an injudicious grouping or weighting of the observations used in

TABLE III

NORMAL PLACES

No.	Phase from Min. P = 9.727	Phase from Max. P = 4.86	Obs. Δm	Comp. Δm	O − C	No. Nights
1	0.23	3.72	+ 0.06	+ 0.06	0	6
2	0.84	4.33	— 0.25	— 0.20	— 5	5
3	1.18	4.67	— 0.40	— 0.40	0	4
4	1.37	0.00	— 0.53	— 0.53	0	3
5	1.70	0.33	— 0.49	— 0.46	— 3	6
6	2.33	0.96	— 0.27	— 0.28	+ 1	13
7	3.15	1.78	— 0.18	— 0.10	— 8	6
8	3.76	2.39	+ 0.07	+ 0.03	+ 4	5
9	4.35	2.98	+ 0.07	+ 0.08	— 1	8
10	5.02	3 65	+ 0.10	+ 0.08	+ 2	4
11	5.28	3.91	— 0.03	— 0.02	— 1	5
12	5.53	4.16	— 0.10	— 0.13	+ 3	3
13	5.82	4.45	— 0.29	— 0.30	+ 1	6
14	6.20	4.83	— 0.53	— 0.53	0	3
15	6.40	0.17	— 0.46	— 0.48	+ 2	3
16	6.80	0.57	— 0.31	— 0.38	+ 7	6
17	7.29	1.06	— 0.28	— 0.26	— 2	8
18	7.88	1.65	— 0.08	— 0.13	+ 5	5
19	8.52	2.29	+ 0.02	+ 0.02	0	5
20	9.24	3.01	+ 0.06	+ 0.08	— 2	5
21	9.58	3.35	+ 0.09	+ 0.09	0	5

determining these points. Of the three nights' observations entering into the normal which fixes Max. I, two have the same phase value, viz., 1.4, while that for the third is 1.3. For Max. II the observations combined all have the same phase value, viz., 6.2. From the data in the table we find :

Interval, Max. II — Max. I = 4.83,
Interval, Max. I — Max. II = 4.90.

A similar result was derived from the curve of the individual observations. After the mean light-curve had been derived, a tracing was made showing that part of the curve near a

maximum. This was repeated on the tracing paper at intervals corresponding to a period of $9^d.727$. The paper was then superposed upon the plot of the mean results by nights and shifted until the curve maxima coincided as nearly as possible with the principal maxima shown by a selected group of observations. The epoch of one of the maxima was then read from the chart. The value thus determined represents, in a measure, the mean maximum of the group, and is uncertain by not more than $0^d.08$. Other groups of observations defining principal maxima were treated in the same way, and when this had been accomplished, tne process was repeated for the secondary maxima. From the epochs thus derived, the following series of values was obtained for the interval Max. II – Max. I: 4.8, 4.6, 4.8, 5.0, and 5.0 days, with a mean of $4^d.84$, which is in close agreement with the value derived from the mean light-curve. The complementary interval, Max. I – Max. II is $4^d.89$. It therefore appears that for the observations in question the difference between the two intervals is well within the uncertainty of the measures themselves.

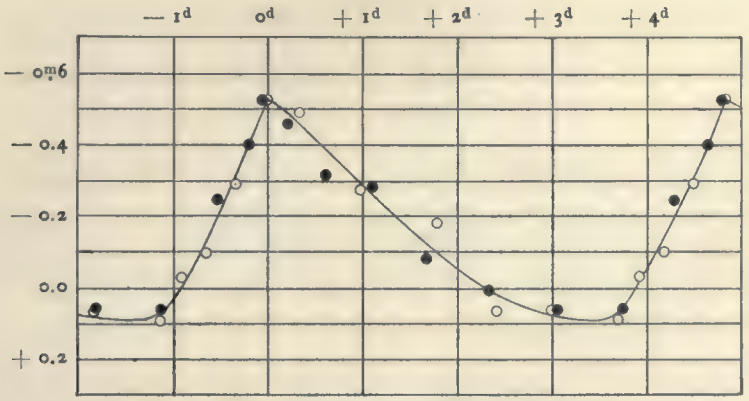

Fig. 3 — Light-curve of VZ Cygni.
Referred to BD. + 42°4246, $8^m.0$.

The similarity of form of the two sections of the curve was then investigated. To accomplish this, the phase values of the normals were calculated on the basis of a period of $4^d.86$. These were referred to Max. I as an initial point, and are to be found in column three of Table III. The values of the magnitude difference in column four were then plotted with the new phase values as abscissæ and a mean light-curve drawn. The ordinates of this curve, corresponding to the phase values in column three, are in column five. The residuals are in column six. The average deviation of the normals lying between Max. I and Max. II is $— 0^m.002$; for those between Max. II and Max. I, $+ 0^m.005$. These numbers are also within the uncertainty of the observations, and do not, therefore, indicate any dissimilarity in the two sections of the curve. The plot of the normals on the assumption that the period is $4^d.86$, and the corresponding curve, are shown in Fig. 3. The circles in outline in this figure indicate the position of those

normals which fall between Max. I and Max. II, as defined by the elements of Blajko, while the full circles show those which lie between Max. II and Max. I.

Finally, a series of mean maxima, read from the plot of the means by nights, by the method explained above, was used to test the accuracy of the period entering into the preceding discussion. An independent solution of the equations of condition afforded by these maxima gave for the period $4^d.864$. This is sensibly one-half the value given by Blajko. The representation of the maxima used is as follows:

Obs. Mean Max.	Comp. Max.	O – C
J. D. 7062.0	J. D. 7062.00	$0^d.00$
7387.8	7387.88	— 0.08
7426.75	7426.79	— 0.04
7461.0	7460.84	+ 0.16
7494 8	7494.89	— 0.09
7533.8	7533.80	0.00
7582.5	7582.44	+ 0.06

In conclusion, therefore, it is evident that the Laws Observatory measures are well satisfied by the elements

$$\text{Max.} = \text{J. D. } 2417062.00 + 4^d.864E, \qquad \text{G. M. T.}, \qquad M - m = 1^d.46.$$

It will be of interest to see whether these elements will represent future observations, or whether it will be necessary to admit the existence of the irregularity in distribution of maxima and minima found by Blajko.

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25. 70.1905 Pegasi—Variability discovered by Mrs. Fleming as a result of an examination of nine H. C. O. plates made during the years 1892 - 1904. The announcement in H. C. O. Cir. 98 gives the approximate photographic range as $8^m.1$ - $9^m.4$, but states further that the visual range is greater. The Laws Observatory measures are as follows:

a) 1905, Aug. 4 - Oct. 26, 18 observations.

b) 1906, June 11 - 1907, Jan. 20, 55 observations.

At the beginning of the 1905 series the variable was $0^m.2$ brighter than $a = \text{BD.} + 21°4658$, $8^m.3$, and increased in brightness $0^m.4$ during the next 17 days. The remaining 1905 observations are too few in number to permit of any conclusion. On Oct. 26 the variable was $0^m.3$ brighter than a. At the beginning of the 1906 series the variable was equal to a. It then decreased to a minimum $0^m.3$ fainter than a, which occurred on J. D. 7400. It was again equal to a about J. D. 7430, and reached a maximum $0^m.3$ brighter than a on J. D. 7490 or thereabout. The date of the maximum is uncertain on account of the flatness of the curve. These results are in substantial agreement with those given by Hartwig, Eph., 1907, p. 310. A second minimum seems

to have occurred about J. D. 7575. The following is therefore a rough approximation for the elements :

$$\text{Min.} = \text{J. D. } 2417400 + 175^{d}\text{E.}$$

---

26. 88.1906 LACERTAE—Variability was discovered here during observations on V Lacertae. Fifty observations and the elements

$$\text{Min.} = \text{J. D. } 2417412.8 + 5^{d}.44\text{E}, \qquad \text{G. M. T.}$$

were published in Bulletin No. 8. At the time of publication an uncertainty existed in the period. This has been removed by 106 additional observations made during the interval 1906, Sept. 7 - 1907, Feb. 8, which show that the published value of the period is correct. The representation of the later observations by the above elements is as follows :

Obs. Min.	Comp. Min.	O – C
J. D. 7467.2	J. D. 7467.20	0d.00
7472.7	7472.64	+ 0.06
7478.2	7478.08	+ 0.12
7526.5	7527.04	— 0.54
7543.3	7543.36	— 0.06
7564.8	7565.12	— 0.32
7598.0	7597.76	+ 0.24

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27. RZ ANDROMEDAE (166.1904)—Variability discovered by Graff, A. N. 3980, who has published observations extending from 1903, Feb. 18, to 1904, Oct. 3. These show a maximum for J. D. 6375. Graff estimates the period to be 8 or 9 months, although the curve of his observations seems to require a somewhat larger value. Fifty-two Laws Observatory observations covering the interval 1906, June 16 - 1907, March 5, show a minimum for J. D. 7500, and a maximum for J. D. 7575. The curve is so flat, however, that these epochs are very uncertain. If Graff's estimate of the period is correct, there must be five cycles included between the maxima on J. D. 6375 and J. D. 7575. The corresponding period is 240^{d}. The assumption of four intermediate cycles gives 300^{d}. The data is insufficient for the derivation of a definite result.

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28. SS ANDROMEDAE (167.1904)—Variability discovered by Graff, A. N. 3980. The Laws Observatory observations are the same in number and cover the same interval as those for RZ Andromedae (166.1904). A maximum occurred about J. D. 7495, and minima are determined for J. D. 7435 and J. D. 7598. These minima were combined with three others derived from the observations of Graff, A. N. 3980, for the determination of the following elements :

$$\text{Min.} = \text{J. D. } 2416280 + 165^{d}.8\text{E}, \qquad m - \text{M} = 108^{d}$$

The representation of the observations is as follows :

Obs. Min.	Comp. Min.	O – C	Observer
J. D. 5946	J. D. 5940	+ 6^d	Hartwig
6280	6272	+ 8	Graff
6603	6603	0	Graff
6757	6769	— 12	Graff
7435	7432	+ 3	Seares & Haynes
7598	7598	0	Haynes

The minimum by Hartwig is from A. N. 4127. It was not used for the determination of the elements, since the epoch as published is uncertain. Several maxima are also given by Hartwig in the same place. The second of these, for J. D. 5815, and the value J. D. 7495 given above, are in agreement with P = 165^d.8. A maximum for J. D. 6660 from the observations by Graff is also satisfied by the above elements. The remaining maxima by Hartwig are not well represented ; but this is to be expected, inasmuch as the curve is flat at the times of greatest light.

The Laws Observatory measures give no indication of the secondary or intermediate minima suggested by Graff, A. N. 3980, and Hartwig, Eph., 1907, p. 310. The range seems to be about one magnitude, although there is some reason for believing that the amplitude is not constant. Minima should occur on 1907, July 7 and Dec. 20.

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29. RU Aquarii (66.1901)—Variability announced in H. C. O. Cir. 54, also in A. N. 3695. Twenty-six observations were secured by Mr. Haynes during the interval 1906, Oct. 11 - 1907, Jan. 21. The star passed a maximum about J. D. 7522. A minimum followed 40 days later. The period appears to be 2 or 2½ months. The range is one magnitude, with a maximum equal to BD. — 18°6297, 8^m.7.

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30. RS Cassiopeiae (108.1904)—Variability discovered by Madame Ceraski from plates by Blajko, A. N. 3953. From 258 observations extending from 1906, July 18, to 1907, Feb. 26, Mr. Haynes has derived the following elements :

$$\text{Min.} = \text{J. D. } 2417412.50 + 6^d.298E, \qquad \text{G. M. T.,} \qquad M - m = 1^d.90.$$

The range is about 0^m.9. At maximum the variable is 0^m.1 fainter than BD. + 61°2486, 9^m.1.

The representation of the observations is as follows :

Obs. Min.	Comp. Min.	O – C	Obs. Min.	Comp. Min.	O – C
J. D. 7412.7	J. D. 7412.5	+ 0d2	J. D. 7481.8	J. D. 7481.8	0d0
7418.8	7418.8	0.0	7494.5	7494.4	+ 0.1
7425.0	7425.1	— 0.1	7513.5	7513.3	+ 0.2
7431.4	7431.4	0.0	7551.2	7551.1	+ 0.1
7437.6	7437.7	— 0.1	7557.2	7557.4	— 0.2
7462.8	7462.9	— 0.1	7563.6	7563.7	— 0.1
7469.2	7469.2	0.0	7601.0	7601.4	— 0.4
7475.4	7475.5	— 0.1	7620.5	7620.3	+ 0.2

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31. 52.1906 ANDROMEDAE—Variability discovered by Mrs. Fleming, H. C. O. Cir. 111. Five H. C. O. plates made between 1890 and 1899 show an approximate range of $8^m.2 - < 10^m.5$. Fifty-two observations were made here during the interval 1906, June 20 - 1907, March 1. At the beginning of the series on J. D. 7383 the variable was $0^m.55$ brighter than b = BD. + 34°4976, $9^m.1$. The star gradually declined to a minimum on J. D. 7545, when it was $0^m.9$ fainter than b. From this date the star steadily increased in brightness. On March 1 it was $0^m.80$ brighter than b. It is impossible at present to estimate the period, although it is apparently greater than 250 days.

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32. RS ANDROMEDAE—The variability of this star was announced in H. C. O. Cir. 24 on the basis of 48 plates which show a range from $9^m.3$ to $9^m.8$. Hartwig, Eph. 1904, p. 246, states that the amplitude, visual presumably, is $0^m.75$, and adds, further, that the period is short. The designation, "Kurze Periode," appears in all the later editions of the *Ephemeriden*. The star was observed here upon 48 nights between the dates 1906, June 21 and Sept. 23. In all 91 observations were secured. During the earlier part of the series the variable was referred on each night to both of the comparison stars, a = BD. + 47°4312, $7^m.5$, and b = BD. + 47°4311, $8^m.4$. The later observations were referred to b alone. Part of the measures indicate a possible fluctuation in light, but the evidence is by no means conclusive. To exhibit the results more clearly the series has been subdivided into four groups. The observations referred to a were reduced to b and the mean Δm formed for each group. The first column of the table shows the limiting Julian Dates for each group; the second gives the corresponding mean magnitude difference in the sense $v - b$; the third, the average deviation of the mean of a single night from the mean of the group; and the last column, the number of nights in each group.

Limiting J. Ds.	Δm	Av. Resid.	No. Nights
7383 — 7415	— 0.18	± 0.06	10
7416 — 7434	— 0.22	0.13	11
7436 — 7447	— 0.17	0.04	7
7449 — 7476	— 0.24	± 0.03	19

The results for all but the second group are such that no variation can legitimately be inferred. The measures included in the second group are less regular. They show an amplitude of variation amounting to $0^m.51$; but if one discordant observation be rejected, this is reduced to $0^m.35$. The variation is perhaps real, although its amount is not greater than possible errors of observation. It should be remarked further that the result for J. D. 7440 has been excluded in forming the means in group three. The observations for this date give $v - b = + 0^m.22$. The values for both preceding and following nights are $- 0^m.15$. The variable was also observed on 1906, Dec. 12 with the result $v - b = - 0^m.30$.

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33. 53.1906 ANDROMEDAE—Suspected of variability by Espin and independently discovered by Mrs. Fleming, H. C. O. Cir. 111. Ten H. C. O. plates made during the years 1891 - 1903 give an approximate range of $8^m.3$ to $9^m.8$. The observations made here are 40 in number and extend from 1906, June 19 to 1907, Feb. 21. They show a gradual and uniform decline in brightness of $0^m.4$ which ended at J. D. 7596. This was followed by a slight but definite increase, so that a minimum probably occurred on or about the date mentioned. It is impossible to estimate the period at present.

Columbia, Missouri, 1907, March 11. *F. H. Seares.*

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BULLETIN NO. 11

THE VARIABLE RS CASSIOPEIAE (108.1904)[1]

BD.+61°2487, $9^m.3$, 1855.0 R.A. $23^h30^m29^s$ Dec.+61°37′.7
1900.0 32 33 52.6

The variability of this star was discovered by Madame Ceraski from plates made by Blakjo at Moscow, and was announced in A. N. No. 3953. The photographs indicated a variation from the ninth to the eleventh magnitude approximately. Observations by Miss Whitney at the Vassar College Observatory, A. N. No. 4050, confirmed the variation and gave a range in brightness of $9^m.7$ to $10^m.4$. The following provisional elements, based upon observations made here, were announced in Laws Observatory Bulletin No. 10.

$$\text{Min.} = \text{J.D.}2417412.50 + 6^d.298E, \qquad \text{G.M.T.}$$

The present paper contains a revision of these elements and a discussion of all the observations thus far obtained.

Measures were made upon 101 nights during the interval 1906, July 18, to 1907, March 5. In all, 262 observations were secured. Of these 68 are by Professor Seares, all made between 1906, July 18, and November 9. The remaining 194 observations are by me. My own observations antedating 1907, January 1 were made with the right eye. Since that time it has been necessary to use the left eye, a circumstance which may have introduced a slight systematic difference into the series.

The instrument used was the equalizing photometer with the Zeiss wedge, described in Bulletin No. 7, attached to the 7½-inch equatorial.

The comparison stars used were a = BD.+61°2486, $9^m.1$, and b = BD.+61°2481, $9^m.4$. As usual, the settings were so arranged as to give not only the differences $v-a$ and $v-b$, but also that of $b-a$, and, at the same time, to eliminate any uniform changes in atmospheric extinction and in the intensity of the comparison star.

[1]The observations and discussion contained in this Bulletin have been made, in part, with the assistance of a grant from the Gould Fund of the National Academy of Sciences.

TABLE I

OBSERVATIONS OF RS CASSIOPEIAE

Date	G. M. T.	Julian Day	Phase	Obs.	Star	Wedge		$b-a$	Δm	Mag.	O − C
1906, July 18	$19^h\ 37^m$	2417410.82	$4^d.51$	S	*a*	24.1	28.1	$+0^m.45$	$+0^m.43$	$9^m.52$	− 18
				S	*b*	28.3	29.0		+ 0.08	9.77	+ 7
20	17 42	7412.74	0.13	H	*a*	21.4	30.3	0.47	+ 0.96	10.05	+ 1
				H	*b*	25.8	30.4		+ 0.50	10.19	+ 15
	20 4	.84	0.23	S	*a*	19.0	27.9	0.61	+ 0.95	10.04	+ 2
				S	*b*	24.8	28.9		+ 0.45	10.14	+ 12
21	16 14	7413.68	1.07	S	*a*	26.0	29.6	0.67	+ 0.40	9.49	+ 2
				S	*b*	32.0	30.0		− 0.23	9.46	− 1
23	17 15	7415.72	3.11	H	*a*	25.6	27.8	0.49	+ 0.24	9.33	− 12
				H	*b*	30.1	28.4		− 0.19	9.50	+ 5
24	18 10	7416.76	4.15	S	*a*	24.6	28.6	0.45	+ 0.44	9.53	− 10
				S	*b*	28.7	28.4		− 0.04	9.65	+ 2
25	17 28	7417.73	5.12	H	*a*	27.6	34.3	0.52	+ 0.74	9.83	0
				H	*b*	32.3	34.4		+ 0.23	9.92	+ 9
26	16 2	7418.67	6.06	S	*a*	31.0	39.8	[0.57]	+ 0.96	10.05	+ 2
	6	.67	6.06	S	*b*	36.1	39.6		+ 0.37	10.06	+ 3
28	18 40	7420.78	1.88	H	*a*	23.4	25.4	0.75	+ 0.20	9.29	0
				H	*b*	30.4	25.1		− 0.58	9.11	− 18
29	17 52	7421.74	2.84	S	*a*	25.0	27.1	0.56	+ 0.23	9.32	− 9
				S	*b*	30.1	26.4		− 0.41	9.28	− 13
30	17 17	7422.72	3.82	H	*a*	27.6	31.5	0.49	+ 0.43	9.52	− 5
				H	*b*	32.0	31.7		− 0.04	9.65	+ 8
Aug. 1	17 57	7424.75	5.85	H	*a*	29.8	37.5	0.44	+ 0.85	9.94	− 6
				H	*b*	33.7	37.4		+ 0.40	10.09	+ 9
3	16 48	7426.70	1.51	S	*a*	27.9	31.5	0.65	+ 0.40	9.49	+ 19
				S	*b*	33.7	31.7		− 0.23	9.46	+ 16
8	15 43	7431.65	0.17	H	*a*	29.6	36.5	0.41	+ 0.77	9.86	− 18
				H	*b*	33.3	36.6		+ 0.37	10.06	+ 2
9	17 22	7432.72	1.24	S	*a*	27.6	29.9	0.43	+ 0.25	9.34	− 5
				S	*b*	31.5	28.5		− 0.33	9.36	− 3
11	18 6	7434.75	3.27	H	*a*	24.9	29.6	0.32	+ 0.52	9.61	+ 13
				H	*b*	27.8	29.9		+ 0.23	9.92	+ 44
13	17 55	7436.75	5.27	S	*a*	26.1	32.1	0.53	+ 0.67	9.76	− 11
				S	*b*	30.9	31.6		+ 0.08	9.77	− 10
14	16 37	7437.69	6.21	H	*a*	26.0	33.9	0.56	+ 0.88	9.97	− 8
				H	*b*	31.1	33.9		+ 0.32	10.01	− 4
15	17 16	7438.72	0.95	S	*a*	25.5	29.8	0.62	+ 0.47	9.56	+ 3
				S	*b*	31.2	29.5		− 0.18	9.51	− 2
16	16 40	7439.69	1.92	H	*a*	20.6	24.0	+ 0.81	+ 0.36	9.45	+ 16
				H	*b*	28.2	23.5		− 0.50	9.19	− 10

Date	G. M. T.	Julian Day	Phase	Obs.	Star	Wedge		$b-a$	Δm	Mag.	O − C
1906, Aug. 17	17h 3m	2417440.71	2d.94	S	*a*	20.4	27.0	+ 0m.97	+ 0m.70	9m.79	+ 36
				S	*b*	29.5	26.8		− 0.29	9.40	− 3
18	15 54	7441.66	3.89	H	*a*	26.2	32.8	0.65	+ 0.74	9.83	+ 25
				H	*b*	32.0	32.7		+ 0.08	9.77	+ 19
22	16 46	7445.70	1.63	S	*a*	21.6	23.7	0.61	+ 0.22	9.31	+ 2
				S	*b*	27.4	24.9		− 0.27	9.42	+ 13
23	17 51	7446.74	2.67	H	*a*	17.0	20.1	0.60	+ 0.36	9.45	+ 6
				H	*b*	22.4	20.4		− 0.21	9.48	+ 9
24	17 56	7447.75	3.68	S	*a*	21.2	27.3	0.57	+ 0.65	9.74	+ 19
				S	*b*	26.5	27.8		+ 0.14	9.83	+ 28
27	16 0	7450.67	0.31	S	*a*	23.3	31.8	0.70	+ 0.93	10.02	+ 3
				S	*b*	29.8	32.3		+ 0.28	9.97	− 2
28	17 6	7451.71	1.35	H	*a*	20.6	24.1	0.76	+ 0.37	9.46	+ 11
				H	*b*	27.7	24.2		− 0.38	9.31	− 4
29	15 59	7452.67	2.31	S	*a*	24.0	26.9	0.69	+ 0.31	9.40	+ 6
				S	*b*	30.4	26.4		− 0.44	9.25	− 9
	16 8	.67	2.31	H	*a*	26.1	28.5	0.61	+ 0.27	9.36	+ 2
				H	*b*	31.6	28.7		− 0.32	9.37	+ 3
30	13 48	7453.58	3.22	S	*a*	30.0	33.9	0.62	+ 0.44	9.53	+ 6
				S	*b*	35.5	33.9		− 0.18	9.51	+ 4
	55	.58	3.22	H	*a*	28.6	33.2	0.63	+ 0.51	9.60	+ 13
				H	*b*	34.3	33.0		− 0.14	9.55	+ 8
Sept. 5	14 12	7459.59	2.94	S	*a*	27.2	31.4	0.82	+ 0.47	9.56	+ 13
				S	*b*	34.5	32.6		− 0.21	9.48	+ 5
	30	.60	2.95	H	*a*	25.6	29.3	0.75	+ 0.40	9.49	+ 6
				H	*b*	32.4	29.7		− 0.30	9.39	− 4
	16 59	.71	3.06	S	*a*	25.6	28.1	0.71	+ 0.27	9.36	− 9
				S	*b*	32.0	28.3		− 0.42	9.27	− 18
	17 7	.71	3.06	H	*a*	24.0	27.9	0.62	+ 0.42	9.51	+ 6
				H	*b*	29.7	27.8		− 0.21	9.48	+ 3
6	14 24	7460.60	3.95	H	*a*	27.8	33.1	0.65	+ 0.59	9.68	+ 9
				H	*b*	33.6	32.5		− 0.13	9.56	− 3
	16 54	.70	4.05	H	*a*	23.5	29.9	0.72	+ 0.69	9.78	+ 17
				H	*b*	30.2	30.2		0.00	9.69	+ 8
7	14 46	7461.62	4.97	H	*a*	26.7	33.0	0.54	+ 0.70	9.79	− 1
				H	*b*	31.6	33.3		+ 0.19	9.88	+ 8
	17 18	.72	5.07	S	*a*	26.2	31.3	0.68	+ 0.57	9.66	− 16
				S	*b*	32.3	32.1		− 0.02	9.67	− 15
8	15 32	7462.65	6.00	S	*a*	22.6	32.2	0.59	+ 1.04	10.13	+ 10
				S	*b*	28.2	32.2		+ 0.45	10.14	+ 11
9	14 16	7463.59	0.65	H	*a*	22.5	27.9	+ 0.49	+ 0.57	9.66	− 8
				H	*b*	27.2	28.7		+ 0.17	9.86	+ 12

Date	G. M. T.	Julian Day	Phase	Obs.	Star	Wedge		$b-a$	Δm	Mag.	O − C
1906, Sept. 10	16h 48m	2417464.70	1d.76	H	*a*	23.5	26.8	+ 0m.36	+ 0m.35	9m.44	+ 15
				H	*b*	26.9	26.8		− 0.01	9.68	+ 39
	17 40	.74	1.80	S	*a*	26.1	28.5	0.56	+ 0.27	9.36	+ 7
				S	*b*	31.2	27.4		− 0.42	9.27	− 2
12	15 28	7466.64	3.70	H	*a*	19.4	21.4	0.47	+ 0.21	9.30	− 25
				H	*b*	23.8	21.6		− 0.23	9.46	− 9
	18 12	.76	3.82	S	*a*	27.3	31.6	0.58	+ 0.48	9.57	0
				S	*b*	32.5	30.5		− 0.23	9.46	− 11
14	16 11	7468.67	5.73	S	*a*	23.1	29.8	0.50	+ 0.72	9.81	− 16
				S	*b*	27.8	30.1		+ 0.25	9.94	− 3
15	15 28	7469.64	0.41	S	*a*	26.0	33.2	0.54	+ 0 80	9.89	− 3
				S	*b*	30.9	33.2		+ 0.26	9.95	+ 3
16	14 17	7470.60	1.37	H	*a*	27.5	30.3	0.58	+ 0.31	9.40	+ 5
				H	*b*	32.6	30.5		− 0.24	9.45	+ 10
17	15 35	7471.65	2.42	H	*a*	26.5	28.8	0.55	+ 0.25	9.34	− 1
				H	*b*	31.5	28.8		− 0.30	9.39	+ 4
18	15 34	7472.65	3.42	S	*a*	22.5	27.2	0.70	+ 0.50	9.59	+ 9
				S	*b*	29.0	28.0		− 0.11	9.58	+ 8
19	15 39	7473.65	4.42	H	*a*	21.6	25.8	0.45	+ 0.44	9.53	− 15
				H	*b*	25.9	25.4		− 0.06	9.63	− 5
20	14 40	7474.61	5.38	H	*a*	21.8	28.6	0.66	+ 0.73	9.82	− 7
				H	*b*	28.0	28.6		+ 0.07	9.76	− 13
	16 43	.70	5.47	S	*a*	26.3	32.3	0.46	+ 0.67	9.76	− 15
				S	*b*	30.5	31.5		+ 0.12	9.81	− 10
21	15 13	7475.63	0.10	S	*a*	25.6	33.1	0.66	+ 0.83	9.92	− 13
				S	*b*	31.6	34.2		+ 0.29	9.98	− 7
	20	.64	0.11	H	*a*	22.4	30.3	0.44	+ 0.85	9.94	− 11
				H	*b*	26.5	30.6		+ 0.45	10.14	+ 9
22	15 10	7476.63	1.10	H	*a*	21.4	24.6	0.56	+ 0.34	9.43	− 2
				H	*b*	26.6	24.7		− 0.21	9.48	+ 3
	20	.64	1.11	S	*a*	24.9	26.5	0.41	+ 0.18	9.27	− 18
				S	*b*	28.6	27.5		− 0.12	9.57	+ 12
23	15 28	7477.64	2.11	H	*a*	18.8	22.6	0.53	+ 0.41	9.50	+ 19
				H	*b*	23.8	23.1		− 0.07	9.62	+ 31
24	15 1	7478.63	3.10	H	*a*	25.0	27.1	0.49	+ 0.23	9.32	− 14
				H	*b*	29.5	27.5		− 0.22	9.47	+ 1
	8	.63	3.10	S	*a*	28.5	32.0	0.62	+ 0.39	9.48	+ 2
				S	*b*	34.1	32.0		− 0.23	9.46	0
25	15 26	7479.64	4.11	S	*a*	27.4	33.7	0.58	+ 0.71	9.80	+ 17
				S	*b*	32.5	33.6		+ 0.12	9.81	+ 18
	33	.65	4.12	H	*a*	26.0	31.0	+ 0.44	+ 0.55	9.64	+ 1
				H	*b*	30.0	30.8		+ 0.09	9.78	+ 15

Date	G. M. T.	Julian Day	Phase	Obs.	Star	Wedge		$b-a$	Δm	Mag.	O − C
1906, Sept. 26	$15^h\ 16^m$	2417480.64	$5^d.11$	H	a	25.2	32.8	$+0^m.62$	$+0^m.85$	$9^m.94$	+11
				H	b	30.8	33.1		+0.26	9.95	+12
Oct. 1	15 30	7485.65	3.83	H	a	21.2	24.5	0.63	+0.35	9.44	−13
				H	b	27.1	24.9		−0.24	9.45	−12
	40	.65	3.83	S	a	24.4	28.1	0.60	+0.40	9.49	−8
				S	b	29.9	27.9		−0.22	9.47	−10
6	13 27	7490.56	2.45	H	a	19.9	22.9	0.46	+0.32	9.41	+5
				H	b	24.3	22.7		−0.16	9.53	+17
	15 59	.67	2.56	H	a	29.0	31.1	0.58	+0.23	9.32	−5
				H	b	34.2	31.3		−0.32	9.37	0
	17 9	.71	2.60	H	a	23.2	26.4	0.61	+0.33	9.42	+4
				H	b	28.9	26.4		−0.28	9.41	+3
	18 2	.75	2.64	S	a	25.7	29.7	0.42	+0.44	9.53	+14
				S	b	29.5	29.6		+0.01	9.70	+31
	10	.76	2.65	H	a	24.3	28.6	0.51	+0.47	9.56	+17
				H	b	29.0	28.3		−0.08	9.61	+22
	19 3	.79	2.68	H	a	24.1	28.6	0.60	+0.49	9.58	+19
				H	b	29.6	28.7		−0.10	9.59	+20
7	17 58	7491.75	3.64	H	a	27.6	32.0	0.47	+0.49	9.58	+4
				H	b	31.8	31.7		−0.02	9.67	+13
9	17 52	7493.74	5.63	S	a	25.9	31.7	0.48	+0.64	9.73	−23
				S	b	30.3	31.9		+0.19	9.88	−8
10	20 18	7494.85	0.45	H	a	30.4	37.0	0.64	+0.74	9.83	−6
				H	b	36.1	37.2		+0.12	9.81	−8
11	16 2	7495.67	1.27	H	a	25.8	29.2	0.62	+0.37	9.46	+8
				H	b	31.4	29.0		−0.27	9.42	+4
12	14 46	7496.62	2.22	H	a	27.8	30.0	0.51	+0.24	9.33	0
				H	b	32.4	30.1		−0.26	9.43	+10
18	18 16	7502.76	2.07	H	a	28.9	31.4	0.52	+0.28	9.37	+6
				H	b	33.6	30.8		−0.31	9.38	+7
19	18 0	7503.75	3.06	H	a	29.8	32.5	0.56	+0.30	9.39	−6
				H	b	34.8	31.5		−0.37	9.32	−13
27	16 57	7511.71	4.72	H	a	28.9	33.1	0.60	+0.47	9.56	−18
				H	b	34.3	33.0		−0.14	9.55	−19
29	13 42	7513.57	0.29	H	a	28.5	35.6	0.51	+0.79	9.88	−11
				H	b	33.1	35.8		+0.30	9.99	0
30	18 28	7514.77	1.49	H	a	30.0	31.0	0.57	+0.11	9.20	−10
				H	b	35.1	30.9		−0.47	9.22	−8
31	17 12	7515.72	2.44	H	a	25.8	28.1	0.56	+0.25	9.34	−2
				H	b	30.9	28.1		−0.31	9.38	+2
Nov. 1	17 10	7516.71	3.43	H	a	25.5	28.3	+0.54	+0.31	9.40	−10
				H	b	30.4	28.4		−0.22	9.47	−3

Date	G. M. T.	Julian Day	Phase	Obs.	Star	Wedge		$b-a$	Δm	Mag.	O−C
1906, Nov. 8	$14^h\ 10^m$	2417523.59	4.02	H	a	29.4	36.1	$+0^m.72$	$+0^m.75$	$9^m.84$	+23
				H	b	35.8	36.1		+0.03	9.72	+11
9	15 51	7524.66	5.09	S	a	32.0	36.2	0.47	+0.46	9 55	−27
				S	b	36.3	37.4		+0.12	9.81	−1
12	18 0	7527.75	1.89	H	a	21.2	21.1	0.61	−0.01	9.08	−21
				H	b	26.9	21.1		−0.62	9.07	−22
21	16 59	7536.71	4.56	H	a	23.3	29.8	0.57	+0.70	9.79	+8
				H	b	28.6	29.7		+0.12	9.81	+10
26	16 21	7541.68	3.24	H	a	22.6	27.6	0.80	+0.53	9.62	+14
				H	b	30.1	28.1		−0.22	9.47	−1
27	15 18	7542 64	4.20	H	a	24.5	28.6	0.66	+0.45	9.54	−10
				H	b	30.5	28.6		−0.21	9.48	−16
Dec. 3	17 13	7548.72	3.98	H	a	32.2	35.2	0.65	+0.33	9.42	−18
				H	b	38.2	35.5		−0.29	9.40	−20
5	15 12	7550.63	5.89	H	a	29.4	37.5	0.63	+0.90	9.99	−2
				H	b	35.0	37.5		+0.27	9.96	−5
6	11 38	7551.48	0.45	H	a	29.6	38.3	0.59	+0.95	10.04	+14
				H	b	34.9	37.9		+0.32	10.01	+11
	21 16	.89	0.86	H	a	23.6	28.2	0.60	+0.49	9.58	−1
				H	b	29.2	28.1		−0.12	9.57	−2
7	13 45	7552.57	1.54	H	a	33.4	36.2	0.74	+0.31	9.40	+11
				H	b	40.3	36.0		−0.45	9.24	−5
	56	.58	1.55	H	a	36.0	38.1	0.81	+0.22	9.31	+2
				H	b	43.4	37.9		−0.61	9.08	−21
11	15 9	7556.63	5.60	H	a	33.6	44.0	[0.73]	+1.14	10.23	+28
	13	.63	5.60	H	b	40.5	45.8		+0.61	10.30	+35
12	14 46	7557.62	0.30	H	a	30.8	41.0	0.74	+1.11	10.20	+20
				H	b	37.5	40.8		+0.35	10.04	+4
17	16 17	7562.68	5.36	H	a	20.2	28.2	0.61	+0.85	9.94	+5
				H	b	26.0	28.6		+0.29	9.98	+9
18	13 39	7563.57	6.25	H	a	15.6	23.8	0.65	+0.94	10.03	−2
				H	b	21.1	24.2		+0.33	10.02	−3
1907, Jan. 4	15 49	7580.66	4.46	H	a	24.2	30.8	0.81	+0.72	9.81	+12
				H	b	31.6	30.4		−0.14	9.55	−14
20	15 36	7596.65	1.58	H	a	22.0	24.0	0.55	+0.21	9.30	+1
				H	b	27.2	23.8		−0.36	9.33	+4
21	16 57	7597.71	2.64	H	a	25.9	29.7	0.75	+0.42	9.51	+12
				H	b	32.6	29.8		−0.32	9.37	−2
23	15 54	7599.66	4.59	H	a	33.5	40.2	0.60	+0.72	9.81	+10
				H	b	39.1	39.4		+0.03	9.72	+1
25	15 0	7601.62	0.26	H	a	22.9	28.4	+0.62	+0.58	9.67	−34
				H	b	28.7	28.0		−0.08	9.61	−40

Date	G. M. T.	Julian Day	Phase	Obs.	Star	Wedge		$b-a$	Δm	Mag.	O − C
1907, Jan. 27	$14^h\ 40^m$	2417603.61	$2^d.25$	H	*a*	26.4	29.4	$+\ 0^m.67$	$+\ 0^m.33$	$9^m.42$	+ 9
				H	*b*	32.4	29.4		− 0.34	9.35	+ 2
Feb. 7	15 18	7614.64	0.69	H	*a*	27.9	33.0	0.69	+ 0.57	9.66	− 5
				H	*b*	34.1	33.0		− 0.12	9.57	− 14
8	13 35	7615.57	1.62	H	*a*	21.4	21.8	0.65	+ 0.05	9.14	− 15
				H	*b*	27.5	22.3		− 0.55	9.14	− 15
9	15 1	7616 63	2.68	H	*a*	25.5	25.5	0.50	0.00	9.09	− 30
				H	*b*	30.0	25.5		− 0.50	9.19	− 20
10	13 16	7617.55	3.60	H	*a*	22.3	23.9	0.67	+ 0.17	9.26	− 27
				H	*b*	28.5	23.9		− 0.50	9.19	− 34
11	12 54	7618.54	4.59	H	*a*	24.2	27.3	0.67	+ 0.33	9.42	− 29
				H	*b*	30.4	27.0		− 0.37	9 32	− 39
12	0 1	7619.00	5.05	H	*a*	28.0	35.5	0.69	+ 0.84	9 93	+ 11
				H	*b*	34.2	35.9		+ 0.19	9.88	+ 6
	13 17	.55	5.60	H	*a*	26.1	33.6	0.68	+ 0.84	9.93	− 2
				H	*b*	32.2	33.5		+ 0.15	9.84	− 11
13	12 45	7620.53	0.29	H	*a*	24.1	33.6	0.60	+ 1.05	10.14	+ 14
				H	*b*	29.6	33.4		+ 0.42	10.11	+ 11
14	12 50	7621.53	1.29	H	*a*	27.4	30.2	0.78	+ 0.31	9.40	+ 3
				H	*b*	34.4	30.5		− 0.44	9.25	− 12
	56	.54	1.30	H	*a*	27.4	29.6	0.60	+ 0.25	9.34	− 3
				H	*b*	32.7	29.7		− 0.34	9.35	− 2
	23 24	.97	1 73	H	*a*	26.3	29.1	0.69	+ 0.31	9.40	+ 11
				H	*b*	32.5	29.4		− 0.35	9.34	+ 5
	33	.98	1.74	H	*a*	28.0	28.9	0.56	+ 0.10	9.19	− 10
				H	*b*	33.0	29.1		− 0.44	9.25	− 4
15	13 12	7622.55	2.31	H	*a*	31.5	32.8	0.65	+ 0.15	9 24	− 9
				H	*b*	37.4	33.1		− 0.47	9.22	− 11
	22	.56	2.32	H	*a*	32.6	33.1	0.51	+ 0.05	9.14	− 20
				H	*b*	37.3	32.0		− 0.58	9.11	− 23
	30	.56	2.32	H	*a*	31.6	32.6	0.53	+ 0.12	9.21	− 13
				H	*b*	36.4	32.4		− 0.44	9.25	− 9
16	12 56	7623.54	3.30	H	*a*	28.5	30.3	0.64	+ 0.20	9.29	− 19
				H	*b*	34.2	30.2		− 0.45	9.24	− 24
	13 5	.55	3.31	H	*a*	28.6	31.3	0.68	+ 0.30	9.39	− 9
				H	*b*	34.7	31.5		− 0.36	9.33	− 15
17	14 1	7624.58	4.34	H	*a*	25.4	31.2	0.63	+ 0.64	9.73	+ 6
				H	*b*	31.1	31.2		+ 0.01	9.70	+ 3
18	13 42	7625.57	5.33	H	*a*	28.7	36.0	0.58	+ 0.81	9.90	+ 2
				H	*b*	33.9	36.2		+ 0.25	9.94	+ 6
	54	.58	5.34	H	*a*	31.6	39.0	+ 0.60	+ 0.80	9.89	+ 1
				H	*b*	37.0	39.4		+ 0.24	9.93	+ 5

Date	G. M. T.	Julian Day	Phase	Obs.	Star	Wedge	$b-a$	Δm	Mag.	O−C
1907, Feb. 21	13 21	2417628.56	2d03	H	*a*	29.3 29.7	+0m61	+0m05	9m14	−16
				H	*b*	34.7 29.4		−0.60	9.09	−21
25	13 23	7632.56	6.03	H	*a*	26.7 34.7	0.64	+0.89	9.98	− 5
				H	*b*	32.5 34.8		+0.26	9.95	− 8
26	23 30	7633.98	1.16	H	*a*	33.5 36.8	0.58	+0.36	9.45	+ 3
				H	*b*	39.0 36.3		−0.28	9.41	− 1
Mar. 1	13 27	7636.56	3.74	H	*a*	26.8 31.2	0.59	+0.48	9.57	+ 1
				H	*b*	32.1 31.2		−0.11	9.58	+ 2
	16 22	.68	3.86	H	*a*	33.3 36.9	0.53	+0.40	9.49	− 9
				H	*b*	38.2 36.8		−0.14	9.55	− 3
5	14 32	7640.61	1.50	H	*a*	30.0 31.7	+0.63	+0.19	9.28	− 2
				H	*b*	35.6 32.1		−0.39	9.30	0

REMARKS

1906
July 20—Sky white, occasional clouds during second observation.
" 23—Haze, images diffuse and ragged.
" 24—Air transparent but unsteady.
" 26—Occasional haze, constant flashing of distant lightning.
" 30, Aug. 1—Sky white, occasional clouds, moon.
Aug. 3—Sky white and hazy, full moon.
" 8, 9—Haze.
" 16, 17, 22—Occasional clouds.
Sept. 7—Images somewhat diffuse.
" 10, 14—Sky thick.
" 26—Sky white, images diffuse.

1906
Oct. 7—Sky white.
" 9—Sky clear but images bad.
" 27, 29, 30, 31, Nov. 1—Sky white, moon.
Dec. 5—Haze.
" 7—Sky thick, some clouds.
" 11—Sky very thick.
" 17—Occasional clouds.

1907
Jan. 21, 23, 25, 27—Sky white, moon.
Feb. 7, 8, 9, 10, 17, 18—Haze.
" 26—Sky white, full moon.
Mar. 5—Sky thick.

The observations are printed in detail in Table I. The first three columns contain the date; the fifth and sixth, the initial of the observer and the comparison stars used. The wedge readings in column seven are in each case the means of four settings, the first number referring to the comparison star, and the second to the variable. The values of $b-a$ and Δm in columns eight and nine, respectively, were obtained by means of Table V, Bulletin No. 7. The occurrence of bracketed numbers in column eight indicates that, owing to haze, the observations referred to *a* and *b* were made independently, and not in a single series as is usually done. These values were given zero weight in forming the mean of the differences $b-a$. Of the remaining 129 values of this difference, 33 are by Seares and 96 by Haynes. The means and probable errors are

Seares $+0^m.591 \pm 0^m.014$,
Haynes $+0.599 \pm 0.007$,
All $+0.596 \pm 0.006$.

The constancy of the comparison star difference is indicated by the numbers in the second and fourth columns of Table II. These are the means of the differences $b-a$ in groups of ten, excepting in the case of the last, which is based upon only nine determinations. The Julian Dates in the first and third columns show the interval covered by each group.

TABLE II

MEAN VALUES OF COMPARISON STAR DIFFERENCE

Interval	$b-a$	Interval	$b-a$
J. D. 2417410–422	$+0^m.55$	J.D.2417491–514	$+0^m.55$
424–440	0.57	515–550	0.62
441–453	0.64	551–597	0.68
459–463	0.66	599–619	0.64
464–473	0.53	620–623	0.62
474–479	0.54	624–640	+0.60
479–490	+0.55	Mean	+0.596

In order to facilitate the reduction of the observations, the preliminary elements were used for the derivation of an approximate mean light-curve. This was drawn on tracing paper and superposed on the plotted observations for the determination of the epochs of minima. The twenty-one minima thus obtained are given in the first three columns of Table III. The corresponding weights are in column four. A least square solution based upon this data gave for the revised elements:

$$\text{Min.} = \text{J. D. } 2417412.607 + 6^d.2919\,E, \qquad \text{G. M. T.}$$

Column six of Table III gives the minima computed with the revised elements, and column seven, the residuals O – C.

In order to derive the final mean light-curve, the observations referred to star b were reduced to a by means of the difference $b-a = +0^m.60$. They were then combined into eighteen normal places in accordance with the phase values given in column four of Table I, which are counted from minima and derived by means of the revised elements. In forming the normals, all observations were given equal weight excepting those of 1906, Dec. 11, and 1907, Jan. 25. These were rejected on account of their very considerable deviation from the approximate mean light-curve mentioned above. The mean phase and the mean Δm are in columns two and three, and eight and nine of Table IV. The number of observations in each normal and the number

of nights involved may be found in columns four and five, and ten and eleven, respectively, while the deviations of the normals from the finally accepted light-curve are in columns six and twelve. The ordinates of the mean light-curve are given in columns two and five of Table V. The corresponding magnitudes for the variable, based on the magnitude of *a* derived below, are in columns three and six of Table V. The curve itself is shown in Fig. I.

TABLE III

MINIMA FOR RS CASSIOPEIAE

Date	G. M. T.	Julian Date	Wt.	E	Comp. Min.	O − C
1906, July 20	$14^h\ 24^m$	2417412.6	2	0	2417412.61	−0d01
26	21 36	7418.9	3	1	7418.90	0.00
Aug. 2	4 48	7425.2	3	2	7425.19	+0.01
8	12 0	7431.5	1	3	7431.48	+0.02
14	16 48	7437.7	1	4	7437.77	−0.07
20	21 36	7443.9	1	5	7444.07	−0.17
27	4 48	7450.2	2	6	7450.36	−0.16
Sept. 9	0 0	7463.0	3	8	7462.94	+0.06
15	9 36	7469.4	3	9	7469.23	+0.17
21	14 24	7475.6	3	10	7475.53	+0.07
27	19 12	7481.8	1	11	7481.82	−0.02
Oct. 10	7 12	7494.3	3	13	7494.40	−0.10
29	7 12	7513.3	2	16	7513.28	+0.02
Nov. 10	21 36	7525.9	1	18	7525.86	+0.04
Dec. 6	2 24	7551.1	3	22	7551.03	+0.07
12	7 12	7557.3	1	23	7557.32	−0.02
18	14 24	7563.6	1	24	7563.61	−0.01
1907, Jan. 25	2 24	7601.1	2	30	7601.36	−0.26
Feb. 13	7 12	7620.3	3	33	7620.24	+0.06
19	14 24	7626.6	2	34	7626.53	+0.07
25	19 12	7632.8	1	35	7632.82	−0.02

The magnitude of comparison star *a* was determined by a reference to seven P.DM. stars, all of which are of nearly the same declination as the variable. One of the reference stars is quite near *a*. The remaining six were chosen in pairs, at approximately equal distances in right ascension on opposite sides of *a*. As a consequence, the effect of extinction has been eliminated from the mean magnitude of *a*, although no corrections were applied to the individual observations. The results of the comparison are given in Table VI. The means of the observed differences between *a* and the reference stars are in column five, and the resulting magnitudes for *a*, in column six. These values show a considerable deviation from the mean, due, partly, to the brightness of the reference stars and the extent of the interval measured. All

of the measures were made with the aperture reduced to four inches, but, even then, the settings were difficult and uncertain on account of the difference in appearance of real and artificial stars. The effect of atmospheric absorption is also such as to increase the deviation from the mean, although, as mentioned above, this has been eliminated from the final result. The mean magni-

TABLE IV

NORMAL PLACES FOR RS CASSIOPEIAE

No.	Phase	Δm	Obs.	Nts.	O − C	No.	Phase	Δm	Obs.	Nts.	O − C
1	$0^d.04$	$+0^m.94$	10	4	$-0^m.02$	10	$3^d.02$	$+0^m.34$	20	7	$-0^m.01$
2	0.26	0.95	12	6	+0.03	11	3.30	0.42	16	6	+0.02
3	0.53	0.74	10	5	0.00	12	3.75	0.42	20	9	−0.05
4	1.02	0.40	10	4	0.00	13	4.05	0.57	18	8	+0.05
5	1.28	0.30	14	6	+0.02	14	4.51	0.59	14	7	−0.02
6	1.55	0.20	16	7	0.00	15	5.07	0.73	12	6	0.00
7	1.82	0.20	14	5	0.00	16	5.36	0.77	12	4	−0.03
8	2.22	0.22	20	7	−0.02	17	5.64	0.87	8	4	0.00
9	2.58	+0.35	22	7	+0.06	18	5.97	+0.94	10	5	+0.01

TABLE V

MEAN LIGHT-CURVE FOR RS CASSIOPEIAE

PHASE FROM MINIMUM

Phase	Δm	Mag.	Phase	Δm	Mag.
$0^d.00$	$+0^m.96$	$10^m.05$	$2^d.50$	$+0^m.27$	$9^m.36$
0.25	0.93	10.02	3.00	0.35	9.44
0.50	0.78	9.87	3.50	0.42	9.51
0.75	0.58	9.67	4.00	0.51	9.60
1.00	0.41	9.50	4.50	0.61	9.70
1.25	0.30	9.39	5.00	0.71	9.80
1.50	0.21	9.30	5.50	0.83	9.92
1.75	0.19	9 28	6.00	0.93	10 02
2 00	+ 0.21	9.30	6 50	+ 0.94	10.03

tude of a from all the stars is 9.09. This combined with the difference $b - a = +0^m.60$, derived above, gives $b = 9^m.69$. The magnitudes of the variable in column ten of Table I were derived by adding the adopted magnitudes of a and b to the quantities in column nine of the same table. The deviation of the individual observations from the mean light-curve is shown in the last column of Table I.

Table VII gives information as to the precision and consistency of the observations. In order to secure the data contained in this table the observations were divided into four groups. Group I comprises all the observations by Seares, excepting those made on 1906, Nov. 9, which are separated from the rest by a considerable interval of time; group II includes the observations made by Haynes during the same interval as that covered by group I; group III, the remaining observations by Haynes made with the right eye; and group IV, the observations with the left eye. Column two of Table VII shows the last three figures of the Julian Dates which limit the various groups. Column three contains the initial of the observer, while columns four and five give the number of observations on variable and comparison stars, respectively. The next three columns contain the average residual for the variable referred to *a*, for the variable referred to *b*, and for all the observations on the variable, respectively. These were obtained by averaging the residu-

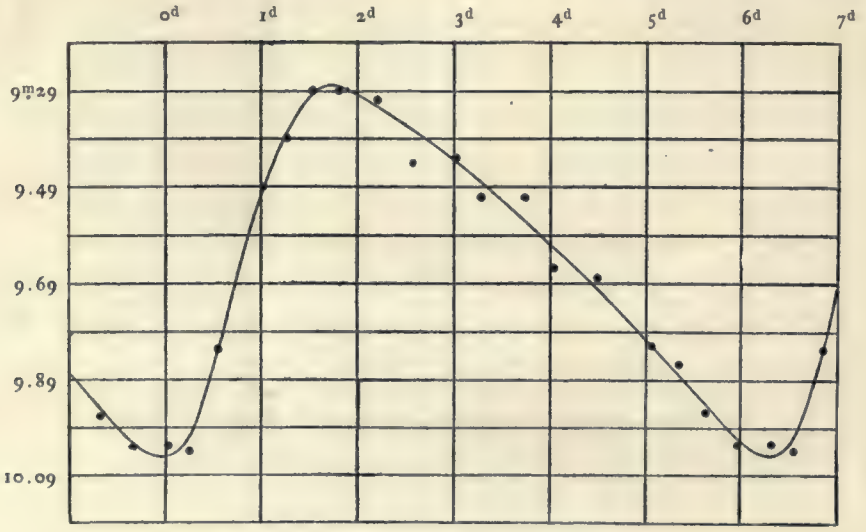

Fig. 1—Light-curve of RS Cassiopeiae

als in the last column of Table I, the algebraic sign being disregarded. The ninth column contains a similar result for the comparison star differences, the residuals being referred to the mean of the series, $b - a = +0^m.60$. The last four columns are similar in arrangement to those just considered. They contain the mean deviations, obtained from the various groups of residuals by regarding the algebraic sign, and exhibit the internal consistency of the observations.

The average residuals for the variable are sensibly the same throughout the entire series. On the other hand, the precision of the measures for $b - a$ is apparently greater for the end of the series than for the beginning. The reason for this is not clear. It is possible that the settings on the comparison stars during the latter part of the series were influenced by a knowledge on the part of the observer of what the difference in brightness should be. The mean deviations in columns ten to twelve suggest irregularities in the light change of the variable. This possibility is

further emphasized by the occurrence of series of uniformly discordant observations such as that extending from 1907, Feb. 7 to Feb. 11. During this interval the residuals for the variable are consistently negative and average — 0m.23. At the same time, the differences $b - a$ determined during this interval exhibit no abnormal errors of observation. It is not likely that these systematic deviations of the variable are due to any error in the assumed epochs of minima, for the irregularities are mostly confined to the descending branch of the curve. This is shown by the fact that the average residual for all observations taken during increasing light is ±0m.082, while

TABLE VI

MAGNITUDE OF COMPARISON STAR *a*. OBSERVER, HAYNES

Star	BD. No.	P. DM. No.	P. DM. Mag.	Mean Δm	Mag. a	No. Obs.	No. Nights
m	+ 61°2444	IV 2073	5.03	+ 3.97	9.00	7	7
n	2533	2117	5.57	3.33	8.90	7	7
p	2433	2068	7.98	1.30	9.28	6	6
q	2551	2122	7.38	1.74	9.12	5	5
r	2427	2066	6.44	2.80	9.24	5	5
s	2562	2126	7.28	1.68	8.96	5	5
t	2490	2098	6.64	+ 2.48	9.12	7	4
					Mean 9.09		

TABLE VII

MEAN RESIDUALS AND MEAN DEVIATIONS

Group	Interval	Obs.	No. Obs.		Mean Residuals				Mean Deviations			
			v	$b-a$	v_a	v_b	v	$b-a$	v_a	v_b	v	$b-a$
I	410–493	S	66	32	0.105	0.091	0.098	0.088	— 0.001	+ 0.014	+ 0.007	— 0.005
II	410–493	H	80	40	0.094	0.111	0.102	0.096	+ 0.018	+ 0.070	+ 0.044	— 0.043
III	493–576	H	50	24	0.102	0.100	0.101	0.065	+ 0.009	— 0.017	— 0.004	+ 0.025
IV	576–640	H	64	32	0.110	0.115	0.113	0.060	— 0.050	— 0.087	— 0.068	+ 0.031

the corresponding mean residual for decreasing light is ±0m.112. These irregularities doubtless account for the greater average residual shown by the observations on the variable, as compared with those resulting from measures of the comparison star difference.

Under the circumstances, one can not properly speak of a probable error, so far as observations on the variable are concerned. A notion of the accidental errors is best obtained from the comparison star differences, although, even here, it is clear that small systematic irregularities are present. From the entire series we find a probable error of ±0m.067 for a single difference based upon eight settings of the wedge, four being made upon each star.

The data for the determination of the systematic difference of the observers is scanty and consequently unreliable. From the first two groups of Table VII we obtain

	Variable	$b - a$
S - C	+ 0^m.007	− 0^m.005,
H - C	+ 0.044	− 0.043,
S - H	− 0.037	+ 0.038.

The systematic difference resulting from the observations on the variable is in agreement with all such differences previously obtained. In this connection, see Laws Observatory Bulletins, No. 7, p. 99; No. 8, pp. 112, 113, 123, 124; No. 9, p. 140. The value S − H = + 0^m.038 obtained from the measures of $b - a$ is in striking contrast with these results. Whether this disagreement is accidental or the result of a definite cause can not now be determined.

Columbia, Missouri, 1907, April 27. *E. S. Haynes.*

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LAWS OBSERVATORY
UNIVERSITY OF MISSOURI

BULLETIN NO. 12

FINDING EPHEMERIDES FOR COMET 1894 IV (E. SWIFT)

Comet 1894 IV is an object of unusual interest on account of its possible identity with the lost comet of De Vico which has not been seen since its appearance in 1844. It belongs to the Jupiter family of short-period comets, and has been subject at various times to perturbations of unusual magnitude as a result of the fact that its aphelion point lies close to the orbit of Jupiter.

The observations made during the apparition of 1894 extend from 1894, Nov. 21, to 1895, January 29, and cover a geocentric arc of three hours in right ascension and twenty-two degrees in declination. The corresponding heliocentric motion in the orbit was only thirty-nine degrees. Definitive elements based upon these observations were published in A. N. 3606. The perturbations produced by Jupiter, Saturn, Earth, and Mars during the interval 1894-1900 were calculated by the method of variation of the constants and printed in A. N. 3656. During 1897 the comet was within one-half an astronomical unit of distance from Jupiter for over six months time. The resulting disturbances in the orbit were very large, the period, for example, being changed by as much as six months. The comet was due to return to perihelion about Feb. 13, 1901, but the conditions for its rediscovery were extremely unfavorable. The maximum theoretical light at this time was only 0.12 of that at the date of discovery in 1894, and less, indeed, than the brightness when the comet passed beyond the limit of vision of the 36-inch refractor at Mt. Hamilton in Jan., 1895. It is not surprising, therefore, that it was not found at this time. During the years 1900-1907, it has constantly been at a considerable distance from both Mars and Jupiter, and, as there is no other planet which can seriously disturb its orbit, the perturbations during the past seven years have been quite secondary in character. The present osculating orbit must, therefore, be nearly the same as that of 1900.

At the present writing the comet is again near perihelion, and, so far as position is concerned, is much more favorably situated for observation than it was in 1900-1901. On the other hand, the only information we have concerning the orbit has been derived from the comparatively short arc observed thirteen years ago. The definitive elements themselves are therefore somewhat uncertain, and this in turn introduces a further uncertainty as to the exact amount of

the large perturbations of 1897. Discovery during the present return may, in consequence, be a very difficult matter. Under the existing conditions the mean daily motion is the most uncertain element, and this is precisely the one which should be known with some precision in order to insure a rediscovery. It is possible, however, that the relatively favorable location with respect to the earth may counteract, in a measure, the considerable uncertainty in position. The indications are that, when nearest the earth, the comet will be nearly, if not quite, as bright as when it was discovered in 1894; and, in view of the fact that it was seen during this apparition, and actually observed, upon one occasion at least, in a 9-inch refractor, its rediscovery during the next few weeks is by no means an impossibility. All this, of course, assumes that no unfavorable change in the intrinsic brilliancy of the object has taken place in the meantime.

It is therefore desirable that a systematic and careful search be made by those who have the proper equipment at their disposal. This is the more important in that a final answer to the question as to identity with the comet of De Vico probably depends upon its observation at the present return, for the elements of both comets De Vico and Swift are too uncertain for a successful bridging of the gap of fifty years which separates their appearances. And, at the same time, it is not likely that Swift's comet will be seen at any future return, except by accident, unless it is found during the next few months. It is only in this way that the periodic time can be determined with sufficient precision to trace back with certainty the changes which occurred in the orbit during the half century preceding the apparition of 1894 and to predict its motion for the future with any approximation to accuracy.

The ephemeris given in columns one to four of the table is based upon the following osculating elements taken from A. N. 3656. With the exception of a correction[1] in notation and a transfer to the equinox of 1907.0 no change has been made.

EPOCH AND OSCULATION 1900, JULY 23.0, B. M. T.

$$\left.\begin{aligned} L &= 317^\circ 16'.2 \\ \pi &= 349\ \ 2.8 \\ \Omega &= 24\ 56.5 \\ i &= 3\ 35.0 \end{aligned}\right\} \ 1907.0$$

$$\varphi = 31\ \ 2.5$$

$$\mu = 554''.3823$$

It would have been better to have used elements free from the perturbations of the years 1900-1907, but for various reasons it has not been possible to carry through the calculation necessary for the determination of these disturbances. It is not likely, however, that any real accuracy would have been gained for the ephemeris by this procedure, for the uncertainty introduced into the predicted positions through the neglect of these perturbations is, in all probability, less than that arising from the inaccuracies in the original definitive elements and in the large perturbations of the year 1897.

[1] In A. N. 3656, p. 129, the element designated by M should have been printed as L.

EPHEMERIDES

B. M. T.		R. A.	Dec.	Br.	R. A.	Dec.	Br.
1907, July	8.5	1h 34.9m	+ 7° 17′	0.42	1h 3.7m	+ 3° 15′	0.48
	12.5	43.8	8 14		12.2	4 12	
	16.5	1 52.5	9 9	0.45	20.5	5 8	0.54
	20.5	2 1.1	10 2		28.7	6 1	
	24.5	9.5	10 53	0.48	36.6	6 53	0.61
	28.5	17.7	11 43		44.3	7 43	
Aug.	1.5	25.7	12 31	0.51	51.7	8 31	0.68
	5.5	33.5	13 17		1 58.9	9 17	
	9.5	40.9	14 0	0.54	2 5.7	10 1	0.74
	13.5	48.0	14 41		12.1	10 43	
	17.5	2 54.7	15 20	0.58	18.1	11 23	0.80
	21.5	3 1.1	15 57		23.6	12 1	
	25.5	7.1	16 31	0.61	28.7	12 36	0.86
	29.5	12.6	17 3		33.3	13 9	
Sept.	2.5	17.6	17 34	0.64	37.3	13 40	0.92
	6.5	22.1	18 2		40.7	14 8	
	10.5	26.0	18 29	0.67	43.6	14 34	0.97
	14.5	29.3	18 53		45.8	14 57	
	18.5	32.1	19 15	0.70	47.3	15 18	1.01
	22.5	34.1	19 35		48.3	15 36	
	26.5	35.6	19 53	0.72	48.5	15 52	1.05
	30.5	36.3	20 10		48.0	16 5	
Oct.	4.5	36.4	20 24	0.73	46.9	16 16	1.05
	8.5	35.8	20 36		45.1	16 24	
	12.5	34.5	20 46	0.73	42.9	16 30	1.03
	16.5	32.7	20 53		40.3	16 34	
	20.5	30.3	20 58	0.72	37.5	16 36	0.98
	24.5	27.3	21 1		34.3	16 36	
	28.5	24.0	21 2	0.69	31.1	16 35	0.91
Nov.	1.5	20.3	21 0		27.7	16 32	
	5.5	16.5	20 57	0.64	24.5	16 29	0.81
	9.5	12.7	20 53		21.3	16 25	
	13.5	8.9	20 48	0.58	18.5	16 21	0.71
	17.5	5.3	20 42		16.2	16 18	
	21.5	3 2.0	+ 20 34	0.50	2 14.3	+ 16 15	0.60

Brightness, 1894, Nov. 21=1.00

The constants to the equator for the year 1907.0 are:

$$x = r\,[9.9999]\sin(\ 79^\circ\ 0' + v)$$
$$y = r\,[9.9511]\sin(349\ 46 + v)$$
$$z = r\,[9.6532]\sin(346\ \ 0 + v)$$

The elements given above indicate a perihelion passage for 1907, July 9. For that date the logarithm of the geocentric distance was 0.1745 and the brightness 0.42 of that at the time of discovery in 1894. The geocentric distance decreases until about October 20, upon which date the logarithm is 9.9942 The decrease in this quantity more than offsets the increasing distance of the comet from the sun, so that the calculated brightness increases until about October 8.

In order to exhibit the effect of an uncertainty in the mean daily motion, a second series of positions was computed on the assumption that the perihelion passage will not occur until July 25. The results are in columns five to seven of the table. This variation of sixteen days in the date of perihelion corresponds to a difference of about two seconds of arc in the mean daily motion when distributed over the entire interval 1894-1907. It will be observed that such a delay would bring the comet into even more favorable position with respect to the earth than is indicated by the elements printed above. Under this assumption the logarithm of the minimum geocentric distance is 9.9388, a value which should be attained about October 12.

In conclusion, it is of interest to note that the present position of Swift's comet is apparently close to that of comet *d* 1907 (Daniel) as shown by calculations made at the Students' Observatory of the University of California and distributed in *Lick Observatory Bulletin* No. 119. The elements given for the latter object seem to preclude, however, any possibility of identity between the two comets.

Columbia, Missouri, 1907, July 10. *F. H. Seares.*

LAWS OBSERVATORY
UNIVERSITY OF MISSOURI

BULLETIN NO. 13

THE ZÖLLNER-MÜLLER PHOTOMETER

The Laws Observatory has recently secured from Toepfer & Son, of Potsdam, an automatic registering photometer of the Zöllner-Müller type. The instrument is practically a duplicate of one constructed by this firm for the Astrophysical Observatory at Potsdam from designs by Müller, and briefly described by Vogel.[1] The design of the instrument, in so far as the arrangement of the colorimeter, nicols, and intensity circle is concerned, is similar to that of the well known Zöllner photometer. It differs from the instrument of Zöllner, however, in the method used for the formation of the artificial star, and through the addition of a device by means of which the settings of the intensity circle can be recorded on a ribbon of paper without removing the eye from the eyepiece.

The source of the light for the artificial star is an eight-volt incandescent lamp requiring a current of 1.3 amperes. The rays from the lamp pass through a ground glass window in the side of the lamp box, and thence through an iris diaphragm whose aperture can be varied from 0.5 mm. to 10.0 mm. They are then reflected from a small silvered hemisphere of 7 mm. radius, situated in the axis of the lateral tube which carries the colorimeter and nicols. From this point the course of the rays is the same as in the photometer of Zöllner. The silvered hemisphere gives a minified image of the opening in the iris diaphragm, far superior in definition to the artificial star as ordinarily formed. The adjustable diaphragm, which can be set instantly, affords a wide range of brightness for the artificial star. When used in connection with proper objectives, the instrument is suitable for the observation of all objects from the brighter planets down to the faintest stars.

The registering device is essentially the same as that designed by E. v. Gothard[2] for use in connection with the extinction wedge photometer, and described by Müller.[3] In addition to the ordinary graduations, the intensity circle of the Zöllner-Müller photometer is provided

[1] *Vierteljahrsschrift der Astronomischen Gesellschaft*, v. 40, p. 168.

[2] *Zeitschrift für Instrumentenkunde*, v. 7, p. 347.

[3] *Photometrie der Gestirne*, pp. 185-6.

with a set of graduations and an index cut in relief. Pressure exerted upon a lever conveniently placed, brings the record ribbon and a strip of carbon paper in contact with the raised graduations and index in such a manner as to print the reading of the circle upon the ribbon of paper. The release of the lever automatically shifts the paper and the carbon strip, and prepares the apparatus for the following setting. The settings can also be read from the ordinary graduations in the usual manner. Both methods give the reading to the nearest degree and, by estimation, to tenths of a degree. The recording apparatus greatly facilitates the work of observation, and presents the very important advantage that the observer is quite in ignorance as to the readings while the measures are being made. It possesses a further advantage in that the observer is spared the strain and fatigue resulting from the constant adaptation of the eye, first to the relatively dark field of view, and then to the graduations of the circle as illuminated by the reading lamp.

The intensity circle is graduated from 0° to 90° in opposite directions from two points 180° apart. The nicols are so adjusted with respect to the graduations that when the reading is 0° the artificial star is completely extinguished. In accordance with the law of Malus, the intensity of the artificial star for any other reading is then proportional to the square of the sine of the reading. If θ_1 and θ_2 represent the mean intensity circle readings corresponding to the settings on two stars, the magnitude difference of the pair, neglecting the effect of extinction, will be given by the equation

$$m_1 - m_2 = -2.5\ (\log \sin^2\theta_1 - \log \sin^2\theta_2),$$

or by

$$\Delta m = 5\log \sin \theta_2 - 5\log \sin \theta_1.$$

The values of Δm can conveniently be calculated by means of Table I, the body of which contains the values of $5\log \sin \theta - [4.00 - 10]$ corresponding to the argument θ. The tabular values are given for every tenth degree of the argument from 9° to 21°, and for every degree from 20° to 80°. As it is desirable that the readings of the circle should always be kept within the limits 10° and 70°, the extent of the table is sufficient for all practical purposes.

In order to adapt the instrument to the observation of objects of unusual brightness, it has been fitted with a lateral ocular in accordance with the suggestion of Ceraski.[1] This ocular receives the rays of the artificial star directly, while those of the real star are reflected from the surface of the glass plate mounted in front of the eyepiece. With the regular ocular the conditions are reversed, the rays of the real star being received directly, while those of the artificial star are reflected from the glass plate. Since the loss in intensity occurring as a result of reflection under an incidence of 45° amounts to three or four magnitudes, the arrangement described makes it possible to observe with the lateral ocular stars six or seven magnitudes brighter than those which can be reached with the ordinary eyepiece.

[1] *Astronomische Nachrichten*, v. 120, p. 217.

TABLE I

FOR THE CONVERSION OF INTENSITY CIRCLE READINGS INTO MAGNITUDES

θ	0	1	2	3	4	5	6	7	8	9
9°	1.97	2.00	2.02	2.04	2.07	2.09	2.11	2.13	2.16	2.18
10	2.20	2.22	2.24	2.26	2.28	2.30	2.32	2.34	2.36	2.38
11	2.40	2.42	2.44	2.46	2.48	2.50	2.52	2.54	2.55	2.57
12	2.59	2.61	2.62	2.64	2.66	2.68	2.69	2.71	2.73	2.74
13	2.76	2.78	2.79	2.81	2.82	2.84	2.86	2.87	2.89	2.90
14	2.92	2.93	2.95	2.96	2.98	2.99	3.01	3.02	3.04	3.05
15	3.06	3.08	3.09	3.11	3.12	3.13	3.15	3.16	3.18	3.19
16	3.20	3.22	3.23	3.24	3.25	3.27	3.28	3.29	3.30	3.32
17	3.33	3.34	3.35	3.37	3.38	3.39	3.40	3.41	3.43	3.44
18	3.45	3.46	3.47	3.48	3.50	3.51	3.52	3.53	3.54	3.55
19	3.56	3.57	3.58	3.60	3.61	3.62	3.63	3.64	3.65	3.66
20	3.67	3.68	3.69	3.70	3.71	3.72	3.73	3.74	3.75	3.76
20	3.67	3.77	3.87	3.96	4.05	4.13	4.21	4.28	4.36	4.43
30	4.50	4.56	4.62	4.68	4.74	4.79	4.85	4.90	4.95	4.99
40	5.04	5.08	5.13	5.17	5.21	5 25	5.28	5.32	5.36	5.39
50	5.42	5.45	5.48	5.51	5.54	5.57	5.59	5.62	5.64	5.67
60	5.69	5.71	5.73	5.75	5.77	5.79	5.80	5.82	5.84	5.85
70	5.86	5.88	5.89	5.90	5.91	5.92	5.93	5.94	5.95	5.96

The photometer is also provided with an objective of 60 mm. aperture and 60 cm. focal length. When used in connection with this, the whole apparatus can be mounted on a stand which permits of motion in azimuth and altitude. A cap for the 60 mm. objective, with adjustable sector shaped openings, also forms part of the equipment. The range of adjustment is such that, when in position, the effective aperture can be reduced by any amount from one-third to two-thirds of its full value.

THE GANS–CRAWFORD TELESCOPE

Through the generosity of Mr. Hanford Crawford of St. Louis it has been possible to secure for the Laws Observatory a 4½-inch equatorial refracting telescope made by the late R. Brown Gans, for many years a resident of Boone County, Missouri. The instrument was originally provided with a portable tripod mounting, but for convenience it has been mounted permanently under the cone in the central part of the obssrvatory building. The moving parts of the new equatorial mounting were obtained from Wm. Gaertner & Co. of Chicago. The supporting

column was made in the university shops from 5-inch wrought iron pipe and fittings. The instrument is provided with circles which read directly to 1° and to 4^m, and, by estimation, to $0^\circ.1$ and $0^m.4$, respectively. The tailpiece of the telescope has been remodelled by the university mechanician so as to receive either the equalizing wedge photometer or the Zöllner-Müller registering photometer. When thus fitted the instrument is suitable for the observation of stars from the fifth to the tenth magnitude. The tube carrying the 4½-inch objective can quickly be removed from the cradle. The insertion of adapters fits the cradle to receive the 60 mm. objective-tube belonging to the Zöllner-Müller photometer. With this arrangement, the instrument is suitable for the photometric observation of stars from the third to the eighth magnitude; and by taking advantage of the objective cap, and the lateral ocular, described above, its range can be extended so as to include the brighter planets.

THE VARIABLE X LACERTAE (88.1906)[1]

1855.0	R.A.	22^h	43^m	9^s	Dec. +55°39′.8
1900.0			44	58	54.0

The following discussion is a continuation of that contained in Bulletins Nos. 8 and 10, pp. 108 and 161, respectively. The star in question is one whose variability was brought to light during the discussion of certain observations on V Lacertae, (110.1904), made in 1904-5. It was used as a comparison star in these observations, and, in consequence, a knowledge of its period and light curve is necessary for the reduction of the measures on V Lacertae.

Fifty observations and a set of preliminary elements were published in Bulletin No. 8. One hundred and fifty-four additional observations are now available for the correction of these elements. These observations, printed in Table II, were secured partly with the equalizing Zeiss-wedge photometer, and partly with the Zöllner-Müller photometer, both instruments being used in connection with the 7½-inch equatorial. Column five of the table of observations gives the initial of the observer, S for Seares and H for Haynes, and indicates the photometer used. The equalizing wedge photometer is designated by W, the Zöllner-Müller, by Z. Column seven contains the mean wedge reading for the observations made with the former instrument, and the mean intensity circle readings, expressed in degrees and tenths, for those secured with the latter. Each mean reading for the wedge is derived from four settings, except for the observation of 1907, May 22, when, owing to haze, only two settings were made upon each star. The means for the Zöllner-Müller photometer are also based upon four settings, one for each of the four quadrants. In all cases, the first number in column seven refers to the comparison star, the second to the variable. The values of $c-b$ in column eight and of Δm in column nine were derived by means of Table V, Bulletin No. 7, and Table I of the present bulletin.

[1] The observations and the reductions given in this bulletin have been made, in part, with the assistance of a grant from the Gould Fund of the National Academy of Sciences.

TABLE II

OBSERVATIONS OF X LACERTAE

Date	G. M. T.	Julian Day	Phase	O, I.	Star	Readings		$c-b$	Δm	Mag.	O-C
1906, Sept. 7	$15^h\ 59^m$	2417461.67	$0^d.03$	H, W	b	14.9	18.6		$+0^m.46$	$8^m.75$	+ 16
				H, W	c	21.2	18.5	$+0^m.75$	— 0.29	8.48	— 11
	16 10	.67	0.03	S, W	b	16.5	19.1		+ 0.31	8.60	+ 1
				S, W	c	21.1	19.3	0.52	— 0.19	8.58	— 1
8	16 4	7462.67	1.03	S, W	b	12.4	15.9		+ 0.48	8.77	+ 24
				S, W	c	14.9	15.3	0.34	+ 0.05	8.82	+ 29
	12	.67	1.03	H, W	b	12.9	15.6		+ 0.36	8.65	+ 12
				H, W	c	18.0	15.3	0.66	— 0.35	8.42	— 11
12	19 5	7466.79	5.15	H, W	b	16.5	18.2		+ 0.20	8.49	— 8
				H, W	c	20.9	18.6	0.50	— 0.25	8.52	— 5
	16	.80	5.16	S, W	b	17.5	18.5		+ 0.12	8.41	— 16
				S, W	c	20.4	18.1	0.33	— 0.26	8.51	— 6
13	16 48	7467.70	0.62	S, W	b	12.4	15.4		+ 0.41	8.70	+ 13
				S, W	c	16.3	15.8	0.53	— 0.07	8.70	+ 13
	58	.71	0.63	H, W	b	18.0	17.3		— 0.08	8.21	— 36
				H, W	c	19.3	18.7	0.15	— 0.07	8.70	+ 13
14	15 37	7468.65	1.57	H, W	b	9.5	9.8		+ 0.03	8.32	— 5
				H, W	c	14.8	9.7	0.70	— 0.68	8.09	— 28
	45	.66	1.58	S, W	b	9.0	9.7		+ 0.09	8.38	+ 1
				S, W	c	12.9	9.4	0.52	— 0.47	8.30	— 7
15	14 58	7469.62	2.54	S, W	b	14.5	14.5		0.00	8.29	+ 6
				S, W	c	18.3	15.4	0.50	— 0.37	8.40	+ 17
16	15 8	7470.63	3.55	H, W	b	16.7	16.7		0.00	8.29	— 2
				H, W	c	20.3	16.4	0.42	— 0.46	8.31	
17	15 18	7471.64	4.56	H, W	b	17.8	19.9		+ 0.24	8.53	+ 3
				H, W	c	20.0	18.9	0.25	— 0.12	8.65	+ 15
	27	.64	4.56	S, W	b	14.3	15.6		+ 0.17	8.46	— 4
				S, W	c	18.6	16.3	0.55	— 0.28	8.49	— 1
18	15 16	7472.64	0.12	H, W	b	14.8	17.4		+ 0.35	8.64	+ 5
				H, W	c	19.1	17.4	0.55	— 0.20	8.57	— 2
	26	.64	0.12	S, W	b	14.3	17.4		+ 0.41	8.70	+ 11
				S, W	c	18.8	17.0	0.58	— 0.22	8.55	— 4
19	15 22	7473.64	1.12	S, W	b	14.0	16.9		+ 0.39	8.68	+ 16
				S, W	c	19.4	17.4	0.68	— 0.23	8.54	+ 2
	30	.65	1.13	H, W	b	13.5	16.1		+ 0.35	8.64	+ 12
				H, W	c	17.5	15.0	0.53	— 0.33	8.44	— 8
20	14 26	7474.60	2.08	H, W	b	13.8	14.1		+ 0.04	8.33	+ 8
				H, W	c	17.6	14.6	0.50	— 0.39	8.38	+ 13
	16 32	.69	2.17	S, W	b	12.7	14.3		+ 0.22	8.51	+ 26
				S, W	c	18.0	14.5	$+0.70$	— 0.46	8.31	+ 6

Date	G. M. T.	Julian Day	Phase	O, I.	Star	Readings		$c-b$	Δm	Mag.	O−C
1906, Sept. 21	$14^h\ 58^m$	2417475.62	$3^d.10$	S, W	b	17.6	17.4	$+0^m.47$	$-0^m.02$	$8^m.27$	+ 2
				S, W	c	21.8	18.3		− 0.38	8.39	+ 14
	15 4	.63	3.11	H, W	b	16.2	15.9	0.44	− 0.04	8.25	[illegible]
				H, W	c	19.9	15.6		− 0.53	8.24	− 1
22	14 49	7476.62	4.10	H, W	b	10.6	11.6	0.52	+ 0.14	8.43	+ 3
				H, W	c	14.5	11.3		− 0.42	8.35	− 5
	59	.62	4.10	S, W	b	15.0	16.1	0.54	+ 0.15	8.44	+ 4
				S, W	c	19.3	16.7		− 0.31	8.46	+ 6
23	14 40	7477.61	5.09	H, W	b	12.1	14.6	0.67	+ 0.34	8.63	+ 7
				H, W	c	17.1	14.5		− 0.34	8.43	− 13
24	14 46	7478.62	0.65	H, W	b	16.6	18.3	0.47	+ 0.21	8.50	− 7
				H, W	c	20.7	18.5		− 0.24	8.53	− 4
	53	.62	0.65	S, W	b	17.9	21.4	0.41	+ 0.38	8.67	+ 10
				S, W	c	21.6	21.5		− 0.02	8.75	+ 18
25	14 54	7479.62	1.65	H, W	b	15.3	15.2	0.59	− 0.01	8.28	− 7
				H, W	c	20.1	15.4		− 0.57	8.20	− 15
	15 2	.62	1.65	S, W	b	13.8	16.4	0.68	+ 0.35	8.64	+ 29
				S, W	c	19.1	16.4		− 0.33	8.44	+ 9
26	15 7	7480.63	2.66	S, W	b	17.7	17.8	0.40	+ 0.02	8.31	+ 8
				S, W	c	21.3	18.2		− 0.34	8.43	+ 20
Nov. 1	16 33	7516.69	0.62	H, W	b	17.5	19.0	0.40	+ 0.18	8.47	− 10
				H, W	c	21.1	19.4		− 0.18	8.59	+ 2
8	17 31	7523.73	2.22	H, W	b	20.5	19.3	0.39	− 0.13	8.16	− 8
				H, W	c	24.2	19.2		− 0.53	8.24	0
9	15 35	7524.65	3.14	S, W	b	20.7	20.5	0.54	− 0.02	8.27	+ 2
				S, W	c	25.8	20.5		− 0.56	8.21	− 4
	42	.65	3.14	H, W	b	19.3	19.9	0.47	+ 0.07	8.36	+ 11
				H, W	c	23.7	19.6		− 0.43	8.34	+ 9
12	16 4	7527.67	0.72	H, W	b	19.7	20.9	0.41	+ 0.12	8.41	− 15
				H, W	c	23.6	20.7		− 0.31	8.46	− 10
21	16 4	7536.67	4.28	H, W	b	16.7	17.5	0.45	+ 0.10	8 39	− 5
				H, W	c	20.6	17.5		− 0.35	8.42	− 2
26	15 49	7541.66	3.82	H, W	b	16.3	15.4	0.37	− 0.12	8.17	− 17
				H, W	c	19.4	15.5		− 0.48	8.29	− 5
27	14 56	7542.62	4.78	H, W	b	16.6	19.6	0.48	+ 0.36	8.65	+ 12
				H, W	c	20.8	19.7		− 0.11	8.66	+ 13
Dec. 3	16 34	7548.69	5.41	H, W	b	21.8	24.1	0.53	+ 0.24	8.53	− 6
				H, W	c	26.8	24.4		− 0.26	8.51	− 8
6	15 35	7551.65	2.93	H, W	b	19.7	19.2	0.46	− 0.06	8.23	− 1
				H, W	c	24.1	19.3		− 0.51	8.26	+ 2
11	16 26	7556.69	2.53	H, W	b	26.7	25.7	[+0.63]	− 0.11	8.18	− 5
	30	.69	2.53	H, W	c	32.4	26.5		− 0.65	8.12	− 11

Date	G. M. T.	Julian Day	Phase	O, I.	Star	Readings		$c-b$	Δm	Mag.	O − C
1906, Dec. 12	$16^h\ 44^m$	2417557.70	$3^d.54$	H, W	b	20.5	20.6	$+0^m.51$	$+0^m.01$	$8^m.30$	0
				H, W	c	25.4	20.8		−0.48	8.29	− 1
17	17 18	7562.72	3.12	H, W	b	12.6	12.1	0.60	−0.07	8.22	− 3
				H, W	c	17.1	12.2		−0.65	8.12	− 13
18	16 56	7563.71	4.11	H, W	b	15.5	17.0	0.65	+0.20	8.49	+ 9
				H, W	c	21.0	17.1		−0.44	8.33	− 7
1907, Jan. 4	15 20	7580.64	4.70	H, W	b	16.8	17.7	0.43	+0.11	8.40	− 11
				H, W	c	20.5	18.1		−0.27	8.50	− 1
5	14 6	7581.59	0.21	H, W	b	22.9	24.4	0.36	+0.15	8.44	− 15
				H, W	c	26.4	24.6		−0.19	8.58	− 1
	12	.59	0.21	H, W	b	17.1	18.6	0.46	+0.18	8.47	− 12
				H, W	c	21.2	18.3		−0.31	8.46	− 13
20	15 8	7596.63	4.37	H, W	b	16.6	17.2	0.47	+0.07	8.36	− 9
				H, W	c	20.7	16.4		−0.50	8.27	− 18
21	16 44	7597.79	5.44	H, W	b	19.4	22.4	0.55	+0.32	8.61	+ 2
				H, W	c	24.6	22.5		−0.21	8.56	− 3
23	15 13	7599.63	1.92	H, W	b	24.2	23.9	0.37	−0.03	8.26	− 2
				H, W	c	27.6	23.8		−0.41	8.36	+ 8
25	14 32	7601.61	3.90	H, W	b	11.4	11.7	0.66	+0.04	8.33	− 3
				H, W	c	16.3	11.9		−0.59	8.18	− 18
27	14 14	7603.59	0.44	H, W	b	17.2	18.8	0.53	+0.20	8.49	− 9
				H, W	c	21.9	18.8		−0.33	8.44	− 14
Feb. 8	13 25	7615.56	1.53	H, W	b	12.4	12.5	0.49	+0.02	8.31	− 7
				H, W	c	16.0	12.4		−0.49	8.28	− 10
May 22	21 25	7718.89	1.45	H, W	b	20.1	22.6	0.38	+0.27	8.56	+ 15
				H, W	c	23.7	22.2		−0.16	8.61	+ 20
June 14	21 6	7741.88	2.66	H, W	b	12.9	12.3	0.47	−0.08	8.21	− 2
				H, W	c	16.4	12.5		−0.52	8.25	+ 2
15	21 4	7742.88	3.66	H, W	b	16.4	16.7	0.45	+0.04	8.33	+ 1
				H, W	c	20.2	16.2		−0.47	8.30	− 2
16	21 0	7743.88	4.66	H, W	b	18.2	18.7	0.33	+0.06	8.35	− 16
				H, W	c	21.2	19.0		−0.23	8.54	+ 3
17	20 22	7744.85	0.19	H, W	b	17.0	20.8	0.39	+0.43	8.72	+ 13
				H, W	c	20.4	19.7		−0.07	8.70	+ 11
27	20 22	7754.85	4.75	H, Z	b	50.9	46.4	0.27	+0.15	8.44	− 8
				H, Z	c	43.3	50.7		−0.26	8.51	− 1
28	20 58	7755 87	0.33	H, Z	b	50.9	38.4	0.81	+0.46	8.75	+ 16
				H, Z	c	32.3	38.4		−0.33	8.44	− 15
July 2	21 10	7759.88	4.34	H, Z	b	56.6	46.5	0.38	+0.31	8.60	+ 15
				H, Z	c	44.6	44.1		+0.02	8.79	+ 34
4	19 24	7761.81	0.82	H, Z	b	45.4	49.3	+0.40	+0.26	8.55	0
				H, Z	c	36.2	39.8		−0.17	8.60	+ 5

Date	G. M. T.	Julian Day	Phase	O, I.	Star	Readings		$c-b$	Δm	Mag.	O − C
1907, July 5	20h 34m	2417762.86	1d87	H, Z	*b*	47.6	43.0	+ 0m48	+0m17	8m46	+ 17
				H, Z	*c*	36.2	49.2		− 0.54	8.23	− 6
6	19 26	7763.81	2.82	H, Z	*b*	46.2	43.4	[1.05]	+ 0.10	8.39	+ 15
				H, Z	*c*	26.4	48.0		− 1.12	[7.65]	[−59]
7	20 6	7764.84	3.85	H, Z	*b*	72.2	78.9	0.16	− 0.07	8.22	− 13
				H, Z	*c*	62.2	76.9		− 0.21	8.56	+ 21
8	20 6	7765.84	4.85	H, Z	*b*	36.3	33.8	0.41	+ 0.13	8.42	− 11
				H, Z	*c*	29.3	34.2		− 0.30	8.47	− 6
11	19 38	7768.82	2.39	H, Z	*b*	42.4	50.0	0.39	− 0.27	8.02	− 21
				H, Z	*c*	34.4	56.5		− 0.84	7.93	− 30
19	19 22	7776.81	4.94	H, Z	*b*	35.8	36.5	0.21	− 0.04	8.25	− 29
				H, Z	*c*	32.1	41.2		− 0.46	8.31	− 23
20	18 43	7777.78	0.47	H, Z	*b*	48.2	40.3	0.65	+ 0.32	8.61	+ 3
				H, Z	*c*	33.6	37.2		− 0.19	8.58	0
21	18 12	7778.76	1.45	H, Z	*b*	49.8	41.9	0.51	+ 0.29	8.58	+ 17
				H, Z	*c*	37.0	43.3		− 0.28	8.49	+ 8
29	18 16	7786.76	4.00	H, Z	*b*	47.4	47.0	0.42	+ 0.02	8.31	− 7
				H, Z	*c*	37.4	41.1		− 0.16	8.61	+ 23
31	19 11	7788.80	0.60	H, Z	*b*	40.8	34.0	0.45	+ 0.33	8.62	+ 5
				H, Z	*c*	32.0	35.6		− 0.20	8.57	0
Aug. 3	19 4	7791.79	3.59	H, Z	*b*	28.4	29.8	0.43	− 0.10	8.19	− 12
				H, Z	*c*	23.0	32.5		− 0.69	8.08	− 23
8	18 3	7796.75	3.11	H, Z	*b*	45.4	44.3	0.62	+ 0.04	8.33	+ 8
				H, Z	*c*	32.3	43.8		− 0.56	8.21	− 4
9	18 20	7797.76	4.12	H, Z	*b*	52.6	47.6	0.56	+ 0.16	8.45	+ 5
				H, Z	*c*	37.8	46.5		− 0.36	8.41	+ 1
10	17 6	7798.71	5.07	H, Z	*b*	41.4	32.0	0.36	+ 0.48	8.77	+ 21
				H, Z	*c*	34.1	39.0		− 0.25	8.52	− 4
11	16 8	7799.67	0.59	H, W	*b*	19.3	20.5	+ 0.36	+ 0.13	8.42	− 15
				H, W	*c*	22.6	20.4		− 0.24	8.53	− 4

REMARKS

1906

Sept. 13—Sky white.

Nov. 1—Sky white, moon.

Dec. 11—Sky thick.

1907

Jan. 23, 25, 27—Sky white, moon.

Feb. 8—Occasional haze.

May 22—Observation interrupted by haze.

June 15—Sky white.

" 16—Sky white, air unsteady.

1907

June 17—Sky white.

" 27—Sky white, haze, moon.

" 28—Sky white, clouds, moon.

July 2—Images diffuse.

" 5—Eyes much fatigued.

" 6—Haze, clouds immediately after observation.

" 11—Air unsteady, images very bad.

" 20—Sky very white.

Aug. 11—Clouds immediately after observation.

The comparison stars are the same as those used for the earlier observations.[1] The present series of measures gives seventy-seven values for the difference $c-b$. Two of these, the values for 1906, Dec. 11, and 1907, July 6, have been excluded on account of bad atmospheric conditions. The others have been combined with the four values derived from the observations in Bulletin No. 8 for the formation of the mean, $c-b = \pm 0^m.48 \pm 0^m.009$. The means for the separate observers and instruments were also calculated. These with the probable errors corresponding to a single observation, and the number of observations included in each mean, are to be found in Table III. The magnitude previously derived for b is 8.29,[2] which, combined with the adopted mean for $c-b$ gives $c = 8^m.77$. These values of the brightness of the comparison stars are the basis of the magnitudes of the variable given in column ten of Table II.

TABLE III

MEANS FOR COMPARISON STAR DIFFERENCES

Observer and Inst.	Mean $c-b$	P. E.	No. Obs.
Seares, Zeiss-Wedge Phot.	$+0^m.51$	$\pm 0^m.073$	18
Haynes, Zeiss-Wedge Phot.	0.47	0.076	44
Haynes, Zöllner-Müller Phot.	0.44	0.098	17
All Observations	+ 0.48	± 0.084	79

The determination of the revised elements was based upon minima, as these appear to be somewhat better defined than the maxima. The observations referred to c were reduced to b by adding the mean comparison star difference $c-b$ to the values of $v-c$. The results were then combined with the directly observed difference $v-b$, thus forming for each night a mean $\Delta m = v-b$. These means were plotted and, from the resulting curve, the epochs of minima were read to the nearest tenth of a day. The Julian dates of the epochs and their corresponding weights are to be found in columns three and four, and eight and nine of Table IV, opposite the numbers 14-30. The fifty observations of Bulletin No. 8, treated in the same manner, gave the values for Nos. 11-13 of this table, while the results for Nos. 1-10 were derived from the observations on V Lacertae in which X Lacertae was used as a comparison star. The method used for disentangling the variation of the two stars is that described in Bulletin No. 8, pp. 109-110, except that, for the final reduction, a second approximation was made. This led to a slight modification of the values given in Bulletin No. 8 for the first two of the minima of Table IV, and to the addition of certain other minima not revealed by the first approximation with sufficient clearness to make them certain.

The thirty minima thus derived were subjected to a least square reduction for the determination of the corrections to the preliminary elements[3] given in Bulletin No. 8. The results are

[1] *Laws Observatory Bulletin* No. 8, pp. 108-9.

[2] *Laws Observatory Bulletin* No. 8, p. 112.

[3] The initial date has been changed to J. D. 2416672.6.

$$\Delta T_o = -0^d.152 \pm 0^d.071$$
$$\Delta P_o = +0^d.00269 \pm 0^d.00049$$

in which the appended quantities are the probable errors. These applied to the preliminary epoch and period, respectively, give for the revised elements

$$\text{Min.} = \text{J. D. } 2416672.45 + 5^d.44269E, \qquad \text{G. M. T.}$$

The representation of the observed minima by the revised elements is exhibited through the residuals in columns five and ten of Table IV. The least square reduction gave for the probable error of a minimum of unit weight $\pm 0^d.21$.

TABLE IV

OBSERVED MINIMA AND THEIR REPRESENTATION

No.	E	Obs. Min.	Wt.	O − C	No.	E	Obs. Min.	Wt.	O − C
1	0	6672^d.6	2	+0^d.15	16	148	7478^d.0	2	+0^d.03
2	1	6678.0	2	+0.11	17	157	7526.8	1	−0.15
3	6	6705.2	2	+0.10	18	160	7543.0	1	−0.28
4	68	7042.7	2	+0.15	19	161	7548.6	1	−0.12
5	70	7053.3	1	−0.14	20	164	7564.8	1	−0.25
6	72	7064.2	2	−0.12	21	167	7581.1	1	−0.28
7	129	7374.2	1	−0.36	22	170	7598.0	2	+0.29
8	130	7379.6	2	−0.40	23	171	7603.6	2	+0.45
9	131	7385.0	2	−0.44	24	197	7744.6	1	−0.06
10	134	7401.5	1	−0.27	25	199	7755.6	1	+0.06
11	136	7412.8	2	+0.15	26	200	7760.8	1	−0.19
12	137	7418.2	2	+0.10	27	201	7766.3	1	−0.13
13	138	7423.5	2	−0.04	28	203	7777.8	1	+0.49
14	146	7467.1	2	+0.02	29	205	7788.6	2	+0.40
15	147	7472.5	2	−0.02	30	207	7799.0	2	−0.08

The phase values referred to the nearest preceding minimum were calculated for each observation with the revised elements, and are to be found in column four of Table II. From a general inspection of the results it seemed likely that the mean light-curve derived from the fifty observations in Bulletin No. 8, and given on pp. 112-3 of that bulletin, would represent satisfactorily the observations in Table II. In consequence, the mean light-curve has not been revised, and the residuals in the last column of Table II are those resulting from a comparison of the observations with the curve derived from the measures in Bulletin No. 8.

In order to determine whether the present series of observations differs systematically in any particular from the mean light-curve of Bulletin No. 8, the residuals were subjected to the following analysis: All residuals, excepting the second for 1907, July 6, falling within the phase limits indicated in column one of Table V were combined into means, attention being given to

TABLE V

AVERAGE DEVIATION OF OBSERVATIONS FROM MEAN LIGHT-CURVE

BY SECTIONS

Phase Limits	No. of Observations				Average Deviation			
	S, W	H, W	H, Z	All	S, W	H, W	H, Z	All
0.00—0.49	4	14	4	22	+0m02	−0m02	+0m01	−0m01
0.50—0.99	4	10	4	18	+ 0.13	− 0.09	+ 0.02	− 0.01
1.00—1.49	4	4	2	10	+ 0.18	+ 0.01	+ 0.12	+ 0.10
1.50—1.99	4	10	2	16	+ 0.08	− 0.03	+ 0.06	+ 0.01
2.00—2.49	2	4	2	8	+ 0.16	+ 0.03	− 0.26	− 0.01
2.50—2.99	4	6	1	11	+ 0.13	− 0.02	+ 0.15	+ 0.05
3.00—3.49	4	6	2	12	+ 0.03	0.00	+ 0.02	+ 0.02
3.50—3.99	0	10	4	14	———	− 0.05	− 0.07	− 0.05
4.00—4.49	2	8	6	16	+ 0.05	− 0.06	+ 0.12	+ 0.02
4.50—4.99	2	8	6	16	− 0.02	+ 0.02	− 0.13	− 0.04
5 00—5.43	2	6	2	10	− 0.11	− 0.06	+ 0.08	− 0.04

TABLE VI

AVERAGE DEVIATION OF OBSERVATIONS FROM MEAN LIGHT-CURVE

ENTIRE CURVE

Observer and Inst.	No. Obs.	Average Dev.	P. E.
Seares, Zeiss-Wedge Phot.	32	+0m076	± 0m087
Haynes, Zeiss-Wedge Phot.	86	− 0.030	0.071
Haynes, Zöllner-Müller Phot.	35	0.000	0.103
All Observations	153	− 0.001	± 0.081

the algebraic signs. The results thus derived express the average deviations of the observations in Table II from the mean light-curve of Bulletin No. 8, considered in sections, each of which covers a phase interval of 0d5. This process was applied not only to the entire series, irrespective of observer and instrument, but also to the measures of each observer, treated separately and with distinction as to the instrument used. The average deviations for Seares, wedge photometer; Haynes wedge photometer; and Haynes, Zöllner-Müller photometer, are in columns six, seven, and eight, respectively, of Table V, while the results for the entire series of measures are in the last column. The corresponding number of observations entering into each average deviation are shown in columns two to five. Table VI shows the averages for the entire curve, and in addition, the probable errors of a single observation for each observer and instrument, and for the whole series of measures.

The internal agreement of the data for the different observers and instruments is as satisfactory as could be expected in view of the small number of observations available for discussion. The series for Seares, wedge photometer, shows an apparently definite positive systematic deviation from the curve. On the other hand, the first of the series for Haynes, gives a small negative deviation. In the remaining series for Haynes considered section by section, the positive signs are in excess, but in the average for the series the positive and negative residuals balance each other exactly. The average deviations in the last column of Table V, derived from all of the observations, are well within the uncertainties of the measures, and indicate that the mean light curve of Bulletin No. 8 satisfactorily represents the present series of observations.

SYSTEMATIC DIFFERENCES

	Variable	$c-b$
S–H, Wedge Phot.	$+ 0^{m}.106$	$+ 0^{m}.036$
W–Z, Haynes	— 0.030	+ 0.031

The data is not extensive enough to afford any very reliable indication as to the systematic differences between the two observers and the two instruments. The algebraic sign of the difference S – H is opposite to that usually obtained hitherto, and, in addition, the value derived from the observations on the variable is considerably larger than would be expected on the basis of our previous experience.

THE VARIABLE V LACERTAE (110.1904)

1855.0	R.A.	22^{h}	42^{m}	$43^{s}.85$	Dec. $+55°$	33′	21″.4 [1]
1900.0			44	32.29		47	34.3

A preliminary announcement concerning the variability of this star was made in Bulletin No. 8, page 114. As explained in that place, the earlier observations were referred to BD.+55°2817, an object which was itself found to be variable when an attempt was made to reduce these observations. It thus became necessary to determine the elements of variation of BD.+55°2817 (X Lacertae, 88.1906), before a complete reduction could be made of the observations on V Lacertae. This has been done and the final results are to be found in pp. 190-198 of the present bulletin. The following pages contain the discussion and results for all of the observations made on V Lacertae.

The star BD.+55°2817 is designated by *a*. As soon as the variability of this object was suspected, a second comparison star, *b*, was introduced, and when the variability of *a* had been confirmed, a third comparison object, *c*, was added. The stars *b* and *c* were also used as comparison objects in the investigation of the variability of *a*. Their BD. numbers and their brightness determined by a photometric reference to stars of the Potsdam DM. are given on pages 108, 109, 112 of Bulletin No. 8, and p. 195 of the present bulletin.

[1] Position from A. N. No. 3953.

TABLE VII

OBSERVATIONS OF V LACERTAE

Date	G. M. T.	Julian Day	Phase a	Phase v	Obs. Star	Wedge	$v-a$	$a-b$	$\Delta m = v-b$	O − C	No. Obs.
1904, June 30	18h 6m	2416662.75	1.19	0.90	S, a	15.4 16.9	+0m.27	+0m.20	+0m.47	− 3	4
July 1	16 58	6663.71	2.15	1.86	"	14.6 18.5	+ 0.72	— 0.04	0.68	− 9	4
	17 57	.75	2.19	1.90	"	14.6 19.0	+ 0.82	— 0.05	0.77	− 1	3
2	17 4	6664.71	3.15	2.86	"	13.5 19.0	+ 0.98	— 0.04	0.94	− 9	4
	58	.75	3.19	2.90	"	12.8 18.0	+ 0.89	— 0.03	0.86	— 19	2
4	15 31	6666.65	5.09	4.80	"	11.9 10.8	— 0.11	+ 0.27	0.16	− 4	4
	16 24	.68	5.12	4.83	"	12.2 11.2	— 0.11	+ 0.27	0.16	− 2	4
7	16 25	6669.68	2.68	2.84	"	16.0 22.1	+ 1.13	— 0.06	1.07	+ 4	3
8	15 28	6670.64	3.64	3.80	"	12.8 17.8	+ 0.85	+ 0.02	0.87	− 8	4
	20 28	.85	3.85	4.01	"	11.0 15.5	+ 0.62	+ 0.06	0.68	— 15	2
9	15 15	6671.64	4.64	4.80	"	16.8 16.2	— 0.11	+ 0.21	0.10	— 10	4
	20 4	.84	4.84	0.02	"	20.8 20.5	— 0.05	+ 0.24	0.19	− 2	4
	52	.87	4.87	0.05	"	21.8 21.9	+ 0.01	+ 0.25	0.26	+ 3	4
10	15 50	6672.66	0.21	0.84	"	19.0 19.4	+ 0.08	+ 0.30	0.38	— 11	4
11	15 24	6673.64	1.19	1.82	"	17.2 20.4	+ 0.62	+ 0.20	0.82	+ 6	4
	16 32	.69	1.24	1.87	"	16.9 20.5	+ 0.70	+ 0.19	0.89	+ 12	4
	20 52	.87	1.42	2.05	"	16.9 21.3	+ 0.84	+ 0.13	0.97	+ 15	4
12	15 15	6674.64	2.19	2.82	"	17.8 24.2	+ 1.04	— 0.05	0.99	− 3	4
	16 13	.68	2.23	2.86	"	17.7 24.2	+ 1.06	— 0.05	1.01	− 1	4
	20 50	.87	2.42	3.05	"	15.2 21.9	+ 1.24	— 0.06	1.18	+ 8	4
13	15 15	6675.64	3.19	3.82	"	16.6 21.8	+ 0.98	— 0.03	0.95	+ 1	4
14	15 48	6676.66	4.21	4.84	"	17.6 17.4	— 0.04	+ 0.13	0.09	− 9	4
	16 46	.70	4.25	4.88	"	16.8 16.6	— 0.04	+ 0.14	0.10	− 6	4
	17 46	.74	4.29	4.92	"	16.2 16.1	— 0.02	+ 0.15	0.13	− 3	4
	18 55	.79	4.34	4.97	"	15.8 16.3	+ 0.09	+ 0.16	0.25	+ 6	4
	19 50	.83	4.38	0.03	"	16.6 16.9	+ 0.06	+ 0.17	0.23	+ 1	4
	20 40	.86	4.41	0.06	"	16.3 16.6	+ 0.05	+ 0.17	0.22	− 2	4
	21 18	.89	4.44	0.09	"	16.8 17.1	+ 0.06	+ 0.18	0.24	− 1	2
15	15 13	6677.63	5.18	0.83	"	16.9 18.0	+ 0.22	+ 0.28	0.50	+ 1	4
16	15 40	6678.65	0.76	1.85	"	11.7 15.0	+ 0.47	+ 0.26	0.73	− 3	4
	19 57	.83	0.94	2.03	"	14.4 17.7	+ 0.59	+ 0.26	0.85	+ 4	4
17	15 18	6679.64	1.75	2.84	"	17.9 24.7	+ 1.08	+ 0.02	1.10	+ 7	4
	16 13	.68	1.79	2.88	"	17.2 23.0	+ 1.02	+ 0.01	1.03	− 1	4
	17 6	.71	1.82	2.91	"	16.2 22.0	+ 1.08	+ 0.01	1.09	+ 4	4
	52	.74	1.85	2.94	"	16.1 22.0	+ 1.10	0.00	1.10	+ 4	4
	18 36	.78	1.89	2.98	"	16.3 22.2	+ 1.09	— 0.01	1.08	+ 1	4
	19 28	.81	1.92	3.01	"	15.6 22.7	+ 1.28	— 0.01	1.27	+ 19	4
	20 14	.84	1.95	5.04	"	15.7 22.1	+ 1.18	— 0.02	1.16	+ 7	4
	53	.87	1.98	3.07	"	15.8 21.7	+ 1.10	— 0.03	1.07	− 3	4
	21 40	.90	2.01	3.10	"	15.4 21.7	+ 1.17	— 0.03	+ 1.14	+ 3	4

Date	G. M. T.	Julian Day	Phase a	Phase v	Obs. Star	Wedge	$v-a$	$a-b$	$\Delta m = v-b$	O − C	No. Obs.
1904, July 17	22h 9m	6679.92	2.03	3.12	S, *a*	15.8 21.7	+1m.10	−0m.03	+1m.07	− 4	4
	26	.93	2.04	3.13	"	15 2 21.5	1.18	— 0.03	1.15	+ 3	1
18	14 56	6680.62	2.73	3.82	"	15.8 21.7	1.10	— 0.06	1.04	+ 10	4
	15 46	.66	2.77	3.86	"	15.0 20.4	1.02	— 0.05	0.97	+ 5	4
23	14 48	6685.62	2.29	3.84	"	15.7 21.4	1.07	— 0.05	1.02	+ 9	4
	15 35	.65	2.32	3.87	"	15.1 20.4	1.00	— 0.05	0.95	+ 3	4
	16 24	.68	2.35	3.90	"	14.7 20.0	0.99	— 0.06	0.93	+ 3	4
24	14 39	6686.61	3.28	4.83	"	17.6 18.3	0.14	— 0.01	0.13	− 5	4
	15 52	.65	3.32	4.87	"	16.8 17.8	0.20	— 0.01	0.19	+ 3	5
	18 27	.77	3.44	0.00	"	15.3 16.8	0.27	0.00	0.27	+ 7	5
28	17 0	6690.71	1.93	3.94	"	12.8 17.3	0.75	— 0.02	0.73	− 15	4
Aug. 1	16 22	6694.68	0.46	2.93	"	14.1 17.9	0.68	+ 0.29	0.97	− 9	4
2	15 57	6695.66	1.44	3.91	"	11.2 15.3	0.57	+ 0.12	0.69	− 20	4
	17 26	.73	1.51	3.98	"	13.6 17.8	0.73	+ 0.10	0.83	− 2	4
3	15 55	6696.66	2.44	4.91	"	14.8 15.8	0.17	— 0.06	0.11	− 3	5
4	16 17	6697.68	3.46	0.95	"	13.6 16.4	0.46	+ 0.01	0.47	− 5	4
8	14 58	6701.62	1.96	4.89	"	14.1 15.0	0.14	— 0.02	0.12	− 3	4
	15 53	.66	2.00	4.93	"	13.2 14.6	0.21	— 0.03	0.18	+ 1	4
10	15 4	6703.63	3.97	1.91	"	18.2 22.5	0.76	+ 0.08	0.84	+ 6	4
	16 8	.67	4.01	1.95	"	18.4 22.6	0.73	+ 0.09	0.82	+ 3	4
	22 36	.94	4.28	2.22	"	18.6 23.1	0.75	+ 0.15	0.90	+ 4	4
11	15 10	6704.63	4.97	2.91	"	19.4 24.2	0.72	+ 0.26	0.98	− 7	4
12	18 51	6705.79	0.69	4.07	"	25.7 31.0	0.55	+ 0.27	0.82	+ 2	4
22	16 33	6715.69	5.14	4.11	"	26.0 31.0	0.52	+ 0.27	0.79	+ 2	4
Oct. 7	13 33	6761.56	2.03	0.05	"	17.3 19.3	0.40	— 0.03	0.37	+ 14	4
	14 9	.59	2.06	0.08	"	17.4 19.7	0.45	— 0.04	0.41	+ 16	4
11	14 49	6765.62	0.64	4.11	"	21.1 25.1	0.52	+ 0.28	0.80	+ 3	4
14	14 30	6768.60	3.62	2.11	"	18.8 23.6	0.77	+ 0.02	0.79	− 4	4
17	13 43	6771.57	1.15	0.10	"	19.7 20.4	0.13	+ 0.22	0.35	+ 9	4
1905, July 12	19 58	7039.83	2.72	4.27	"	11.5 17.0	0.74	— 0.06	0.68	+ 1	4
13	15 40	7040.65	3.54	0.11	"	10.2 12.3	0.28	+ 0.01	0.29	+ 3	2
	16 12	.68	3.57	0.14	H, *a*	13.0 15.2	0.30	+ 0.02	0.32	+ 4	4
14	16 30	7041.69	4.58	1.15	H, *a*	5.1 9.0	0.46	+ 0.21	0.67	+ 9	2
	33	.69	4.58	1.15	S, *a*	10.7 14.0	0.44	+ 0.21	0.65	+ 7	3
	18 25	.77	4.66	1.23	S, *a*	9.4 12.5	0.42	+ 0.22	0.64	+ 3	4
15	16 17	7042.68	0.13	2.14	H, *a*	6.4 10.0	0.43	+ 0.30	0.73	− 11	4
	17 24	.72	0.17	2.18	S, *a*	8.2 12.4	0.55	+ 0.30	0.85	0	2
16	15 13	7043.63	1.08	3.09	S, *a*	9.5 17.5	1.06	+ 0.23	1.29	+ 18	2
	30	.65	1.10	3.11	H, *a*	4.6 10.6	0.72	+ 0.23	0.95	− 16	2
	43	.65	1.10	3.11	S, *a*	8.4 14.9	0.86	+ 0.23	1.09	− 2	2
	56	.66	1.11	3.12	H, *a*	3.4 10.0	+ 0.76	+ 0.23	+ 0.99	− 12	2

Date	G. M. T.	Julian Day	Phase a	Phase v	Obs. Star	Wedge		$v-a$	$a-b$	$\Delta m = v-b$	O − C	No. Obs.
1905, July 17	16h 2m	2417044.67	2.12	4.13	S, a	6.7	14.8	+1m04	−0m04	+1m00	+ 24	2
	18	.68	2.13	4.14	H, a	3.5	10.7	0.85	— 0.04	0.81	+ 5	2
18	14 34	7045.61	3.06	0.09	S, a	10.4	12.6	0.29	— 0.04	0.25	0	2
	48	.62	3.07	0.10	H, a	7.3	9.6	0.28	— 0.04	0.24	— 2	2
22	15 59	7049.67	1.68	4.15	S, a	15.4	22.6	0.84	+ 0.05	0.89	+ 14	4
23	18 10	7050.76	2.77	0.26	"	6.1	9.8	0.44	— 0.05	0.39	+ 7	2
25	15 36	7052.65	4.66	2.15	"	15.0	19.5	0.57	+ 0.22	0.79	— 5	2
28	16 32	7055.69	2.25	0.20	"	9.7	11.7	0.27	— 0.05	0.22	— 8	2
29	17 7	7056.71	3.27	1.22	"	17.2	22.9	0.64	— 0.02	0.62	+ 1	2
31	17 27	7058.73	5.29	3.24	"	9.8	16.6	0.92	+ 0.29	1.21	+ 8	2
Aug. 4	17 58	7062.75	3.87	2.28	"	19.2	26.9	0.82	+ 0.06	0.88	+ 1	2
5	14 19	7063.60	4.72	3.13	"	26.0	34.2	0.91	+ 0.22	1.13	+ 1	3
	17 57	.75	4.87	3.28	"	29.9	38.0	0.89	+ 0.25	1.14	+ 1	2
7	14 46	7065.62	1.30	0.17	"	11.4	12.6	0.16	+ 0.17	0.33	+ 4	4
	20 22	.85	1.53	0.40	"	12.6	15.2	0.35	+ 0.09	0.44	+ 9	4
8	16 46	7066.70	2.38	1.25	"	9.9	16.3	0.86	— 0.06	0.80	+ 18	2
9	14 6	7067.59	3.27	2.14	"	13.0	21.3	1.02	— 0.02	1.00	+ 16	2
	20 42	.86	3.54	2.41	"	10.8	17.8	0.93	+ 0.01	0.94	+ 4	2
11	15 4	7069.63	5.31	4.18	"	17.7	22.3	0.51	+ 0.29	0.80	+ 7	2
14	16 56	7072.71	2.94	2.28	"	11.3	19.8	1.09	— 0.05	1.04	+ 17	2
25	18 20	7083.76	3.11	3.36	"	12.3	22.3	1.22	— 0.04	1.18	+ 6	2
26	17 32	7084.73	4.08	4.33	"	11.8	17.1	0.71	+ 0.11	0.82	+ 20	2
1906, Feb. 15	14 2	7257.58	2.76	2.79	"	13.6	22.5	1.08	— 0.05	1.03	+ 2	4
Mar. 9	13 46	7279.57	2.98	4.84	"	9.1	11.8	0.36	— 0.05	0.31	+ 13	4
June 8	18 38	7370.78	1.67	1.38	"	15.8	20.8	0.59	+ 0.05	0.64	— 1	2
	50	.78	1.67	1.38	H, a	14.2	18.2	0.51	+ 0.05	0.56	+ 11	2
9	16 16	7371.68	2.57	2.28	S, a	21.2	27.7	0.70	— 0.06	0.64	— 23	2
	35	.69	2 58	2.29	H, a	19.6	27.4	0.82	— 0.06	0.76	— 12	2
10	16 18	7372.68	3.57	3.28	S, a	22.5	27.2	0.49	+ 0.02	0.51	[—62]	2
	26	.68	3.57	3.28	H, a	18.8	23.9	0.54	+ 0.02	0.56	[—57]	2
11	17 30	7373.73	4.62	4.33	S, a	24.6	26.8	0.24	+ 0.21	0.45	— 17	2
	40	.74	4.63	4.34	H, a	24.6	25.7	0.12	+ 0.21	0.33	— 28	2
12	17 44	7374.74	0.18	0.36	S, a	24.8	26.4	0.17	+ 0.30	0.47	+ 13	2
	56	.75	0.19	0.37	H, a	19.5	20.7	0.12	+ 0.30	0.42	+ 8	2
15	14 44	7377.61	3.05	3.23	S, a	23.7	31.3	0.83	— 0.05	0.78	[—35]	2
	58	.62	3.06	3.24	H, a	24.7	32.5	0.86	— 0.04	0.82	[—31]	2
	19 22	.81	3.25	3.43	S, a	15.7	22.0	0.73	— 0.02	0.71	[—39]	2
	34	.82	3.26	3.44	H, a	16.2	22.4	0.70	— 0.02	0.68	[—42]	2
16	15 56	7378.66	4.10	4.28	S, a	16.6	23.9	0.81	+ 0.11	0.92	+ 26	2
	16 13	.68	4.12	4.30	H, a	19.2	25.6	0.68	+ 0.11	0.79	+ 14	2
	18 41	.78	4.22	4.40	S, a	14.8	20.0	0.65	+ 0.13	0.78	+ 22	2
	19 2	.79	4.23	4.41	H, a	14.8	20.4	+ 0.69	+ 0.14	+ 0.83	+ 28	2

Date	G. M. T.	Julian Day	Phase a	Phase v	Obs. Star	Wedge		$v-a$	$a-b$	$\Delta m = v-b$	O − C	No. Obs.
1906, June 17	15h 10m	2417379.63	5.07	0.27	S, a	23.1	22.3	−0m09	+0m27	+0m18	− 14	2
	24	.64	5.08	0.28	H, a	23.5	22.8	− 0.07	+ 0.27	0.20	− 12	2
19	15 13	7381.63	1.63	2.27	H, a	12.6	21.4	+ 1.08	+ 0.06	1.14	+ 27	2
	24	.64	1.64	2.28	S, a	7.6	16.0	+ 1.10	+ 0.06	1.16	+ 29	2
	20 51	.87	1.87	2.51	S, a	8.0	15.7	+ 1.02	0.00	1.02	+ 9	2
	21 0	.88	1.88	2.52	H, a	9.5	17.3	+ 1.05	− 0.01	1.04	+ 11	2
20	16 28	7382.69	2.69	3.33	S, a	11.1	22.2	+ 1.36	− 0.06	1.30	+ 18	2
	38	.69	2.69	3.33	H, a	11.6	21.6	+ 1.24	− 0.06	1.18	+ 6	2
21	14 45	7383.62	3.62	4.26	S, a	18.2	24.3	+ 0.66	+ 0.02	0.68	0	2
	15 0	.62	3.62	4.26	H, a	15.4	20.6	+ 0.62	+ 0.02	0.64	− 4	2
23	17 56	7385.75	0.31	1.41	S, a	16.0	18.5	+ 0.31	+ 0.30	0.61	− 4	2
	18 4	.75	0.31	1.41	H, a	9.3	11.8	+ 0.33	+ 0.30	0.63	− 2	2
24	15 43	7386.65	1.21	2.31	S, a	22.6	28.6	+ 0.64	+ 0.20	0.84	− 4	2
25	15 23	7387.64	2.20	3.30	S, a	15.0	25.7	+ 1.22	− 0.05	1.17	+ 4	2
	32	.65	2.21	3.31	H, a	11.7	21.6	+ 1.23	− 0.05	1.18	+ 5	2
26	14 47	7388.62	3.18	4.28	H, a	10.4	15.7	+ 0.71	− 0.03	0.68	+ 2	2
27	14 37	7389.61	4.17	0.28	S, a	14.0	15.6	+ 0.21	+ 0.12	0.33	+ 1	2
	48	.62	4.18	0.29	H, a	10.9	12.3	+ 0.18	+ 0.13	0.31	− 2	2
	21 34	.90	4.46	0.57	S, a	12.6	14.4	+ 0.24	+ 0.19	0.43	+ 3	2
	42	.90	4.46	0.57	H, a	10.0	11.2	+ 0.17	+ 0.19	0.36	− 4	2
July 1	14 50	7393.62	2.74	4.29	S, a	13.2	19.4	+ 0.79	− 0.06	0.73	+ 7	2
	58	.62	2.74	4.29	H, a	13.2	18.9	+ 0.74	− 0.06	0.68	+ 2	2
2	14 42	7394.61	3.73	0.30	S, a	15.0	16.6	+ 0.22	+ 0.04	0.26	− 7	2
	50	.62	3.74	0.31	H, a	16.3	18.4	+ 0.26	+ 0.04	0.30	− 3	2
3	14 38	7395.61	4.73	1.30	S, a	14.0	18.4	+ 0.57	+ 0.23	0.80	+ 17	2
	46	.62	4.74	1.31	H, a	15.2	19.1	+ 0.49	+ 0.23	0.72	+ 9	2
5	17 39	7397.74	1.41	3.43	S, a	11.3	20.2	+ 1.13	+ 0.14	1.27	+ 17	2
	46	.74	1.41	3.43	H, a	10.7	18.6	+ 1.03	+ 0.14	1.17	+ 7	2
8	16 11	7400.67	4.34	1.38	H, a	13.7	16.8	+ 0.41	+ 0.16	0.57	− 8	1
	17	.68	4.35	1.39	S, a	15.9	19.8	+ 0.47	+ 0.16	0.63	− 2	1
10	15 55	7402.66	0.89	3.37	H, a	13.4	20.4	+ 0.87	+ 0.26	1.13	+ 1	2
	21 22	.89	1.12	3.60	H, a	10.5	17.3	+ 0.90	+ 0.22	1.12	+ 9	2
11	14 44	7403.61	1.84	4.32	S, a	24.4	30.4	+ 0.65	0.00	0.65	+ 2	1
	49	.62	1.85	4.33	H, a	21.8	27.7	+ 0.63	0.00	0.63	+ 1	1
16	14 50	7408.62	1.41	4.35	S, a	27.2	31.5	+ 0.48	+ 0.14	0.62	+ 2	1
					S, b	25.8	32.0			0.69	+ 9	1
	15 0	.62	1.41	4.35	H, a	23.5	27.8	+ 0.46	+ 0.14	0.60	0	1
					H, b	21.0	27.3			0.67	+ 7	1
17	14 40	7409.61	2.40	0.35	S, a	17.4	20.0	+ 0.30	− 0.06	0.24	− 10	1
					S, b	18.0	20.4			0.27	− 7	1
	14 49	.62	2.41	0.35	H, a	14.1	17.8	+ 0.49	− 0.06	0.43	+ 9	1
					H, b	14.5	17.4			+ 0.39	+ 5	1

Date	G. M. T.	Julian Day	Phase a	Phase v	Obs. Star	Wedge	$v-a$	$a-b$	$\Delta m = v-b$	O − C	No. Obs.
1906, July 17	$20^h\ 53^m$	2417409.87	2.66	0 61	H, *a*	14.4 16.6	$+0^m.30$	$-0^m.06$	$+0^m.24$	− 17	1
					H, *b*	13.6 17.1			0.47	+ 6	1
	21 1	.88	2.67	0.62	S, *a*	11.1 14.4	0.44	− 0.06	0.38	− 4	1
					S, *b*	11.5 15.0			0.47	+ 5	1
18	14 35	7410.61	3.40	1.35	H, *a*	18.9 23.3	0.47	0.00	0.47	− 17	1
					H, *b*	18.5 23.2			0.51	− 13	1
	44	.61	3.40	1.35	S, *a*	19.8 24.5	0.49	0.00	0.49	− 15	1
					S, *b*	19.7 25.5			0.60	− 4	1
	21 50	.91	3.70	1.65	H, *a*	12.4 17.3	0.66	+ 0.03	0.69	− 3	1
					H, *b*	12.0 17.5			0.73	+ 1	1
	58	.92	3.71	1.66	S, *a*	13.4 18.0	0.60	+ 0.03	0.63	− 9	1
					S, *b*	13.4 17.9			0.59	− 13	1
20	14 47	7412.62	5.41	3 36	H, *a*	17.0 22.7	0.64	+ 0.30	0.94	− 18	1
					H, *b*	15.0 22.5			0.89	− 23	1
	59	.62	5.41	3.36	S, *a*	20.0 26.7	0.71	+ 0.30	1.01	− 11	1
					S, *b*	17.6 26.6			0.98	− 14	1
	19 22	.81	0.16	3.55	H, *a*	13.6 20.0	0.81	+ 0.30	1.11	+ 5	1
					H, *b*	11.2 20.0			1.02	− 4	1
	33	.81	0.16	3.55	S, *a*	13.8 20.4	0.82	+ 0.30	1.12	+ 6	1
					S, *b*	11.9 20.4			1.07	+ 1	1
21	14 31	7413.60	0.95	4.34	S, *a*	17.9 21.4	0.38	+ 0.26	0.64	+ 3	1
					S, *b*	16.3 22.2			0.67	+ 6	1
	41	.61	0.96	4.35	H, *a*	17.9 21.4	0.28	+ 0.25	0.53	− 7	1
					H, *b*	16.3 21.5			0.59	− 1	1
	18 54	.79	1 14	4.53	S, *a*	18.3 19.4	0.12	+ 0.22	0.34	− 9	1
					S, *b*	16.4 19.0			0.32	− 11	1
	19 4	.79	1.14	4.53	H, *a*	16.7 19.1	0.29	+ 0.22	0.51	+ 8	1
					H, *b*	15.2 18.5			0.42	− 1	1
22	15 32	7414.65	2.00	0.41	S, *a*	19.2 23.1	0.42	− 0.03	0.39	+ 4	1
					S, *b*	19.1 23.6			0.48	+ 13	1
	42	.65	2.00	0.41	H, *a*	18.7 21.3	0.28	− 0.03	0.25	− 10	1
					H, *b*	18.6 20.9			0.25	− 10	1
	55	.66	2.01	0.42	S, *a*	18.0 22.1	0.45	− 0.03	0.42	+ 7	1
					S, *b*	18.8 21.5			0.28	− 7	1
23	14 44	7415.61	2.96	1.37	H, *a*	21.3 27.0	0.61	− 0.05	0.56	− 8	1
					H, *b*	21.6 27.1			0.58	− 6	1
	15 0	.62	2.97	1.38	S, *a*	19.4 25.8	0.68	− 0.05	0.63	− 2	1
					S, *b*	20.4 25.2			0.50	− 15	1
24	14 14	7416.59	3.94	2.35	H, *a*	16.2 23.1	0.78	+ 0.08	0.86	− 3	1
					H, *b*	16.1 23.1			0.80	− 9	1
	26	.60	3.95	2.36	S, *a*	19.1 25.2	+ 0.74	+ 0.08	0.82	− 7	1
					S, *b*	18.9 25.9			+ 0.74	− 15	1

Date	G. M. T.	Julian Day	Phase *a*	Phase *b*	Obs. Star	Wedge		*v* − *a*	*a* − *b*	$\Delta m = v - b$	O − C	No. Obs.
1906, July 24	19h 18m	2417416.80	4.15	2.56	S, *a*	15.0	21.7	+0m.80	+0m.12	+0m.92	− 2	1
					S, *b*	14.5	22.3			0.93	− 1	1
	28	.81	4.16	2.57	H, *a*	17.9	24.1	+ 0.67	+ 0.12	0.79	− 15	1
					H, *b*	17.9	24.7			0.76	− 18	1
25	15 5	7417.63	4.98	3.39	S, *a*	19.5	27.5	+ 0.84	+ 0.26	1.10	− 1	1
					S, *b*	17.7	28.0			1.12	+ 1	1
	16	.64	4.99	3.40	H, *a*	21.1	28.7	+ 0.82	+ 0.26	1.08	− 3	1
					H, *b*	19.0	28.7			1.04	− 7	1
26	15 6	7418.63	0.53	4.39	S, *a*	27.2	29.8	+ 0.29	+ 0.29	0.58	+ 1	1
					S, *b*	25.4	31.1			0.63	+ 6	1
	16 24	.68	0.58	4.44	H, *a*	21.2	23.5	+ 0.25	+ 0.28	0.53	+ 1	1
					H, *b*	19.0	23.4			0.47	− 5	1
28	14 24	7420.60	2.50	1.38	S, *a*	16.7	23.6	+ 0.77	− 0.06	0.71	+ 6	1
					S, *b*	18.1	23.7			0.61	− 4	1
	32	.61	2.51	1.39	H, *a*	17.3	22.9	+ 0.62	− 0.06	0.56	− 9	1
					H, *b*	17.8	22.7			0.54	− 11	1
	21 6	.88	2.78	1.66	S, *a*	15.0	20.8	+ 0.70	− 0.05	0.65	− 7	1
					S, *b*	15.2	21.0			0.69	− 3	1
	15	.89	2.79	1.67	H, *a*	14.8	20.6	+ 0.71	− 0.05	0.66	− 6	1
					H, *b*	15.9	21.1			0.60	− 12	1
29	15 48	7421.66	3.56	2.44	H, *a*	16.0	22.4	+ 0.73	+ 0.02	0.75	− 16	1
					H, *b*	15.9	22.2			0.72	− 19	1
30	14 17	7422.60	4.50	3.38	S, *a*	25.1	33.7	+ 0.96	+ 0.20	1.16	+ 5	1
					S, *b*	23.2	32.2			0.98	− 13	1
	18 57	.79		3.57	S, *b*	17.4	24.9			0.81	− 23	1
					S, *b*	16.5	26.3			1.07	+ 3	1
	19 13	.80	4.70	3.58	H, *a*	20.6	26.3	+ 0.60	+ 0.22	0.82	− 22	1
					H, *b*	18.3	26.1			0.83	− 21	1
Aug. 1	14 59	7424.62	1.08	0.42	H, *a*	23.2	22.1	− 0.12	+ 0.23	0.11	− 25	1
					H, *b*	19.7	22.4			0.28	− 8	1
	16 2	.67	1.13	0.47	S, *a*	20.5	20.1	− 0.04	+ 0.22	0.18	− 19	1
					S, *b*	18.1	19.9			0.21	− 16	1
	19 44	.82	1.28	0.62	S, *a*	19.4	20.6	+ 0.13	+ 0.18	0.31	− 11	1
					S, *b*	17.0	20.6			0.41	− 1	1
	52	.83	1.29	0.63	H, *a*	19.8	19.9	+ 0.01	+ 0.17	0.18	− 24	1
					H, *b*	17.0	21.2			+ 0.47	+ 5	1

TABLE VIII

OBSERVATIONS OF V LACERTAE. CONTINUED

Date	G. M. T.	Julian Day	Phase	Obs. Star	Readings		Δm	Mag.	O−C
1906, Aug. 2	$14^h\ 23^m$	2417425.60	$1^d.40$	S, *b*	23.3	29.5	$+\ 0^m.66$	$8^m.95$	+ 1
				S, *c*	28.3	28.7	+ 0.05	8.82	— 12
3	15 28	7426.64	2.44	H, *b*	20.8	27.2	+ 0.68	8.97	— 23
				H, *c*	23.4	27.6	+ 0.45	9.22	+ 2
	42	.65	2.45	S, *b*	19.6	26.4	+ 0.71	9.00	— 21
				S, *c*	22.8	25.9	+ 0.32	9.09	— 12
7	14 16	7430.59	1.40	S, *b*	17.4	22.0	+ 0.51	8.80	— 14
				S, *c*	21.2	22.2	+ 0.11	8.88	— 6
Sept. 7	15 59	7461.67	2.59	H, *b*	14.9	24.1	+ 1.06	9.35	+ 11
				H, *c*	21.2	23.9	+ 0.29	9.06	— 18
8	16 4	7462.67	3.59	S, *b*	12.4	21.3	+ 1.10	9.39	+ 6
				S, *c*	14.9	21.4	+ 0.77	9.54	+ 21
12	19 5	7466.79	2.72	H, *b*	16.5	23.1	+ 0.74	9.03	— 25
				H, *c*	20.9	23.1	+ 0.24	9.01	— 27
	16	.80	2.73	S, *b*	17.5	24.2	+ 0.73	9.02	— 26
				S, *c*	20.4	24.3	+ 0.41	9.18	— 10
17	15 18	7411.64	2.59	H, *b*	17.8	23.6	+ 0.63	8.92	— 32
				H, *c*	20.0	23.8	+ 0.40	9.17	— 7
18	15 26	7472.64	3.59	S, *b*	14.3	24.8	+ 1.21	9.50	+ 17
				S, *c*	18.8	23.6	+ 0.51	9.28	— 5
19	15 30	7473.65	4.60	H, *b*	13.5	14.9	+ 1.19	8.48	— 17
				H, *c*	17.5	14.7	— 0.37	8.40	— 25
20	14 26	7474.60	0.57	H, *b*	13.8	16.1	+ 0.31	8.60	— 9
				H, *c*	17.6	16.7	— 0.11	8.66	— 3
1907, May 22	21 25	7718.89	0.71	H, *b*	20.1	22.0	+ 0.20	8.49	— 25
				H, *c*	23.7	23.1	— 0.06	8.71	— 3
June 14	21 6	7741.88	3.77	H, *b*	12.9	20.2	+ 0.92	9.21	— 4
				H, *c*	16.4	20.8	+ 0.51	9.28	+ 3
15	21 4	7742.88	4.77	H, *b*	16.4	17.6	+ 0.15	8.44	— 7
				H, *c*	20.2	17.8	— 0.27	8.50	— 1
16	21 0	7743.88	0.78	H, *b*	18.2	21.0	+ 0.31	8.60	— 16
				H, *c*	21.2	20.9	— 0.03	8.74	— 2
17	20 22	7744.85	1.75	H, *b*	17.0	21.6	+ 0.52	8.81	— 22
				H, *c*	20.4	20.2	— 0.02	8.75	— 28
27	20 22	7754.85	1.79	H, *b*	50.9	36.3	+ 0.69	8.98	— 6
				H, *c*	43.3	31.8	+ 0.57	9.34	+ 30
28	20 58	7755.87	2.81	H, *b*	50.9	26.5	+ 1.21	9.50	+ 20
				H, *c*	32.3	27.3	+ 0.34	9.11	— 19
July 2	21 10	7759.88	1.84	H, *b*	56.6	34.8	+ 0.83	9.12	+ 7
				H, *c*	44.6	34.7	+ 0.45	9.22	+ 17

Date	G. M. T.	Julian Day	Phase	Obs. Star	Readings		Δm	Mag.	O − C
1907, July 4	$19^h\ 24^m$	2417761.81	$3^d.77$	H, *b*	45.4	25.4	+ 1.10	9.39	+ 14
				H, *c*	36.1	27.8	+ 0.52	9.29	+ 4
5	20 34	7762.86	4.82	H, *b*	47.6	41.5	+ 0.24	8.53	+ 5
				H, *c*	36.2	42.3	− 0.28	8.49	+ 1
6	19 26	7763.81	0.78	H, *b*	46.2	35.9	+ 0.45	8.74	− 2
				H, *c*	26.4	33.1	[− 0.45]		[−44]
7	20 6	7764.84	1.81	H, *b*	72.2	44.2	+ 0.67	8.96	− 8
				H, *c*	62.2	46.5	+ 0.43	9.20	+ 16
8	20 6	7765.84	2.81	H, *b*	36.3	23.5	+ 0.86	9.15	− 15
				H, *c*	29.4	21.1	+ 0.68	9.45	+ 15
11	19 38	7768.82	0.81	H, *b*	42.4	28.7	+ 0.74	9.03	+ 26
				H, *c*	34.4	33.6	+ 0.04	8.81	+ 4
19	19 22	7776.81	3.82	H, *b*	35.7	20.4	+ 1.12	9.41	+ 18
				H, *c*	32.1	23.1	+ 0.66	9.43	+ 20
20	18 43	7777.78	4.79	H, *b*	48.2	42.3	+ 0.23	8.52	+ 2
				H, *c*	33.6	36.9	− 0.17	8.60	+ 10
21	18 12	7778.76	0.78	H, *b*	49.8	36.9	+ 0.51	8.80	+ 4
				H, *c*	37.0	37.0	0.00	8.77	+ 1
29	18 16	7786.76	3.80	H, *b*	47.4	25.1	+ 1.20	9.49	+ 25
				H, *c*	37.4	26.2	+ 0.70	9.47	+ 23
31	19 11	7788.80	0.86	H, *b*	40.8	26.8	+ 0.80	9.09	+ 31
				H, *c*	32.0	27.7	+ 0.28	9.05	+ 27
Aug. 3	19 4	7791.79	3.85	H, *b*	28.4	17.6	+ 0.99	9.28	+ 7
				H, *c*	23.0	18.2	+ 0.49	9.26	+ 5
8	18 3	7796.75	3.83	H, *b*	45.4	26.2	+ 1.04	9.33	+ 10
				H, *c*	32.3	28.5	+ 0.24	9.01	− 22
9	18 20	7797.76	4.84	H, *b*	52.6	40.4	+ 0.44	8.73	+ 26
				H, *c*	37.8	45.7	− 0.33	8.44	− 3
10	17 6	7798.71	0.80	H, *b*	41.4	32.7	+ 0.44	8.73	− 4
				H, *c*	34.1	32.9	+ 0.07	8.84	+ 7
11	16 8	7799.67	1.76	H, *b*	19.3	23.1	+ 0.41	8.70	− 33
				H, *c*	22.6	22.8	+ 0.02	8.79	− 24

REMARKS

All observations previous to 1905, July 12, were obtained with the equalizing photographic-wedge photometer. Observations from 1905, July 12, to 1907, June 17, inclusive, and the two on 1907, Aug. 11, were made with the equalizing Zeiss-wedge photometer. The remaining observations were obtained with the Zöllner-Müller photometer.

In addition to the remarks concerning atmospheric conditions appended to Table II, the following are to be noted:

1904
July 1 — Aperture reduced to 4 inches.
2 — Observations stopped by haze.
8 — Observations stopped by clouds.
14 — Occasional clouds. Observations stopped by clouds.
15 — Occasional clouds.
17 — Daylight at end.
24 — Observations stopped by clouds.
1904
28 — Observations stopped by haze.
Aug. 2 — Aperture reduced to 4½ inches.
3 — Haze and clouds. Aperture 4½ inches.
11 — Interrupted by haze.
Oct. 11 — Image bad. Star in zenith. Settings difficult.
1905
July 17 — Observations stopped by haze.
Aug. 4 — Interrupted by haze.

TABLE IX

OBSERVED MAXIMA AND THEIR REPRESENTATION.

No.	E	Obs. Max.	Wt.	O – C	No.	E	Obs. Max.	Wt.	O – C
1	0	6666d7	1	— od14	15	143	7379d7	1	+ od34
2	1	6671.7	2	— 0.12	16	144	7384.4	2	+ 0.06
3	2	6676.8	2	0.00	17	145	7389.4	2	+ 0.08
4	4	6686.8	1	+ 0.03	18	146	7394.2	2	— 0.11
5	6	6696.7	2	— 0.03	19	147	7399.2	1	— 0.09
6	7	6701.6	1	— 0.11	20	149	7409.3	2	+ 0.04
7	20	6766.6	1	+ 0.11	21	150	7414.2	2	— 0.04
8	75	7040.5	1	— 0.04	22	151	7419.4	2	+ 0.18
9	76	7045.6	1	+ 0.08	23	152	7424.4	2	+ 0.20
10	77	7050.6	1	+ 0.10	24	162	7474.0	1	— 0.03
11	78	7055.5	1	+ 0.01	25	216	7743.0	1	— 0.10
12	80	7065.4	2	— 0.05	26	220	7762.9	2	— 0.13
13	123	7279.7	1	— 0.01	27	223	7777.8	2	— 0.17
14	142	7374.6	1	+ 0.22	28	227	7797.8	1	— 0.11

The observations were secured with two different instruments, the equalizing wedge photometer and the Zöllner-Müller photometer, both being used in connection with the 7½-inch equatorial. Strictly speaking, three instruments were employed, for both the photographic and Zeiss wedges were used with the equalizing wedge photometer.[1] All of the earlier observations were made with the photographic wedge. Those secured during 1905, 1906, and a part of the year 1907 were made with the Zeiss wedge. The Zöllner-Müller photometer was used only at the end of the series. The total number of observations made is 618, of which 446 are by Seares and 172 by Haynes. The results for all of the measures are shown in Tables VII and VIII, although the observations for 1906, June 10 and 15, and the last observation for 1907, July 6, thirteen in all, were excluded from the determination of the mean light-curve. The residuals

[1] *Laws Observatory Bulletin* No. 7.

for the first two of these dates are systematic in character. The discordances are doubtless due to some difficulty with the instrument, for the measures of both observers are similarly affected, and observations of other stars made upon these nights show similar deviations. The large error for 1907, July 6, is probably due to atmospheric conditions.

The measures secured with the photographic wedge were among the first made at this observatory with the equalizing wedge photometer. The number of settings per night was purposely increased beyond that necessary for a good determination of the brightness of the variable, in order to afford a basis for the determination of the relative magnitude of the errors de-

TABLE X

NORMAL PLACES

Δm REFERRED TO STAR *b*, 8^m.29

No.	Phase	Δm	No. Obs.	Curve	O − C
1	0.03	+ 0^m.26	25	+ 0^m.22	+ 4
2	0.18	0.30	40	0.29	+ 1
3	0.48	0.36	36	0.37	− 1
4	0.84	0.48	29	0.49	− 1
5	1.33	0.63	43	0.64	− 1
6	1.80	0.73	37	0.75	− 2
7	2.09	0.86	36	0.83	+ 3
8	2.42	0.87	36	0.91	− 4
9	2.82	0.99	31	1.02	− 3
10	2.94	1.06	30	1.06	0
11	3.09	1.12	32	1.11	+ 1
12	3.33	1.13	24	1.12	+ 1
13	3 53	1.08	20	1.07	+ 1
14	3.82	0.99	36	0.94	+ 5
15	4.01	0.80	32	0.84	− 4
16	4.25	0.75	28	0.69	+ 6
17	4.41	0.55	28	0.55	0
18	4.82	0.18	32	0.19	− 1
19	4.91	+0.15	30	+0.16	− 1

rived from a consideration of the results of a single night, and those given by a comparison with the mean light-curve, which is based upon the entire series. To save space in printing, these observations, when made by the same observer and referred to the same comparison star, have been combined into means. The number of observations entering into each mean is indicated in the last column of Table VII, an observation, in each case, consisting of four settings on both the variable and the comparison star. In Table VIII the column containing the readings gives, in all cases, the means of the settings of the wedge and of the intensity circle for a single observation.

For the determination of the period and the mean light-curve, observations not directly referred to star *b* were reduced to this object. The difference in brightness, $a-b$, was obtained from the mean light-curve for *a* given on page 112 of Bulletin No. 8. The phase values necessary for this interpolation, calculated with the elements on page 196, are to be found in column four of Table VII. The values of the difference $a-b$ are in column nine. These combined with the differences $v-a$ from the preceding column give the values of $v-b$ in column ten. The observations referred to *c*, all of which are in Table VIII, were reduced to *b* with the value $c-b = +0^{m}.48$ from page 195. These results are not printed, but instead, there is given in column eight of Table VIII the magnitudes of the variable. On the other hand, the magnitudes of the variable do not appear in Table VII. They can be obtained, however, by adding the quantities in column ten of this table to the magnitude of *b*, which is 8.29.

After having been reduced to *b*, the observations were plotted and the epochs of maxima determined in the usual manner. The results are in columns three and eight of Table IX. The corresponding values of E and the weights are given in adjacent columns. This data together with the preliminary elements

$$\text{Max.} = \text{J. D. } 2416666.74 + 4^{d}.98347\text{E}, \qquad \text{G. M. T.}$$

formed the basis for the least square reduction. The corrections found for the provisional elements are

$$\Delta T_{o} = +0^{d}.096 + 0^{d}.029,$$
$$\Delta P_{o} = -0.00078 \pm 0.00022,$$

the appended quantities being the probable errors. The revised elements are therefore

$$\text{Max.} = \text{J. D. } 2416666^{d}.84 + 4^{d}.98269\text{E}, \qquad \text{G. M. T.}$$

The representation of the observed maxima by these elements is indicated by the residuals, O – C, in columns five and ten of Table IX. The probable error of an observed maximum of unit weight is $\pm 0^{d}.10$.

The phase values based upon the revised elements are given in column five of Table VII, and column four of Table VIII, respectively. The normal places used for the determination of the mean light-curve are given in Table X, the mean phase and the mean Δm referred to star *b* being in columns two and three, respectively, while column four shows the number of observations included in each normal. The resulting curve is illustrated by Fig. 1, its ordinates being given in Table XI for every tenth day of the phase.

An inspection of the curve indicates that the maxima fall $0^{d}.08$ earlier than the dates defined by the revised elements. This arises from the fact that a tracing of an approximation for the light-curve was used in reading the epochs of maxima from the plot,[1] the maximum of the

[1] *Lawa Observatory Bulletin* No. 10, pp. 158-9.

approximation differing slightly in position from that of the curve finally derived. This circumstance in no wise affects the accuracy of the value derived for the period, for the difference enters systematically into all of the observed maxima. Its effect is eliminated by subtracting $0^d.08$ from the initial epoch given by the least square reduction. We thus have for the final elements

$$\text{Max.} = \text{J. D. } 2416666.76 + 4^d.98269E, \quad \text{G. M. T.}, \quad M - m = 1^d.65.$$

It is to be noted, however, that the phase values which appear in Table X refer to the epochs defined by the revised elements given above, and not to those of the final elements.

The residuals in the normal places given by a comparison with the mean light-curve are in the last column of Table X, while those for the observations in Tables VII and VIII are to be found in the last columns of these tables. The systematic deviations from the mean light-curve for the different observers and instruments are to be found in Table XII. These were

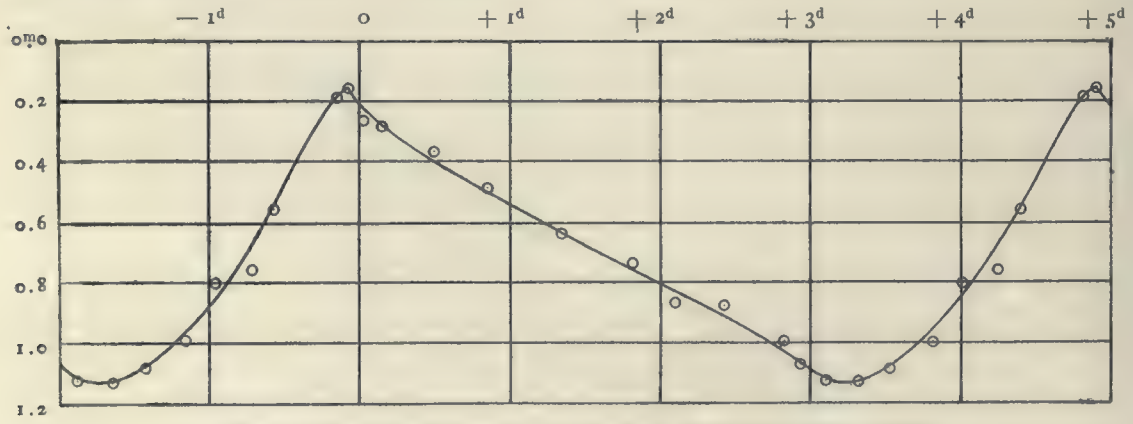


Fig. 1—Mean Light-Curve of V Lacertae. Referred to Star *b*, $8^m.29$

obtained by separating all the observations made with each instrument by each observer, and dividing them into groups in accordance with the corresponding phase values as in the formation of normal places. The mean residual for each group, regard being paid to the algebraic sign, indicates the systematic deviation of the group from the mean light-curve. The average of the residuals without regard to the sign gives a measure of the precision of the observations, excepting for those cases in which the systematic deviations are appreciable. For these the average residual represents a combination of accidental and systematic error, from which the magnitude of the former can be inferred, approximately at least, provided the latter is known. Each subdivision of Table XII gives the mean phase value for each group of residuals, the systematic or average deviation of the group, the number of observations included in each group, and, finally, the average of the residuals without regard to algebraic sign. The last line of the table gives the averages and the total number of observations for each observer and instrument.

So far as systematic differences are concerned, an inspection of the results reveals the folfowing: The magnitude scales of the photographic and Zeiss wedges appear to be in satisfactory agreement. The small systematic difference of $0^m.02$ between the results obtained with the two wedges by Seares agrees in algebraic sign with that derived from the observation of selected pairs of Pleiades stars;[1] but the magnitude of the difference is so small that its reality is open to some doubt. The observations by Haynes seem to indicate a considerable difference between the scales of the Zeiss wedge and the Zöllner-Müller photometer, but in this case the number of observations secured with the latter instrument is too small to admit of any definite conclusion.

TABLE XI

MEAN LIGHT-CURVE

PHASE FROM MAXIMUM. Δm REFERRED TO STAR *b*, $8^m.29$

Phase	Δm	Mag.	Phase	Δm	Mag.	Phase	Δm	Mag.
$0^d.00$	$+0^m.20$	$8^m.49$	$1^d.70$	$+0^m.73$	$9^m.02$	$3^d.40$	$+1^m.11$	$9^m.40$
0.10	0.26	8.55	1.80	0.75	9.04	3.50	1.08	9.37
0.20	0.30	8.59	1.90	0.78	9.07	3.60	1.03	9.32
0.30	0.33	8.62	2.00	0.80	9.09	3.70	1.00	9.29
0.40	0.35	8.64	2.10	0.83	9.12	3.80	0.95	9.24
0.50	0.38	8.67	2.20	0.85	9.14	3 90	0.90	9.19
0.60	0.41	8.70	2.30	0.88	9.17	4.00	0.84	9.13
0.70	0.45	8.74	2.40	0.90	9.19	4.10	0.78	9.07
0.80	0.48	8.77	2.50	0.93	9.22	4.20	0.72	9.01
0.90	0.50	8.79	2.60	0.95	9.24	4.30	0.65	8.94
1.00	0.54	8.83	2.70	0.98	9.27	4.40	0.56	8.85
1.10	0.57	8.86	2.80	1.01	9.30	4.50	0.46	8.75
1.20	0.60	8.89	2.90	1.05	9.34	4.60	0.36	8.65
1.30	0.63	8.92	3.00	1.08	9.37	4.70	0.27	8.56
1.40	0.65	8.94	3.10	1.11	9.40	4 80	0.20	8.49
1.50	0.68	8.97	3.20	1.13	9.42	4.90	0.15	8.44
1.60	+ 0.70	8.99	3.30	+ 1.13	9.42	5.00	+ 0.21	8.50

The systematic difference for the two observers derived from the observations with the Zeiss wedge is more reliable. The means indicate that the magnitude differences determined by Haynes are, on the average, $0^m.06$ smaller than those by Seares. This result agrees in sign, at least, with that derived from the measures on X Lacertae and given on page 198. For the present, the significance of these systematic differences, as well as those derived in previous bulletins, must remain more or less negative in character. Thus far no attempt has been made to trace their connection with the magnitude of the interval measured, or with the color of the objects observed. Until this can be done, the only conclusion to be drawn is that the results of

[1] *Laws Observatory Bulletin* No. 7, p. 104.

the two observers are in substantial agreement, for the systematic differences are, generally speaking, small.

In judging the precision of the measures it is to be noted that the average residuals in column four of Table XII correspond to the mean of four observations, those in columns eight and twelve, to the mean of two observations, and those in the last column, to a single observation. Further, as pointed out above, the average residuals, in the last three divisions at least, include the effect of both systematic and accidental errors.

TABLE XII

SYSTEMATIC DEVIATIONS FROM MEAN LIGHT-CURVE AND AVERAGE RESIDUALS

Seares, Photographic Wedge				Seares, Zeiss Wedge				Haynes, Zeiss Wedge				Haynes, Zöllner Phot.			
Phase	Av. Dev.	No. Obs.	Av. v	Phase	Av. Dev.	No. Obs.	Av. v	Phase	Av. Dev.	No. Obs.	Av. v	Phase	Av. Dev.	No. Obs.	Av. v
$0^d.05$	+ 5	45	± 6	$0^d.21$	− 2	18	± 5	$0^d.26$	0	16	± 5	0.81	+ 10	9	± 12
0.88	− 4	16	5	0.47	0	20	7	0.59	− 9	16	9	1.81	+ 9	6	14
1.88	+ 2	27	6	1.29	+ 3	21	7	1.47	− 7	23	12	2.81	0	4	17
2.10	+ 5	16	7	1.49	− 6	11	6	2.45	− 8	24	15	3.81	+ 10	10	15
2.87	− 3	33	6	2.24	+ 4	16	12	3.27	− 6	12	10	4.82	+ 7	6	± 8
3.04	+ 3	37	6	2.54	− 5	16	9	3.56	− 2	12	7				
3.84	+ 3	28	6	3.24	+ 5	19	8	4.27	+ 3	11	4				
4.02	− 6	26	8	3.50	+ 4	14	8	4.47	− 3	16	± 12				
4.83	− 4	25	5	4.23	+ 10	19	10								
4.92	− 1	25	4	4.44	+ 5	18	± 12								
	0.00	268	± 0.06		+0.02	172	± 0.08		−0.04	130	± 0.10		+0.07	35	± 0.13

As suggested above, the numerous measures made upon certain nights permit of an evaluation of the errors which are peculiar to a night and affect systematically all of the observations obtained upon that night. By comparing the individual observations with the mean for the night upon which they were made, the average residual for a single observation referred to that mean is found to be $\pm 0^m.074$. From this it appears that the average residual for the mean of four observations referred to the mean of the night upon which they were made is $\pm 0^m.037$. On the other hand, the same measures give for the average residual of the mean of four observations referred to the mean light-curve, that is, to the mean of the whole series, $\pm 0^m.060$. A part of the excess of the latter residual over the former is doubtless due to uncertainties in the elements used for the reduction; but, in the main, it is to be thought of as the result of a variation in circumstances affecting the observations, which are constant, or sensibly so, for any given night, but change appreciably from night to night. Among these are to be included the physical condition of the observer, the state of the atmosphere, the appearance and color of the arti-

ficial star, which may vary as a result of changes in focus and in the voltage of the current used, and, finally, the the brightness of the sky background. A comparison of the two residuals indicates that the residual peculiar to a night and affecting systematically the work of that night is, on the average, $\pm 0^{m}.047$. A comparison of this quantity with the residuals resulting from the accidental errors corresponding to any given set of observing conditions affords a basis for an estimation of the number of observations which it is desirable to make upon any given night. The residual for a single observation referred to the mean of the night has been found to be $\pm 0^{m}.074$. The corresponding residual for the mean of two observations is $\pm 0^{m}.052$, and for the mean of four observations, $\pm 0^{m}.037$. That for the mean of two observations is only slightly in excess of $\pm 0^{m}.047$, the residual peculiar to a night. The latter, however, does not include the effect of systematic differences which are peculiar to an observer and to the particular instrument employed, for its value has been derived from the measures made by a single observer with a single instrument. Bearing this in mind, it would seem doubtful if any advantage is to be derived by increasing the number of observations made upon any given night beyond two, unless the additional observations can be secured by a second observer or with a different instrument. This conclusion applies, of course, only to those stars whose variation during a single night is inappreciable, or, at least, small. Although it is based solely upon the results obtained by Seares with the photographic wedge, an inspection of the results by Haynes, and by both observers with other instruments, seems to indicate that it is valid for all of the photometric work done at this observatory.

Columbia, Missouri, 1907, Dec. 31. *F. H. Seares.*

LAWS OBSERVATORY
UNIVERSITY OF MISSOURI

BULLETIN NO. 14

THE VARIABLE RV TAURI (45.1905)[1]

A. G. Camb. 2168 1855.0 R. A. $4^h38^m11^s.82$ Dec. $+25°54'43''.2$
1900.0 4° 57'.58 59 53.4

The variability of this object, BD.+25°732, $8^m.9$, was discovered by Mme. Ceraski from photographs by Blajko taken at various times during the years 1895 to 1905.[2] The star was placed upon the observing list of the Laws Observatory in November, 1906. Since that date 160 photometric observations have been made upon 73 nights. With the exception of eight, all of the observations are by Haynes. On the basis of measures of two maxima and one minimum secured previously to 1907, March 5, the following approximate elements were announced in Bulletin No. 10:

$$\text{Max.} = \text{J. D. } 2417545 + 40^d\text{E.}$$

The star was followed until 1907, April 20, when it was lost in the rays of the sun. Upon plotting the observations it was found that the minima were not of equal depth, and that the curve appeared to be of the Beta Lyrae type with a period double that given above. Since the reappearance of the star it has been observed whenever possible, but long periods of continuously cloudy weather have seriously interfered with the accurate determination of the minima, especially the principal minima. Nevertheless, the difference in amplitude of the adjacent minima is clearly shown by the observations now available. In addition, there is the interesting circumstance that the maxima and the secondary minima, and possibly the principal minima as well, show variations of amplitude among themselves. The star therefore appears to be subject to a secondary variation, which is perhaps irregular, or, at any rate, quite different in period from that of the Beta Lyrae variation. For example, the secondary minimum for J. D. 7965 was a full magnitude brighter than that which occurred on J. D. 7649. The maxima adjacent to the former minimum were also brighter than those immediately preceding and following the latter,

[1]The observations in this bulletin antedating 1907, June 1, were secured with the assistance of a grant from the Gould Fund of the National Academy of Sciences.

[2]*Astronomische Nachrichten*, v. 168, p. 29.

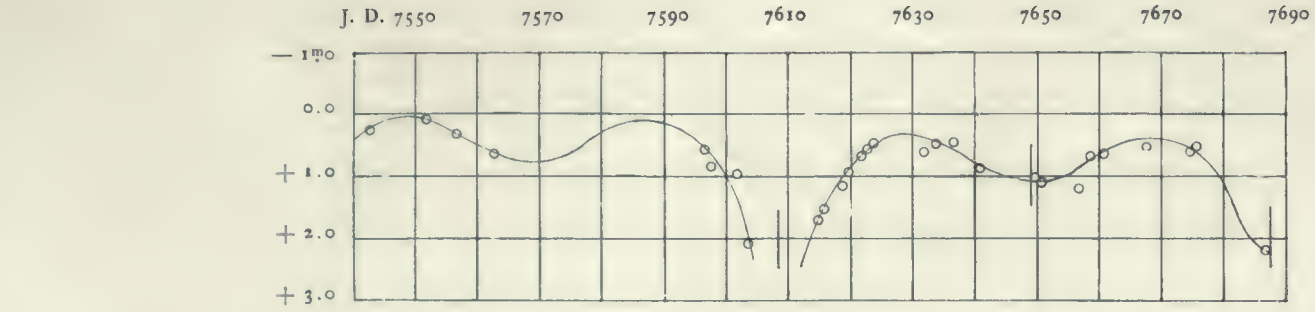
TABLE I

OBSERVATIONS ON RV TAURI

G. M. Date	J. D.	Star	Readings		$b-a$	Δm
1906, Nov. 27, 16^h	2417542.7	a	32.6	34.0	$+0^m.16$	$+0^m.15$
		b	34.1	33.5		-0.06
Dec. 6, 19	7551.8	a	25.6	26.5	0.49	$+0.10$
		b	30.1	26.6		-0.38
11, 19	7556.8	a	27.3	31.1	0.62	$+0.42$
		b	32.8	30.8		-0.23
17, 20	7562.8	a	23.8	31.3	0.80	$+0.82$
		b	31.2	31.3		$+0.02$
1907, Jan. 20, 19	7596.8	a	24.7	29.7	0.40	$+0.55$
		b	28.4	29.4		$+0.11$
21, 18	7597.8	a	24.7	31.9	0.34	$+0.80$
		b	27.8	32.0		$+0.47$
25, 18	7601.8	a	24.4	33.2	0.42	$+0.97$
		b	28.3	33.1		$+0.54$
27, 17	7603.7	a	24.1	43.4	0.62	$+2.12$
		b	29.8	44.2		$+1.59$
Feb. 7, 18	7614.8	a	29.9	46.2	0.63	$+1.80$
		b	35.5	46.4		$+1.19$
8, 16	7615.7	a	18.5	33.1	0.51	$+1.59$
		b	23.2	32.6		$+1.03$
11, 17	7618.7	a	35.1	45.5	0.50	$+1.15$
		b	39.8	46.0		$+0.70$
12, 15	7619.6	a	30.0	38.6	0.42	$+0.94$
		b	33.7	38.9		$+0.55$
14, 17	7621.7	a	34.3	40.7	0.41	$+0.69$
		b	38.1	40.3		$+0.23$
15, 15	7622.6	a	34.8	39.5	0.33	$+0.50$
		b	37.9	39.9		$+0.21$
16, 16	7623.7	a	33.1	36.9	0.34	$+0.42$
		b	36.2	37.2		$+0.11$
24, 17	7631.7	a	29.2	34.8	0.50	$+0.63$
		b	33.6	35.3		$+0.18$
26, 16	7633.7	a	32.3	35.8	0.24	$+0.39$
		b	34.5	36.0		$+0.17$
March 1, 15	7636.6	a	30.4	34.6	0.39	$+0.48$
		b	33.8	34.3		$+0.05$
5, 17	7640.7	a	36.9	45.1	0.50	$+0.90$
		b	41.6	45.6		$+0.46$
14, 15	7649.6	a	29.6	38.7	$+0.34$	$+0.99$
		b	32.6	38.6		$+0.64$

G. M. Date	J. D.	Star	Readings	$b-a$	Δm
1907, March 15, 14^h	2417650.6	a	31.2 40.9	$+0^m.31$	$+1^m.06$
		b	33.9 40.8		+0.74
21, 16	7656.7	a	40.0 50.8	0.46	+1.25
		b	44.0 50.6		+0.76
23, 14	7658.6	a	38.0 44.9	0.58	+0.77
		b	43.2 44.8		+0.18
25, 15	7660.6	a	36.2 41.8	0.36	+0.61
		b	39.6 41.9		+0.26
April 1, 15	7667.6	a	41.0 46.0	0.44	+0.57
		b	44.8 45.8		+0.11
8, 16	7674.7	a	32.8 38.5	0.34	+0.61
		b	35.9 38.1		+0.23
9, 15	7675.6	a	34.2 39.6	0.46	+0.58
		b	38.5 39.3		+0.08
20, 15	7686.6	a	36.2 ——	0.38	>2.17
		b	39.8 ——		>1.79
July 31, 22	7788.9	a	37.6 48.7	0.33	−0.45
		b	31.6 51.5		−0.86
Aug. 3, 20	7791.8	a	29.7 32.0	0.07	−0.14
		b	28.7 37.4		−0.51
10, 21	7798.9	a	41.1 38.5	0.65	+0.11
		b	29.0 41.6		−0.68
12, 22	7800.9	a	25.3 26.4	0.50	+0.12
		b	29.8 26.2		−0.40
18, 21	7806.9	a	35.4 33.8	0.39	+0.08
		b	28.9 33.1		−0.27
Sept. 12, 23	7831.9	a	40.7 40.8	0.17	0.00
		b	36.9 44.9		−0.35
18, 19	7837.8	a	37.0 33.2	0.07	+0.21
		b	35.7 33.6		+0.11
Oct. 4, 19	7853.8	a	44.2 38.9	0.36	+0.23
		b	36.2 36.2		0.00
5, 20	7854.8	a	43.1 34.0	0.35	+0.43
		b	35.5 38.4		−0.15
8, 19	7857.8	a	36.2 42.9	0.27	−0.31
		b	31.5 40.9		−0.49
11, 21	7860.9	a	29.1 28.7	0.64	−0.04
		b	34.8 28.3		−0.73
12, 20	7861.8	a	30.5 31.3	0.69	+0.09
		b	36.7 31.7		−0.56
18, 0	7867.0	a	26.3 24.5	+0.45	−0.19
		b	30.4 24.3		−0.66

G. M. Date	J. D.	Star	Readings	$b-a$	Δm
1907, Oct. 21, 17^h	2417870.7	a	24.8 22.1	$+0^m.33$	$-0^m.28$
		b	27.8 21.4		-0.69
23, 18	7872.8	a	24.8 21.4		-0.36
Nov. 4, 0	7884.0	a	21.5 24.1	0.45	$+0.28$
		b	25.7 24.1		-0.17
25, 20	7905.8	a	22.4 22.9	0.50	$+0.06$
		b	27.1 22.8		-0.55
Dec. 4, 22	7914.9	a	24.8 24.9	0.37	$+0.01$
		b	28.2 25.0		-0.35
5, 20	7915.8	a	23.9 25.9	0.46	$+0.21$
		b	28.2 26.2		-0.22
30, 20	7940.8	a	21.2 19.7	0.30	-0.15
		b	24.0 19.6		-0.46
1908, Jan. 1, 17	7942.7	a	21.9 21.0	0.57	-0.10
		b	27.3 21.5		-0.62
4, 19	7945.8	a	23.4 21.4	0.46	-0.22
		b	27.7 20.9		-0.73
8, 17	7949.7	a	22.6 21.5	0.25	-0.13
		b	25.1 22.3		-0.29
14, 13	7955.6	a	16.3 14.1	0.37	-0.30
		b	19.3 14.0		-0.67
16, 13	7957.6	a	24.3 22.9	0.14	-0.14
		b	25.6 23.2		-0.25
17, 17	7958.7	a	23.1 22.5	0.30	-0.06
		b	26.0 22.5		-0.36
18, 13	7959.6	a	18.8 18.4	0.31	-0.05
		b	21.7 18.4		-0.36
21, 13	7962.6	a	17.4 17.8	0.43	$+0.05$
		b	21.3 17.9		-0.37
23, 13	7964.5	a	21.6 22.8	0.42	$+0.13$
		b	25.6 22.4		-0.34
24, 14	7965.6	a	24.7 25.3	0.40	$+0.06$
		b	28.4 25.3		-0.34
27, 14	7968.6	a	21.7 21.3	0.23	-0.05
		b	23.9 21.1		-0.30
29, 16	7970.7	a	28.2 27 6	0.26	-0.06
		b	30.5 27.5		-0.33
Feb. 6, 17	7978.7	a	16.9 14.9	0 29	-0.27
		b	19.4 15.4		-0.49
16, 15	7988.6	a	23.7 22.4	$+0.34$	-0.14
		b	26.9 22.2		-0.50



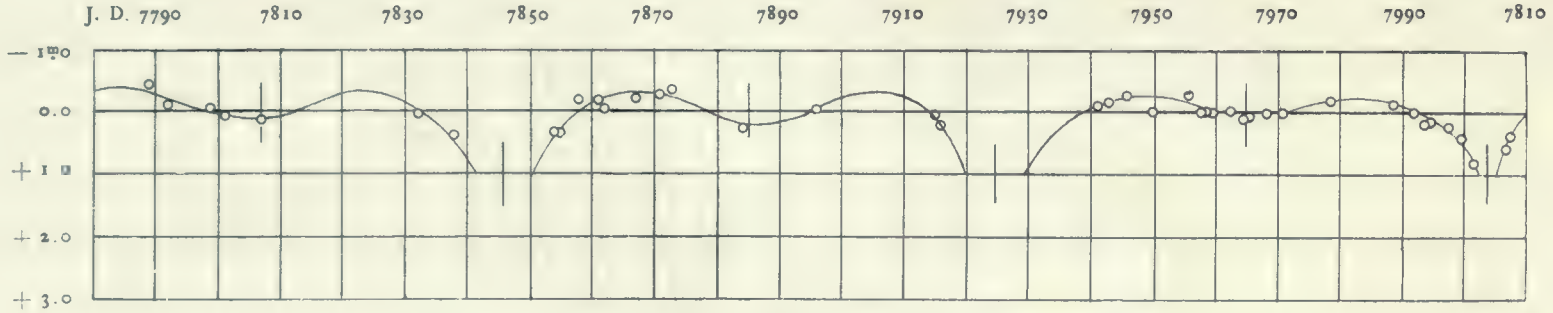


Plate I—Light Variation of R V Tauri 1906, Nov. 27 to 1908, March 6.

Referred to Comparison Star a = BD.+26°746, 9m1.

G. M. Date	J. D.	Star	Readings		$b-a$	Δm
Feb. 19, 16^h	2417991.7	a	20.6	20.9	$+0^m48$	$+0^m03$
		b	25.2	21.2		-0.42
21, 14	7993.6	a	19.9	21.4	0.36	$+0.15$
		b	23.3	21.6		-0.18
22, 14	7994.6	a	24.4	25.4	0.31	$+0.10$
		b	27.3	25.3		-0.22
25, 14	7997.6	a	20.6	22.7	[0.43]	$+0.23$
		b	24.7	23.0		-0.17
27, 14	7999.6	a	25.4	29.8	0.51	$+0.49$
		b	30.0	29.6		-0.04
29, 13	8001.6	a	20.8	28.5		$+0.83$
		a	22.0	29.8		$+0.84$
14	.6	a	22.8	30.4		$+0.81$
15	.6	a	23.9	31.5	[0.46]	$+0.83$
		b	28.2	32.2		$+0.45$
March 5, 13	8006.6	a	24.2	29.0		$+0.52$
14	.6	a	23.2	28.9	[0.37]	$+0.61$
		b	26.7	28.6		$+0.21$
		b	26.0	28.1	[0.21]	$+0.23$
		a	24.0	29.1		$+0.55$
		a	26.8	32.6		$+0.65$
6, 14	8007.6	a	21.6	25.7	0.44	$+0.43$
		b	25.8	25.7		-0.01
		a	24.8	27.3	0.26	$+0.27$
		b	27.2	26.7		-0.05
9, 14	8010.6	a	23.9	23.8	0.43	-0.01
		b	27.9	23.9		-0.43
		a	28.1	26.9	0.40	-0.13
		b	31.7	27.1		-0.51
10, 14	8011.6	a	24.3	22.5	0.44	-0.18
		b	28.4	22.8		-0.59
16	.7	a	34.2	31.3	0.20	-0.32
		b	36.0	31.2		-0.54
11, 14	8012.6	a	23.3	20.1	0.42	-0.34
		b	27.3	20.5		-0.72
		a	28.5	24.2	$+0.25$	-0.47
		b	30.8	24.6		-0.68

REMARKS.

The last two observations for each of the last four dates are by Seares. All others are by Haynes.

The observations of 1907, July 31, Aug. 3, 10, and of Aug. 18 to Oct. 8, inclusive, were made with the Zöllner-Müller photometer; all others, with the Zeiss-Wedge equalizing photometer.

The G. M. Date has been abbreviated to the nearest hour from the original record, which is given to minutes. The decimal of the J. D. has also been derived from the recorded time of observation. It is possible, therefore, that in some cases this decimal differs by one unit from that corresponding to the printed hour of observation.

The atmospheric conditions were generally unsatisfactory. The following require special comment:

1907, Jan. 27—Sky white. Moon. Variable at limit of visibility. It may have been fainter than the measures indicate.

1907, April 20—Sky white. Moon. Variable invisible. Wedge readings for the variable certainly greater than 55 divisions.

1908, March 5—Haze. Rapidly changing transparency.

The results for the last three nights are not shown by the plate.

although the difference does not appear to have exceeded $0^m.6$. Again, it seems improbable that the principal minimum just passed on J. D. 8003.5 should have reached the depth of those of J. D. 7608 and J. D. 7687.

The peculiar light variation thus indicated, or at least its general character, has been derived quite independently by Mr. Sigurd Enebo of Dombaas, Norway, from a discussion of observations of his own, as appears from a letter received from him a few days ago. Mr. Enebo has found the following preliminary elements, which, it will be noted, are in close agreement with those derived in this bulletin:

$$\text{Prin. Min.} = \text{J. D. } 2417610 + 78^d.5E$$
$$\text{Sec. Min.} = \text{J. D. } 2417649 + 78.5E$$
$$\text{Max.} = \text{J. D. } 2417630 + 39.25E$$

The Laws Observatory measures were secured with the Zeiss-Wedge and the Zöllner-Müller photometers used in connection with the 7½-inch equatorial. They are printed in detail in Table I and are shown graphically in the accompanying plate. The fourth column of the table contains the means for the wedge and the intensity circle readings corresponding to the settings on the comparison star and the variable, those for the comparison star being printed first. These have been combined to form the magnitude difference, variable minus comparison star, by means of Table V, Bulletin No. 7, and Table I, Bulletin No. 13. The results are to be found in the sixth column of the table. The differences for the comparison stars are in column five. The comparison stars are a = BD. +26°746, $9^m.1$, and b = BD. +26°747, $9^m.5$. The measures give 73 values for the difference $b - a$. The bracketed values are not included, as the corresponding settings on a and b do not fall within the same set of measures. The mean is $b - a = +0^m.40$ with a probable error of $\pm 0^m.011$. The plate shows the mean result for each night. The observations referred to star b were reduced to a by means of the difference $b - a$. The resulting Δm was combined with that derived from the observations referred directly to a for the formation of the mean for the night.

Table II contains the epochs of the maxima and minima as read from the plot of the observations. From this data the following elements have been derived:

$$\text{Prin. Min.} = \text{J. D. } 2417608.5 + 79^d.0E, \quad \text{G. M. T.}$$
$$\text{Sec. Min.} = \text{J. D. } 2417649.0 + 79.0E,$$
$$\text{Max.} = \text{J. D. } 2417550.5 + 39.5E,$$

The differences between the observed epochs of maxima and minima and those computed with these elements are given in the third, sixth, and ninth columns of Table II. Although the observations are satisfactorily represented by a period of $79^d.0$, it is likely that a more extended series will lead to a somewhat smaller value. The photographic observations by Blajko indicate that the variable was fainter than the tenth magnitude on 1898, Jan. 16, 1900, March 18 and 29,

TABLE II

EPOCHS OF MAXIMA AND MINIMA.

Prin. Min.			Sec. Min.			Max.		
E	J. D.	O–C	E	J. D.	O–C	E	J. D.	O–C
0	7608.5	$0^d.0$	0	7649	$0^d.0$	0	7549	$-1^d.5$
5	8003.5	0.0	2	7807	0.0	2	7629	−0.5
			3	7885	−1.0	3	7668	−1.0
			4	7965	0.0	8	7868	+1.5
						10	7948	+2.5
						11	7984	−1.0

and on 1905, Feb. 23, 24, and 25. It must therefore have been near minima upon these dates. The 1905 observations are certainly near a principal minimum, and it is probable that those of 1898 and 1900 are also near principal minima. If this assumption is correct, the period of $79^d.5$ derived by Enebo must be very near the true value.

The unusual character of the variation recorded by these observations recalls that of the stars R Sagittae and V Vulpeculae, both of which have been classed as of the Beta Lyrae type, and both of whose light-curves have undergone very curious changes. In the case of the former, reversals of minima seem to have occurred in 1874 and 1883, so that between these dates the secondary minima were fainter than the principal minima.[1] In the case of V Vulpeculae, the observations of Williams made during 1903-4 gave a curve of a pronounced Beta Lyrae type, with a period of $75^d.3$.[2] On the other hand, Laws Observatory measures extending from July, 1905, to September, 1906, led to a mean light-curve of the ordinary continuous variation type, with

[1] Chandler, *Third Catalogue of Variable Stars*, A. J. No. 379.

[2] *Astronomische Nachrichten*, v. 164, p. 305.
Journal British Astronomical Association, v. XV, p. 200.

a period one-half that of Williams, which represents satisfactorily all of these observations.[1] The minimum of this curve agrees in amplitude with the secondary minimum of the original curve of Williams. The star was also observed by Williams during 1905-6 with results similar to those obtained here. In other words, during the interval November, 1904, to July, 1905, the character of the variation seems to have altered completely. Further observations made here during 1906 and 1907 indicate that the curve has again changed in form.

Thus far it has not been possible to formulate a satisfactory theory accounting for the alterations of light-curve in either R Sagittae or V Vulpeculae. The discovery of a third object, presenting some points of resemblance to these stars, is therefore a matter of considerable interest.

[1] *Laws Observatory Bulletin* No. 8, p. 115. Williams' second article cited above was unknown to me when the paper in Bulletin No. 8 was prepared. *F. H. S.*

For further indications as to irregularity of variation see *Second Catalogue of Variable Stars*, Annals H. C. O., v. LV, p. 60.

Columbia, Missouri, 1908, March 11.

F. H. Seares.
E. S. Haynes.

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BULLETIN NO. 15

THE ALGOL VARIABLE RW MONOCEROTIS (24.1907)

AG. Leipz. II 3033 1855.0 $6^h\ 26^m\ 50^s.48$ $+8°\ 56'\ 1''.9$
1900.0 29 18.10 54 11.7

The variability of this star, BD. + 8°1402, $9^m.0$, was discovered by Mme. Ceraski from an examination of 22 plates made by Blajko at Moscow during the interval 1896-1907. In announcing the discovery, Prof. Ceraski stated[1] that the variation was possibly of the Algol type. The correctness of this conjecture was established by Esch[2] and by Ichinohe[3]. Esch, at Valkenburg, observed the star during increasing light on 1907, April 1. Ichinohe, at the Yerkes Observatory, determined a minimum on 1907, April 14.653, G.M.T., and derived the elements

$$\text{Min.} = \text{J. D. } 2417680.653 + 1^d.90686E, \quad \text{G.M.T.}$$

The star has been on the observing list of the Laws Observatory since 1907, April 9. In all, 395 observations on 39 nights have been secured. Measures during changing light were made on nine nights, and well defined portions of the light-curve were determined on five nights. Of the 395 observations, 79 were made by Prof. Seares. The remainder are my own. The measures of 1907, October 5 and 8, were obtained with the Zöllner-Müller photometer. All others were made with the Zeiss-Wedge photometer. Both instruments were used in connection with the 7½-inch equatorial. Each observation consists of eight settings, four upon the comparison star, and four upon the variable, arranged so as to eliminate the effect of uniform changes in the transparency of the atmosphere and in the brightness of the artificial star.

The individual observations are printed in Tables I and II. Table I contains 70 observations, all by Haynes, made during normal light. The third and ninth columns of this table give the means of the settings for each observation, the first number referring to the comparison star

[1] *Astronomische Nachrichten*, v. 174, p. 143.
[2] *Astronomische Nachrichten*, v. 174, p. 283.
[3] *Astronomische Nachrichten*, v. 174, p. 349.

TABLE I

OBSERVATIONS ON RW MONOCEROTIS DURING CONSTANT LIGHT

Julian Date	Star	Readings		Comp. Stars	Δm	O−C	Julian Date	Star	Readings		Comp. Stars	Δm	O−C
7675.62	*a*	29.2	31.5	+0m.68	+0m.26	+ 8	7959.79	*a*	15.6	17.8	+0m.68	+0m.19	− 15
	b	35.3	30.2		− 0.57	− 7		*b*	22.3	17.8		− 0.49	+ 1
7679.66	*b*	39.0	35.6		− 0.35	+ 15	7962.57	*a*	17.9	21.6	+0.91	+ 0.41	+ 7
7686.67	*a*	30.3	33.1	0.69	+ 0.32	+ 14		*b*	26.4	21.6		− 0.50	0
	b	36.5	32.8		− 0.40	+ 10	7965.59	*a*	17.6	21.8	+0.87	+ 0.47	+ 13
7692.64	*a*	36.1	39.4		+ 0.34	+ 16		*b*	25.6	21.2		− 0.47	+ 3
7697.62	*a*	31.8	35.4		+ 0.39	+ 21		*b*	25.7	21.2	−0.09	− 0.48	+ 2
7706.61	*b*	37.6	31.4		− 0.68	− 18		*c*	24.9	21.5		− 0.36	+ 5
7854.93	*a*	33.1	26.4	0.96	+ 0.45	− 13	7970.59	*b*	23.8	19.5	−0.08	− 0.46	+ 4
	b	20.6	27.2		− 0.57	− 7		*c*	23.0	19.1		− 0.42	− 1
7860.89	*a*	22.6	27.3	1.21	+ 0.49	− 9	7978.74	*b*	24.7	19.4	−0.23	− 0.56	− 6
	b	33.7	27.8		− 0.66	− 16		*c*	22.5	19.6		− 0.30	+ 11
7870.82	*a*	16.9	19.6	0.91	+ 0.32	− 26	7991.72	*b*	24.2	20.2	−0.14	− 0.42	+ 8
	b	25 3	19.5		− 0.60	− 10		*c*	22.8	20.3		− 0.27	+ 14
7872.76	*a*	19.3	23.3	1.03	+ 0.43	− 15	7993.72	*b*	25.7	21.8	−0.16	− 0.41	+ 9
	b	28.9	23.4		− 0.59	− 9		*c*	24.2	21.9		− 0.24	+ 17
7883.86	*a*	12.3	16.1	1.15	+ 0.51	− 7	7994.80	*b*	26.8	22.2	−0.20	− 0.49	+ 1
	b	21.6	15.9		− 0.66	− 16		*c*	25.0	22.5		− 0.25	+ 16
7949.63	*a*	18.6	20.9	0.75	+ 0.25	− 9	8007.67	*b*	28.0	24.0	−0.13	− 0.43	+ 7
	b	25.6	21.0		− 0.49	+ 1		*c*	26.8	24.0		− 0.30	+ 11
7954.62	*a*	17.6	19.8	0.88	+ 0.26	− 8	8010.62	*b*	25.9	21.6	−0.07	− 0.45	+ 5
	b	25.7	19.9		− 0.61	− 11		*c*	25.3	20.9		− 0.46	− 5
.85	*a*	14.5	17.9	0.89	+ 0.45	+ 11	.69	*b*	27.4	23.2	−0.13	− 0.44	+ 6
	b	21.9	18.0		− 0.43	+ 7		*c*	26.2	23.0		− 0.33	+ 8
7955.56	*a*	13.0	14.8	0.87	+ 0.24	− 10	8012.64	*b*	29.4	25.1	−0.11	− 0.47	+ 3
	b	19.8	14.8		− 0.63	− 13		*c*	28.4	25.2		− 0.35	+ 6
7957.59	*a*	18.4	20.5	0.72	+ 0.23	− 11	8014.63	*b*	29.8	25.4	−0.11	− 0.49	+ 1
	b	25.2	20.9		− 0.45	+ 5		*c*	28.8	24.9		− 0.43	− 2
.80	*a*	20.5	23.5	0.73	+ 0.32	− 2	8016.64	*b*	25.8	21.0	−0.09	− 0.51	− 1
	b	27.4	23.4		− 0.42	+ 8		*c*	25.0	21.2		− 0.40	+ 1
7958.63	*a*	11.1	13.9	0.90	+ 0.38	+ 4	8024.70	*b*	30.1	25.2	−0.17	− 0.54	− 4
	b	17.9	14.4		− 0.46	+ 4		*c*	28.5	25.4		− 0.35	+ 6
7959.56	*a*	12.5	15.1	0.90	+ 0.34	0	8035.63	*b*	28.3	23.3	−0.18	− 0.53	− 3
	b	19.5	14.7		− 0.61	− 11		*c*	26.6	22.3		− 0.46	− 5
.61	*a*	12.5	14.9	+ 0.84	+ 0.32	− 2							
	b	19.0	14.9		− 0.52	− 2							

TABLE II

OBSERVATIONS ON RW MONOCEROTIS DURING CHANGING LIGHT

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	Comp. Stars	Δm	O − C
1907, Oct. 8	19h 43m	2417857.821	−0d.092	H, *a*	30.0 19.1	+1m.20	+0m.93	+13
				H, *b*	16.8 18.7		−0.22	+6
12	20 18	7861.846	+0.121	H, *a*	19.9 27.1	+1.10	+0.76	−14
				H, *b*	30.2 27.5		−0.29	−11
1908, Jan. 2	14 9	7943.590	−0.092	H, *a*	21.0 23.5	+0.84	+0.27	−10
				H, *b*	28.8 23.7		−0.55	−27
	59	.624	−0.058	H, *a*	18.5 23.8	+0.51	+0.57	−26
				H, *b*	23.2 24.1		+0.09	−9
	15 7	.630	−0.052	H, *b*	24.3 27.0		+0.29	+1
	11	.633	−0.049	H, *b*	26.2 30.2		+0.44	+10
	15	.635	−0.047	H, *b*	26.5 29.9		+0.37	−1
	20	.639	−0.043	H, *b*	26.0 30.0		+0.44	−3
	30	.646	−0.036	H, *b*	28.1 32.8		+0.53	−13
	34	.649	−0.033	H, *b*	28.2 34.4		+0.69	−6
	38	.651	−0.031	H, *b*	26.6 34.2		+0.84	+2
	42	.654	−0.028	H, *b*	27.4 35.0		+0.85	−8
	46	.657	−0.025	H, *b*	26.3 34.6		+0.93	−12
	50	.660	−0.022	H, *b*	26.2 36.9		+1.19	+7
	16 16	.678	−0.004	H, *b*	26.0 36.8		+1.20	−3
	20	.680	−0.002	H, *b*	27.7 38.5		+1.18	−5
	24	.683	+0.001	H, *b*	27.3 38.2		+1.20	−3
	28	.686	+0.004	H, *b*	26.7 38.9		+1.33	+10
4	12 43	7945.529	−0.059	H, *b*	30.2 31.4		+0.14	−3
	46	.532	−0.056	H, *b*	30.6 33.2		+0.29	+7
	51	.535	−0.053	H, *b*	29.6 33.1		+0.39	+12
	54	.537	−0.051	H, *b*	29.1 32.0		+0.33	+3
	58	.540	−0.048	H, *b*	29.1 31.8		+0.31	−5
	13 0	.542	−0.046	H, *b*	28.6 32.4		+0.42	+2
	4	.544	−0.044	H, *b*	29.3 32.7		+0.39	−6
	8	547	−0.041	H, *b*	28.4 32.2		+0.43	−9
	10	.549	−0.039	H, *b*	29.1 33.3		+0.47	−11
	14	.551	−0.037	H, *b*	28.0 33.8		+0.65	+2
	22	.557	−0.031	H, *b*	26.2 33.5		+0.82	0
	24	.558	−0.030	H, *b*	26.0 34.0		+0.89	+4
	28	.561	−0.027	H, *b*	25.7 33.3		+0.84	−13
	32	.564	−0.024	H, *b*	24.9 33.9		+1.00	−8
	36	.567	−0.021	H, *b*	27.2 36.6		+1.05	−9
	40	.569	−0.019	H, *b*	26.6 35.3		+0.96	−20
	44	.572	−0.016	H, *b*	26.0 37.2		+1.24	+5

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	Comp. Stars	Δm	O-C
1908, Jan. 4	$13^h\ 48^m$	2417945.575	−0.013	H, *b*	27.4 37.9		+1.16	− 4
	58	.582	−0.006	H, *b*	25.8 34.8		+1.00	−23
	14 0	.583	−0.005	H, *b*	24.8 34.3		+1.05	−18
	4	.586	−0.002	H, *b*	26.3 35.6		+1.04	−19
	8	.589	+0.001	H, *b*	28.2 36.6		+0.94	−29
	12	.592	+0.004	H, *b*	28.3 37.2		+0.99	−24
	14	.593	+0.005	H, *b*	27.2 36.3		+1.01	−22
	18	.596	+0.008	H, *b*	28.4 37.4		+1.00	−22
	22	.599	+0.011	H, *b*	28.0 38.6		+1.16	− 6
	28	.603	+0.015	H, *b*	27.9 36.9		+1.00	−21
	31	.605	+0.017	H, *b*	26.6 34.6		+0.89	−32
	34	.607	+0.019	H, *b*	28.0 37.2		+1.02	−18
	38	.610	+0.022	H, *b*	27.1 36.3		+1.02	−17
	40	.611	+0.023	H, *b*	27.8 35.9		+0.90	−29
	44	.614	+0.026	H, *b*	27.1 35.6		+0.95	−23
	48	.617	+0.029	H, *b*	28.1 35.9		+0.87	−29
	51	.619	+0.031	H, *b*	28.4 36.4		+0.89	−26
	15 10	.632	+0.044	H, *b*	26.6 31.5		+0.54	−46
	12	.633	+0.045	H, *b*	27.5 32.0		+0.51	−47
	16	.636	+0.048	H, *b*	26.2 31.1		+0.54	−39
	18	.638	+0.050	H, *b*	25.4 30.6		+0.58	−31
	21	.640	+0.052	H, *b*	25.4 30.5		+0.57	−27
	23	.641	+0.053	H, *b*	25.6 30.0		+0.48	−34
	26	.643	+0.055	H, *b*	24.8 29.3		+0.49	−28
	29	.645	+0.057	H, *b*	26.1 29.8		+0.41	−30
	33	.648	+0.060	H, *b*	27.1 31.1		+0.44	−19
	36	.650	+0.062	H, *b*	27.1 29.9		+0.31	−25
	39	.652	+0.064	H, *b*	26.2 29.1		+0.32	−18
	41	.654	+0.066	H, *b*	27.1 29.3		+0.24	−20
	44	.656	+0.068	H, *b*	27.0 28.6		+0.18	−22
	46	.657	+0.069	H, *b*	28.1 30.0		+0.21	−17
	48	.658	+0.070	H, *b*	27.9 29.1		+0.13	−23
	54	.662	+0.074	H, *b*	26.4 27.0		+0.07	−22
	59	.666	+0.078	H, *b*	26.0 26.0		0.00	−22
	16 2	.668	+0.080	H, *b*	25.7 25.7		0.00	−19
	4	.670	+0.082	H, *b*	24.8 24.1		−0.07	−24
	8	.672	+0.084	H, *b*	24.5 24.8		+0.03	−11
	11	.674	+0.086	H, *b*	25.2 24.6		−0.06	−18
	14	.676	+0.088	H, *b*	25.4 24.6		−0.08	−18
	17	.678	+0.090	H, *b*	25.2 24.3		−0.09	−17
	20	.680	+0.092	H, *b*	26.3 25.0		−0.14	−20

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	Comp. Stars	Δm	O − C
1908, Jan. 4	16^h28^m	2417945.686	+ 0d.098	H, *b*	24.5 22.7		− 0m.18	− 18
	31	.688	+ 0.100	H, *b*	24.9 23.1		− 0.18	− 17
	34	.690	+ 0.102	H, *b*	25.2 23.0		− 0.22	− 19
	37	.692	+ 0.104	H, *b*	24.7 22.8		− 0.19	− 14
	42	.696	+ 0.108	H, *a*	18.4 22.3		+ 0.42	− 15
				H, *b*	24.7 22.3	+ 0.67	− 0.25	− 17
	47	.699	+ 0.111	H, *b*	24.7 22.3		− 0.25	− 14
	51	.702	+ 0.114	H, *b*	26.7 24.3		− 0.26	− 14
	56	.705	+ 0.117	H, *a*	20.4 23.6		+ 0.34	− 17
				H, *b*	26.1 23.5	+ 0.60	− 0.27	− 13
	17 2	.710	+ 0.122	H, *a*	19.6 23.1		+ 0.37	− 10
				H, *b*	26.2 23.2	+ 0.69	− 0.31	− 13
	10	.715	+ 0.127	H, *a*	19.9 22.8		+ 0.31	− 12
				H, *b*	25.8 23.1	+ 0.62	− 0.28	− 6
	16	.720	+ 0.132	H, *a*	20.2 22.6		+ 0.26	− 14
				H, *b*	25.9 22.8	+ 0.60	− 0.32	− 7
	23	.724	+ 0.136	H, *a*	19.6 21.5		+ 0.20	− 17
				H, *b*	25.8 22.1	+ 0.65	− 0.39	− 11
	28	.728	+ 0.140	H, *a*	18.6 19.9		+ 0.15	− 20
				H, *b*	24.6 20.4	+ 0.64	− 0.46	− 16
	36	.733	+ 0.145	H, *a*	17.2 19.7		+ 0.30	− 2
	39	.735	+ 0.147	H, *a*	17.8 20.4		+ 0.29	− 2
	42	.738	+ 0.150	H, *a*	16.4 19.4		+ 0.36	+ 7
	44	.739	+ 0.151	H, *a*	17.4 19.3		+ 0.22	− 6
	50	.743	+ 0.155	H, *a*	15.8 18.2		+ 0.30	+ 4
	52	.744	+ 0.156	H, *a*	16.6 19.7		+ 0.37	+ 11
	56	.747	+ 0.159	H, *a*	19.0 21.0		+ 0.21	− 3
				H, *b*	24.6 20.6	+ 0.59	− 0.42	− 1
	19 8	.797	+ 0.209	H, *a*	19.1 21.2		+ 0.22	+ 7
				H, *b*	24.4 21.4	+ 0.56	− 0.32	+ 18
	20 14	.843	+ 0.255	H, *a*	17.9 21.7		+ 0.42	+ 27
				H, *b*	25.7 21.6	+ 0.84	− 0.43	+ 7
	21 26	.893	+ 0.305	H, *a*	20.0 22.0		+ 0.21	+ 6
Jan. 23	12 34	7964.524	− 0.125	H, *a*	18.4 21.6		+ 0.35	0
				H, *b*	26.0 21.8	+ 0.81	− 0.44	+ 5
	50	.535	− 0.114	H, *a*	17.7 20.6		+ 0.33	− 5
				H, *b*	25.4 20.7	+ 0.83	− 0.49	− 3
	13 6	.546	− 0.103	H, *a*	18.0 21.6		+ 0.40	− 5
				H, *b*	24.9 21.4	+ 0.74	− 0.37	+ 2
	26	.560	− 0.089	S, *a*	17.2 21.2		+ 0.45	− 14
				S, *c*	22.9 22.9	+ 0.64	0.00	+ 16

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	Comp. Stars	Δm	O − C
1908, Jan. 23	13h 32m	2417964.564	− 0d085	H, *a*	16.4 21.2	+0m81	+ 0m55	− 9
				H, *b*	23.6 21.1		− 0.27	− 7
	43	·572	− 0.077	S, *b*	27.8 26.9	− 0.21	− 0.10	0
				S, *c*	25.9 26.7		− 0.07	− 6
	48	·575	− 0.074	H, *b*	26.7 26.9	− 0.14	− 0.02	+ 4
	51	·577	− 0.072	H, *c*	25.5 26.5		+ 0.12	+ 6
	54	·579	− 0.070	S, *b*	27.9 29.2	− 0.08	+ 0.14	+ 14
	56	·581	− 0.068	S, *c*	27.2 29.1		+ 0.21	+ 9
	14 0	·583	− 0.066	H, *b*	26.2 27.0	− 0.01	+ 0.09	+ 3
	1	·584	− 0.065	H, *c*	26.1 27.3		+ 0.13	− 3
	4	·586	− 0.063	S, *b*	26.3 27.9	− 0.16	+ 0.18	+ 8
	7	·588	− 0.061	S, *c*	24.8 27.7		+ 0.32	+ 10
	10	·590	− 0.059	H, *b*	25.6 26.7	− 0.12	+ 0.12	− 5
	12	·592	− 0.057	H, *c*	24.9 27.4		+ 0.27	− 2
	15	·594	− 0.055	S, *b*	24.2 25.3	+ 0.08	+ 0.11	− 12
	18	·596	− 0.053	S, *c*	25.0 27.3		+ 0.25	− 11
	21	·598	− 0.051	H, *b*	25.3 27.7	− 0.14	+ 0.27	− 3
	23	·599	− 0.050	H, *c*	23.9 26.7		+ 0.30	− 11
	26	.601	− 0.048	S, *b*	25.2 29.5	− 0.12	+ 0.47	+ 11
	28	.603	− 0.046	S, *c*	24.0 30.6		+ 0.72	+ 23
	31	.605	− 0.044	H, *b*	25.4 28.8	− 0.01	+ 0.38	− 7
	32	.606	− 0.043	H, *c*	25.3 28.8		+ 0.39	− 17
	35	.608	− 0.041	S, *b*	26.6 31.3	+ 0.07	+ 0.52	0
	38	.610	− 0.039	S, *c*	26.0 30.6		+ 0.51	− 16
	41	.612	− 0.037	H, *b*	25.3 30.3	0.00	+ 0.55	− 8
	43	.613	− 0.036	H, *c*	25.3 30.1		+ 0.53	− 22
	46	.615	− 0.034	S, *b*	25.8 32.7	− 0.15	+ 0.77	+ 5
	49	.617	− 0.032	S, *c*	24.4 32.0		+ 0.84	− 4
	52	.619	− 0.030	H, *b*	24.1 32.3	− 0.06	+ 0.90	+ 5
	54	.621	− 0.028	H, *c*	23.5 31.1		+ 0 82	− 20
	58	.624	− 0.025	S, *b*	25.7 35.7	− 0.23	+ 1.11	+ 6
	15 0	.625	− 0.024	S, *c*	23.5 35.6		+ 1.33	+ 16
	4	.628	− 0.021	H, *b*	24.0 34.2	− 0.18	+ 1.12	− 1
	6	.629	− 0.020	H, *c*	22.3 33.8		+ 1.26	+ 2
	10	.632	− 0.017	S, *b*	24.3 37.3	− 0.09	+ 1.43	+ 25
	15	.635	− 0.014	S. *c*	23.4 36.9		+ 1.48	+ 19
	18	.637	− 0.012	H, *b*	24.4 36.1	− 0.08	+ 1.29	+ 8
	20	.639	− 0.010	H, *c*	23.6 35.9		+ 1.35	+ 4
	24	.642	− 0.007	S, *b*	25.4 35.6	− 0.02	+ 1.14	− 9
	27	.644	− 0.005	S, *c*	25.2 37.7		+ 1.38	+ 6
	31	.647	− 0.002	H, *b*	24.9 35.3	− 0.20	+ 1.15	− 8
	32	.647	− 0.002	H, *c*	22.9 34.8		+ 1.30	− 2

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	Comp. Stars	Δm	O−C
1908, Jan. 23	15h 36m	2417964.650	+ 0d001	S, *b*	24.7 37.1	−0m04	+1m37	+ 14
	40	.653	+ 0.004	S, *c*	24.3 35.7		+ 1.26	− 6
	44	.656	+ 0.007	H, *b*	24.3 33.9	+ 0.10	+ 1.06	− 17
	46	.657	+ 0.008	H, *c*	25.3 35.5		+ 1.14	− 17
	50	.660	+ 0.011	S, *b*	24.6 34.7	− 0.12	+ 1.12	− 10
	54	.663	+ 0.014	S, *c*	23.4 34.1		+ 1.17	− 13
	57	.665	+ 0.016	H, *b*	22.5 33.7	− 0.05	+ 1.22	+ 1
	59	.666	+ 0.017	H, *c*	22.1 32.7		+ 1.16	− 14
	16 2	.668	+ 0.019	S, *b*	24.6 35.9	− 0.15	+ 1.25	+ 5
	6	.671	+ 0.022	S, *c*	23.1 36 1		+ 1.42	+ 14
	9	.673	+ 0.024	H, *b*	23.9 35.8	+ 0.04	+ 1.31	+ 13
	12	.675	+ 0.026	H, *c*	24.3 34.6		+ 1.14	− 13
	15	.677	+ 0.028	H, *b*	24.6 35.8	− 0.11	+ 1.24	+ 7
	18	.679	+ 0.030	H, *c*	23.5 34.9		+ 1.25	0
	21	.681	+ 0.032	H, *b*	24.5 35.0	− 0.12	+ 1.16	+ 1
	23	.683	+ 0.034	H, *c*	23.3 33.9		+ 1.16	− 7
	28	.686	+ 0.037	S, *b*	25.1 35.3	− 0.06	+ 1.13	+ 3
	32	.689	+ 0.040	S, *c*	24.5 35.2		+ 1.18	+ 3
	35	.691	+ 0.042	H, *b*	24.4 34.1	− 0.01	+ 1.07	+ 4
	37	.692	+ 0.043	H, *c*	24.3 34.5		+ 1.12	+ 2
	41	.695	+ 0.046	S, *b*	25.0 33.4	− 0.07	+ 0.93	− 3
	44	.697	+ 0.048	S, *c*	24.3 33 8		+ 1.05	+ 3
	47	.699	+ 0.050	H, *b*	22.4 30.8	+ 0.09	+ 0.91	+ 2
	49	.701	+ 0.052	H, *c*	23.2 30.7		+ 0.81	− 12
	52	.703	+ 0.054	S, *b*	24.5 32.0	− 0.07	+ 0.83	+ 3
	55	.705	+ 0.056	S, *c*	23.8 30.8		+ 0.76	− 7
	58	707	+ 0.058	H, *b*	24.2 30.4	− 0.10	+ 0.67	− 2
	17 0	.708	+ 0.059	H, *c*	23.2 29.9		+ 0.72	− 3
	3	.710	+ 0.061	S, *b*	25.6 30.0	− 0.19	+ 0.48	− 12
	6	.712	+ 0.063	S, *c*	23.8 28.6		+ 0.52	− 10
	9	.715	+ 0.066	H, *b*	25.0 29.4	− 0.12	+ 0.48	+ 3
	11	.716	+ 0.067	H, *c*	23.8 27.6		+ 0.41	− 10
	14	.718	+ 0.069	S, *b*	21.7 26.2	− 0.02	+ 0.47	+ 9
	16	.719	+ 0.070	S, *c*	21.5 25.2		+ 0.38	− 7
	19	.722	+ 0.073	H, *b*	22.4 25.8	+ 0.07	+ 0.36	+ 5
	21	.723	+ 0.074	H, *c*	23.0 25.4		+ 0.24	− 14
	23	.724	+ 0.075	S, *b*	24 4 26.2	− 0.22	+ 0.19	− 8
	26	.726	+ 0.077	S, *c*	22.3 27.4		+ 0.54	+ 21
	29	.728	+ 0.079	H, *b*	23.9 26.0	+ 0.05	+ 0.22	+ 1
	31	.730	+ 0.081	H, *c*	24.4 25.7		+ 0.14	− 13
	34	.732	+ 0.083	S, *b*	22.6 23.8	− 0.29	+ 0.12	− 3
	36	.733	+ 0.084	S, *c*	19.9 23.2		+ 0.35	+ 12

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	Comp. Stars	Δm	O−C
1908, Jan. 23	17h 39m	2417964·735	+0d086	H, *b*	22.5 22.8		+0m03	− 9
	41	·737	+ 0.088	H, *c*	22.4 23.3	−0m02	+ 0.10	− 9
	44	·739	+ 0.090	S, *b*	20.8 21.3		+ 0.05	− 3
	47	·741	+ 0.092	S, *c*	21.7 22.2	+ 0.10	+ 0.05	− 10
	50	·743	+ 0.094	H, *b*	21.8 21.7		− 0.01	− 5
	52	·744	+ 0.095	H, *c*	21.1 21.7	− 0.08	+ 0.07	− 5
	54	·746	+ 0.097	S, *b*	21.5 21.8		+ 0.04	+ 3
	57	·748	+ 0.099	S, *c*	21.0 21.5	− 0.05	+ 0.05	− 4
	18 0	·750	+ 0.101	H, *b*	21.4 21.0		− 0.04	− 2
	2	·751	+ 0.102	H, *c*	20.0 20.6	− 0.14	+ 0.06	0
	5	·753	+ 0.104	S, *b*	21.7 22.0		+ 0.03	+ 8
	7	·755	+ 0.106	S, *c*	20.0 20.0	− 0.18	0.00	− 2
	19	·763	+ 0.114	H, *b*	21.1 20.3		− 0.08	+ 4
	20	·764	+ 0.115	H, *c*	21.5 21.4	+ 0.04	− 0.01	+ 3
	23	·766	+ 0.117	S, *b*	25.2 23.9		− 0.13	+ 2
	26	·768	+ 0.119	S, *c*	21.8 22.4	− 0.35	+ 0.06	+ 13
	29	·770	+ 0.121	H, *b*	21.7 21.1		− 0.07	+ 11
	30	·771	+ 0.122	H, *c*	21.6 21.1	− 0.01	− 0.06	+ 3
	33	·773	+ 0.124	S, *b*	23.3 21.5		− 0.20	− 1
	36	·775	+ 0.126	S, *c*	22.4 21.6	− 0.10	− 0.08	+ 4
	40	·778	+ 0.129	H, *b*	22.4 21.8		− 0.06	+ 17
	42	·779	+ 0.130	H, *c*	23.0 21.4	+ 0.07	− 0.18	− 3
	19 4	·794	+ 0.145	H, *a*	17.4 21.7		+ 0.48	− 3
				H, *b*	25.6 21.7	+ 0.89	− 0.41	− 8
	10	·799	+ 0.150	S, *a*	16.5 21.2		+ 0.53	+ 5
				S, *b*	24.3 21.1	+ 0.86	− 0.34	+ 2
Jan. 27	13 30	7968.562	+ 0.101	H, *b*	23.2 23.0		− 0.02	0
				H, *c*	22.3 22.6	− 0.10	+ 0.04	− 3
	50	·576	+ 0.115	H, *b*	23.6 21.9		− 0.18	− 5
				H, *c*	22.7 22.1	− 0.09	− 0.07	− 3
	14 19	·596	+ 0.135	H, *a*	14.3 20.8		+ 0.79	+ 22
				H, *b*	23.9 21.3	+ 1.12	− 0.28	− 1
	39	·610	+ 0.149	H, *b*	24.0 20.6		− 0.36	− 1
				H, *c*	23.0 20.3	− 0.10	− 0.29	− 3
Mar. 9	12 59	8010.541	+ 0.144	H, *b*	27.1 22.5		− 0.48	− 16
				H, *c*	26.1 23.0	− 0.11	− 0.32	− 9
	13 7	·547	+ 0.150	S, *b*	27.3 23.1		− 0.44	− 8
				S, *c*	25.2 23.4	− 0.23	− 0.18	+ 9
	14	·551	+ 0.154	H, *b*	27.5 23.0		− 0.47	− 9
				H, *c*	26.9 22.9	− 0.06	− 0.42	− 13
	20	·556	+ 0.159	S, *b*	27.5 24.3		− 0.34	+ 7
				S, *c*	26.8 24.3	− 0.07	− 0.27	+ 5

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	Comp. Stars	Δm	O − C
1908, Mar. 24	13h 34m	2418025.565	−0d083	H, *b*	30.9 30.2	[+0m05]	−0m08	+ 10
	37	.567	− 0.081	H, *c*	31.3 30.6		− 0.08	− 2
	40	.569	− 0.079	H, *b*	32.0 31.1		− 0.11	+ 2
	43	.571	− 0.077	H, *b*	32.5 31.8		− 0.07	+ 3
	53	.578	− 0.070	H, *b*	32.0 32.7		+ 0.08	+ 8
	56	.581	− 0.067	H, *b*	32.2 32.1		− 0.01	− 5
	59	.583	− 0.065	H, *b*	32.5 33.3		+ 0.09	+ 2
14	2	.585	− 0.063	H, *b*	33.5 35.0		+ 0.16	+ 6
	6	.587	− 0.061	H, *b*	33.5 35.0		+ 0.16	+ 3
	10	.590	− 0.058	H, *b*	33.4 35.3		+ 0.21	+ 3
	13	.592	− 0.056	H, *b*	33.4 36.8		+ 0.38	+ 17
	16	.594	− 0.054	H, *b*	33.2 37.5		+ 0.47	+ 22
	30	.604	− 0.044	H, *b*	32.9 37.4		+ 0.49	+ 4
	32	.606	− 0.042	H, *b*	34.5 38.5		+ 0.43	− 7
	36	.608	− 0.040	H, *b*	34.8 39.1		+ 0.45	− 10
	39	.610	− 0.038	H, *b*	34.2 39.8		+ 0.60	0
	48	.617	− 0.031	H, *b*	35.5 41.8		+ 0.68	− 14
	52	.619	− 0.029	H, *b*	34.4 42.4		+ 0.88	− 1
	56	.622	− 0.026	H, *b*	34.0 43.1		+ 1.00	− 1
	59	.624	− 0.024	H, *b*	34.0 44.1		+ 1.11	+ 3
15	2	.626	− 0.022	H, *b*	34.4 45.0		+ 1.17	+ 5
	6	.629	− 0.019	H, *b*	36.3 46.8		+ 1.16	0
	10	.632	− 0.016	H, *b*	33.9 45.4		+ 1.26	+ 7
	14	.635	− 0.013	H, *b*	34.4 44.6		+ 1.13	− 8
	22	.640	− 0.008	H, *b*	36.5 47.4		+ 1.20	− 2
	26	.643	− 0.005	H, *b*	35.4 46.2		+ 1.19	− 4
	30	.646	− 0.002	H, *b*	34.8 46.6		+ 1.30	+ 7
	34	.649	+ 0.001	H, *b*	35.7 46.9		+ 1.23	0
	38	.651	+ 0.003	H, *b*	35.9 48.0		+ 1.33	+ 10
	42	.654	+ 0.006	H, *b*	37.8 49.4		+ 1.30	+ 7
	46	.657	+ 0.009	H, *b*	35.7 47.7		+ 1.32	+ 10
	50	.660	+ 0.012	H, *b*	35.7 47.7		+ 1.32	+ 10
	58	.665	+ 0.017	H, *b*	35.0 45.9		+ 1.20	− 1
16	1	.667	+ 0.019	H, *b*	34.4 45.3		+ 1.20	0
	6	.671	+ 0.023	H, *b*	34.0 44.8		+ 1.19	0
	14	.676	+ 0.028	S, *b*	32.9 42.7		+ 1.07	− 10
	18	.679	+ 0.031	S, *b*	34.1 44.9		+ 1.19	+ 4
	22	.682	+ 0.034	S, *b*	37.4 46.6		+ 1.02	− 12
	26	.685	+ 0.037	H, *b*	35.6 45.7		+ 1.11	+ 1
	30	.688	+ 0.040	H, *b*	34.5 44.9		+ 1.15	+ 9
	34	.690	+ 0.042	S, *b*	32.8 41.9		+ 0.99	− 4

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	Comp. Stars	Δm	O − C
1908, Mar. 24	16h 38m	2418025.693	+0m045	S, *b*	32.6 43.0		+1m14	+ 16
	44	.697	+ 0 049	H, *b*	34·3 43·3		+ 0.99	+ 8
	46	.699	+ 0.051	H, *b*	34.0 42.1		+ 0.88	+ 1
	51	.702	+ 0.054	S, *b*	34·7 43·8		+ 1.00	+ 20
	55	.705	+ 0.057	S, *b*	35·8 43·7		+ 0.87	+ 16
	17 0	.708	+ 0.060	H, *b*	36.2 42.3		+ 0.67	+ 4
	2	.710	+ 0.062	H, *b*	35.1 39.8		+ 0.50	— 7
	6	.712	+ 0.064	S, *b*	35.2 39.8		+ 0.49	— 1
	10	.715	+ 0.067	S, *b*	34·4 38.7		+ 0.46	+ 3
	14	.718	+ 0.070	H, *b*	35·9 39.0		+ 0.32	— 4
	16	.719	+ 0.071	H, *b*	36.2 39.2		+ 0.31	— 3
	20	.722	+ 0.074	S, *b*	37.1 39.8		+ 0.28	— 1
	24	.725	+ 0.077	S, *b*	38.3 40.6		+ 0.25	+ 1
	28	.728	+ 0.080	H, *b*	39·3 40.5		+ 0.13	— 6
	30	.729	+ 0.081	H, *b*	40.1 41.0		+ 0.10	— 8
	33	.731	+ 0.083	S, *b*	45·9 46.3		+ 0.04	— 11
Apr. 12	14 34	8044.607	— 0.104	H, *b*	30.9 27.9	— 0.07	— 0.33	+ 7
				H, *c*	30.3 27.6		— 0.29	+ 2
	58	.624	— 0.087	H, *b*	35·4 33·9		— 0.16	+ 6
	15 0	.625	— 0.086	H, *b*	33·4 30.6		— 0.31	— 10
	6	.629	— 0.082	S, *b*	38.6 38.6		0.00	+ 16
	10	.632	— 0.079	S, *b*	39.1 39.6		+ 0.06	+ 19
	14	.635	— 0.076	H, *b*	34.0 33.1		— 0.10	— 2
	16	.636	— 0.075	H, *b*	33·3 32.6		— 0.07	0
	20	.639	— 0.072	S, *b*	37.2 38.0		+ 0.08	+ 11
	24	.642	— 0.069	S, *b*	35.0 35·9		+ 0.10	+ 9
	29	.645	— 0.066	H, *b*	31·7 31·5		— 0.02	— 8
	32	.647	— 0.064	H, *b*	31·3 32.0		+ 0.08	— 1
	36	.650	— 0.061	S, *b*	34.0 35.2		+ 0.13	0
	38	.651	— 0.060	S, *b*	34.6 35·4		+ 0.08	— 7
	42	.654	— 0.057	H, *b*	30.1 31·5		+ 0.16	— 4
	45	.656	— 0.055	H, *b*	30·5 32.0		+ 0.17	— 6
	49	.659	— 0.052	S, *b*	34·4 37·1		+ 0.30	+ 2
	52	.661	— 0.050	S, *b*	37.0 40.3		+ 0.34	+ 2
	58	.665	— 0.046	H, *b*	33.8 37.2		+ 0.37	— 3
	16 1	.667	— 0.044	H, *b*	31·9 36.7		+ 0.53	+ 8
	5	.670	— 0.041	S, *b*	37·3 41.6		+ 0.46	— 6

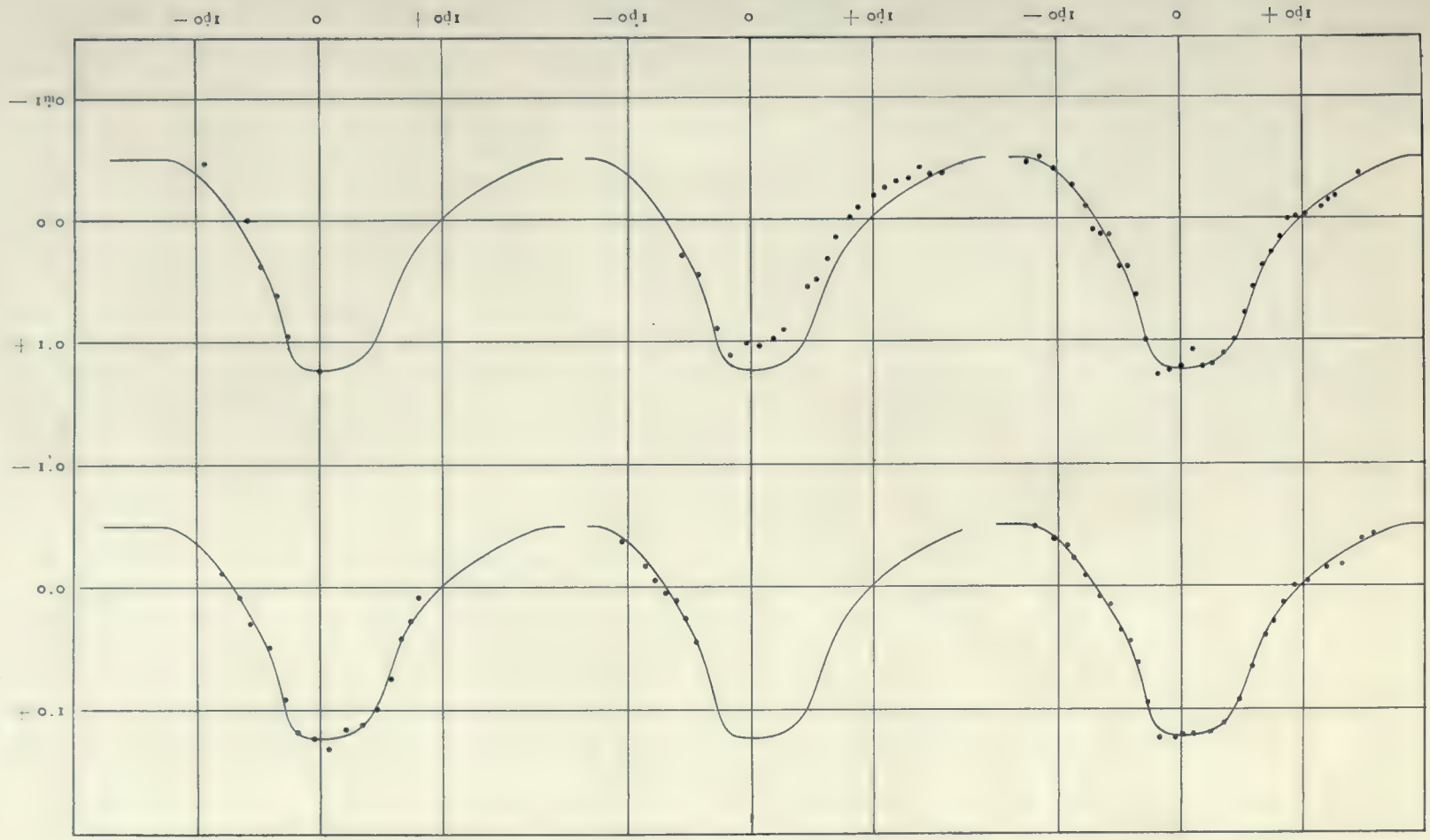

PLATE II.—Agreement of the mean light-curve of RW Monocerotis with the observations of 1908, Jan. 2, 4, 23, Mar. 24, and Apr. 12, in order from left to right. Each point represents the mean of three or four consecutive observations. The last curve of the lower series shows the normal places from which the mean curve was derived. Zero of magnitudes corresponds to star *b* = BD. + 8°1397, 9m.4.

REMARKS

The first two observations of 1908, Mar. 24, were made separately on account of rapidly changing transparency. The resulting value for the difference $c-b$ was therefore given zero weight in the formation of the mean $c-b$.

1907, Apr. 20—Sky white, moon.
26—Sky white, occasional clouds, full moon.
May 1—Sky white.
10—Sky white, current unsteady.
Oct. 5—Bad definition.
21—Occasional thin haze, full moon.
Nov. 3—Considerable floating haze.
1908, Jan. 2—Varying transparency.
4—Sky very white.
8—Sky white.
16—Sky white, full moon.
17—Sky very white, full moon.
19—Gradually thickening haze.
23—Definition bad, sky white.
Feb. 6—Definition bad, sky white.
19—Sky white, moon.
22—Slight haze, moon.
Mar. 11—Sky very white, moon.
13—Sky very white, moon.
15—Sky very white, full moon.
24—Atmosphere unsteady, definition bad.
Apr. 12—Varying transparency.

and the second to the variable. The numbers in the fourth and tenth columns are the comparison star differences, $b-a$ and $c-b$. Columns five and eleven contain the magnitude difference, variable minus comparison star. These differences were derived from the readings in columns three and nine by means of Table V, Bulletin No. 7, and Table I, Bulletin No. 13.

The 320 observations made during changing light are entered in Table II. This table also contains five observations made on 1908, January 4, after the variable had regained normal brightness. With the exception of the additional columns for the calendar date, G.M.T., and the phase values, and of the insertion of the observer's initial in column five, the arrangement here is similar to that of Table I.

The comparison stars used are $a =$ BD. $+9°1282$, $8^m.7$, $b =$ BD. $+8°1397$, $9^m.4$, and $c =$ BD. $+8°1390$, $9^m.3$. Columns four and ten of Table I and column seven of Table II furnish 40 and 74 values of the differences, $b-a$ and $c-b$, respectively. The results of a discussion of these differences are shown in Table III. Lines one to four contain the results for $b-a$ grouped according to the intervals shown in the third column, while line five contains the results derived from all of the measures of $b-a$. The means are entered in column four. It is impossible to assign with certainty the cause of the systematic differences between the means of the four groups. It may be said, however, that settings on a are rendered difficult and perhaps uncertain by the close proximity of the relatively bright star, BD. $+9°1284$, $8^m.0$. The possibility of a fluctuation in the brightness of the comparison stars seems to be restricted to a, for the measures of the difference $c-b$ show no irregularity. The use of the Zöllner-Müller photometer is not responsible for the discordant results, for only two observations of the comparison star difference were secured with it. These are both included in group two, the mean of which would not be changed by excluding the corresponding values. Lines six and seven give the results for the difference $c-b$ based upon the work of the two observers separately, while line eight contains those derived from the whole series.

The magnitudes of the comparison stars were determined by referring star *a* to two PD. stars. The results of this comparison are summarized in Table IV. Star *q* is also No. 2362 of the *Revised Harvard Photometry*, in which its magnitude is given as 6.48. Although the internal agreement of the measures of Δm by the individual observers is satisfactory, the systematic difference between the values in columns six and seven is abnormally large as compared with that for similar measures for other stars. Further observations would have been made but for the fact that the stars had approached conjunction before the difference was noted. The application

TABLE III

COMPARISON STAR DIFFERENCES

No.	Diff.	Limiting Julian Days	Mean Δm	P. E. Mean	P. E. 1 Obsn.	No. Obsns.	Obsr.
1	*a* − *b*	7675—7686	−0.68	——	——	2	H
2	*a* − *b*	7854—7883	−1.08	±0.033	±0.089	7	H
3	*a* − *b*	7943—7945	−0.65	0.018	0.064	12	H
4	*a* − *b*	7949—7968	−0.84	0.014	0.061	19	H
5	*a* − *b*	7675—7968	−0.82	0.019	0.120	40	H
6	*c* − *b*	7964—8010	−0.11	0.014	0.074	27	S
7	*c* − *b*	7964—8044	−0.08	0.008	0.054	47	H
8	*c* − *b*	7964—8044	−0.09	±0.007	±0.063	74	S & H

TABLE IV

MAGNITUDE OF COMPARISON STAR *a*

Star	Number		Magnitude		Δm			Mag. of *a*		No. Obs.	
	BD.	PD.	PD.	HD.	S	H	Mean	PD.	HD.	S	H
p	+8°1379	4033	7.30	6.86	+1.75	+1.36	+1.49	8.79	8.35	2	4
q	+9°1259	4001	6.92	6.62	+2.18	+1.79	+1.92	8.84	8.54	2	4
							Mean	8.81	8.44		

of $b - a = +0.82$ to the values derived for the brightness of *a* gives for *b*, 9.63 and 9.26 according to the Potsdam and Harvard systems, respectively. These results are probably uncertain by two or three tenths of a magnitude, owing to the systematic difference between the observers and the unreliability of the value used for $b - a$.

All observations referred to *a* and *c* were reduced to *b*, using Nos. 1 to 4 from the fourth column of Table III, for $b - a$, and No. 8 from the same table, for $c - b$.

The combination of the 70 values for the difference $v - b$ thus obtained from Table I gave for normal light

$$v - b = -0^m.50 \pm 0^m.008,$$

where the appended quantity is the probable error of the mean, that for a single observation being $\pm 0^m.066$. The residuals, O − C, referred to this mean are in the sixth and twelfth columns of Table I. For the purpose of discovering any possible intermediate variation, these residuals were arranged in order of phase and plotted, but no evidence of a secondary minimum was found.

TABLE V

EPOCHS FOR WHICH $v - b = +0^m.50$ ON DESC. BR.

E	Obs. Geoc. Epochs	Red. to Sun	Obs. Helioc. Epochs	O − C
0	$7943^d.640$	$+0^d.005$	$7943^d.645$	$0^d.000$
1	7945.546	+0.005	7945.551	0.000
11	7964.607	+0.005	7964.612	0.000
43	8025.606	0.000	8025.606	0.000
53	8044.669	−0.002	8044.667	0.000

TABLE VI

REDUCTION TO THE SUN

Date	Red.	Date	Red.	Date	Red.
Jan. 0	$+0^d.0055$	Apr. 29	$-0^d.0029$	Aug. 27	$-0^d.0031$
10	54	May 9	37	Sept. 5	23
20	51	19	44	16	14
30	47	29	49	26	−0.0004
Feb. 9	41	June 8	53	Oct. 6	+0.0005
19	34	18	56	16	15
29	26	28	57	26	24
Mar. 10	17	July 8	56	Nov. 6	32
20	+0.0008	18	54	16	39
30	−0.0002	28	50	26	45
Apr. 9	12	Aug. 7	45	Dec. 5	50
19	21	17	39	15	53
29	−0.0029	27	−0.0031	25	+0.0055

For the determination of the period, the differences $v - b$ obtained from Table II were combined into groups of three or four consecutive observations, each group extending over an interval of about $0^d.01$. The means thus derived were plotted thus giving well defined portions of the light-curve corresponding to five minima. These results are shown in Plate II. The point on the descending branch for which $v - b = +0^m.50$ is common to all of the five segments. And, as the light change is very rapid at this point, the corresponding epochs were chosen for the determination of the period. The geocentric times for $v - b = +0^m.50$, expressed in Julian days

and decimals of G.M.T., were read directly from the curves and entered in the second column of Table V. The reductions to the sun taken from Table VI are in column three. The resulting heliocentric epochs in column four gave the following elements:

$$\text{Desc. Br. } v-b = +0^{m}.50 = \text{J. D. } 2417943.645 + 1^{d}.90607E, \quad \text{G.M.T.}$$

These elements reproduce exactly the observed heliocentric epochs, as may be seen from the column of residuals. In deriving the above elements, the minimum due to Ichinohe was not used. His observations have not been published, and in consequence it was impossible to determine the instant given by his measures for which $v-b = +0^{m}.50$. By basing the determination of the period upon epochs of minima, Ichinohe's result could have been included; but it was believed that the uncertainty in the epochs arising from the asymmetry of the light-curve and the slowness of the light change near minimum would more than offset any advantage resulting from the use of the longer interval.

TABLE VII

NORMAL PLACES

No.	Phase	$v-b$	No. Obs.	O−C	No.	Phase	$v-b$	No. Obs.	O−C
1	−0.078	−0.48	4	0	15	+0.051	+1.20	8	−2
2	−0.062	−0.38	4	+2	16	+0.065	+1.19	12	0
3	−0.050	−0.32	4	−4	17	+0.078	+1.12	8	0
4	−0.044	−0.22	7	−1	18	+0.090	+0.93	12	0
5	−0.036	−0.08	10	+3	19	+0.101	+0.66	8	0
6	−0.025	+0.09	15	+5	20	+0.110	+0.40	7	0
7	−0.015	+0.16	13	−4	21	+0.117	+0.29	7	+2
8	−0.007	+0.36	11	+2	22	+0.124	+0.13	7	−3
9	0.000	+0.45	11	−5	23	+0.134	0.00	8	−6
10	+0.007	+0.62	6	−7	24	+0.144	−0.03	6	0
11	+0.015	+0.96	12	−2	25	+0.160	−0.15	11	0
12	+0.024	+1.23	8	+5	26	+0.173	−0.17	5	+8
13	+0.036	+1.24	8	+1	27	+0.189	−0.38	10	−4
14	+0.042	+1.22	8	−1	28	+0.199	−0.42	4	−2

For the formation of the mean light-curve, geocentric epochs for $v-b = +0^{m}.50$, Desc. Br., were calculated on the basis of the above elements for all observations during changing light. Phase values referred to these epochs were formed for each of the means used in drawing the curves for the individual nights, with the exception of those of 1908, January 4, and for similar means of the scattered observations during changing light. The observations of 1908, January 4, were excluded because, as may be seen from Plate II, these measures differ systematically, especially in the ascending branch of the curve, from the results obtained on other nights.

The means for $v-b$ were arranged according to phase values and combined to form the twenty-eight normal places shown in Table VII. Columns four and nine of this table indicate

the number of observations included in each normal, while columns five and ten contain the residuals referred to the mean light-curve, expressed in hundredths of a magnitude.

The minimum of the curve thus defined corresponds to phase $+0^d.042$. Modifying the above elements so as to refer to minimum, we have finally,

$$\text{Min.} = \text{J. D. } 2417943.687 + 1^d.90607E, \quad \text{G.M.T.}$$

The ordinates of the mean light-curve referred to star b are given in Table VIII for every $0^d.005$, zero phase corresponding to minimum. The curve itself is shown by Plate II. It is noticeably asymmetrical. The period of light change is about $7^h\ 34^m$. Only $3^h\ 7^m$ are required for the descent to minimum, while the ascent extends over $4^h\ 27^m$. The range in brightness is from

TABLE VIII

Mean Light-Curve. Phase from Minimum.

Phase	$v-b$	Phase	$v-b$	Phase	$v-b$	Phase	$v-b$
$-0^d.130$	$-0^m.50$	$-0^d.050$	$+0^m.32$	$+0^d.030$	$+1^m.16$	$+0^d.110$	$-0^m.10$
0.125	0.49	0.045	0.42	0.035	1.13	0.115	0.13
0.120	0.48	0.040	0.55	0.040	1.06	0.120	0.17
0.115	0.46	0.035	0.69	0.045	0.98	0.125	0.20
0.110	0.44	0.030	0.85	0.050	0.89	0.130	0.24
0.105	0.41	0.025	1.05	0.055	0.77	0.135	0.27
0.100	0.37	0.020	1.15	0.060	0.63	0.140	0.30
0.095	0.32	0.015	1.20	0.065	0.47	0.145	0.33
0.090	0.26	0.010	1.22	0.070	0.36	0.150	0 36
0.085	0.20	−0.005	1.23	0.075	0.27	0 155	0.39
0.080	0.14	0.000	1.23	0.080	0.19	0.160	0.41
0.075	−0.07	+0.005	1.23	0.085	0.13	0.165	0.43
0.070	0.00	0.010	1.22	0.090	0.08	0.170	0.45
0.065	+0.07	0.015	1.21	0.095	+0.03	0.175	0.47
0.060	0.15	0.020	1.20	0.100	−0.01	0.180	0.49
−0.055	+0.23	+0.025	+1.18	+0.105	0.06	+0.185	−0.50

$v-b = -0^m.50$ to $v-b = +1^m.23$, giving an amplitude of $1^m.73$. Referring to the magnitudes given for b on page 234, the range according to the Potsdam system is seen to be from $9^m.13$ to $10^m.86$, while according to the Harvard system it is from $8^m.76$ to $10^m.49$.

The agreement of the final elements with the minima observed at Moscow and at the Yerkes Observatory is shown by Table IX. The first three lines refer to single observations, while the last contains results based upon measures extending over both branches of the curve. The geocentric times of observation expressed in Julian days and decimals of G.M.T. are entered in column three. The corresponding heliocentric times appear in column five. Column six contains the residuals referred to the epochs of minima derived from the final elements.

It will be seen that the first two observations fall within the calculated interval of light change. In view of the fact that the variable was only $0^m.3$ fainter than the normal brightness shown by the Moscow plates, the residuals are satisfactory. The magnitude for the third observation shows that the star must have been near a minimum. The corresponding residual is small. The agreement with the minimum of Ichinohe is also satisfactory, especially when the asymmetry of the curve and the slow rate of light change near minimum is taken into consideration.

TABLE IX

REPRESENTATION OF EARLIER OBSERVATIONS

E	Mag.	Geoc. Obsn.	Red. to Sun	Helioc. Obsn.	O−C	Observer
— 1679	9.3	4743.278	0.000	4743.278	— 0.117	Blajko
— 731	9.3	6550.243	+ 0.002	6550.245	— 0.105	Blajko
— 358	10.5	7261.315	+ 0.003	7261.318	+ 0.004	Blajko
— 138	10.8	7680.653	— 0.002	7680.651	+ 0.002	Ichinohe

TABLE X

AVERAGE SYSTEMATIC DEVIATIONS AND PROBABLE ERRORS OF OBSERVATIONS DURING CHANGING LIGHT

J. D.	Desc. Branch			Asc. Branch		
	Sys. Dev.	P. E.	No. Obs.	Sys. Dev.	P. E.	No. Obs.
7943	$-0^m.064$	$\pm 0^m.079$	16	$+0^m.035$	$\pm 0^m.078$	2
7945	— 0.054	0.075	21	— 0.158	0.156	70
7964	— 0.001	0.071	50	— 0.009	0.059	68
7968	——	——	0	+ 0.008	0.043	8
8010	——	——	0	— 0.042	0.086	8
8025	+ 0.018	0.050	27	+ 0 017	± 0.053	30
8044	+ 0.017	± 0.053	21	——	——	0

The phase values for the individual observations during changing light, referred to the nearest geocentric minimum, are in the fourth column of Table II. The residuals referred to the mean light-curve, expressed in hundredths of a magnitude, are in the last column of this table. The mean systematic deviations and the probable errors of a single observation for the measures of each night are entered in Table X, the results for the ascending and descending branches of the curve being shown separately. The agreement with the mean light-curve for the different nights is also shown in Plate II, where the curve is superposed upon the plot of the observations.

The previously mentioned large systematic deviation of the observations on 1908, January 4 = J. D. 2417945 is plainly shown both by the table and the plate. The agreement of the observations during decreasing light with the mean light-curve is satisfactory; but the curve of the

observations does not attain normal depth, and its ascending branch lies above the mean light-curve throughout the greater part of its length. The observations agree well among themselves, however, and the residuals referred to the smooth curve defined by them are very small.

It is at present impossible to account for the occurrence of this abnormal series of measures. Alternately dissimilar minima cannot be accepted as an explanation, for both odd and even minima are included among those used in deriving the mean light-curve. A variation in either of the comparison stars of such a nature as to produce the disagreement is extremely improbable. Fatigue on the part of the observer would be expected to produce irregularities in the measures instead of a systematic deviation. It should be said, however, that on the night in question the sky was very white, making observations near minimum, which are always difficult on account of the presence of a tenth magnitude star very near the variable, even more difficult than usual. This might account for bad observations near minimum, but hardly for their continuation throughout increasing light.

Excluding the discordant observations of 1908, January 4, the average systematic deviations and the probable errors of a single observation during changing light are

	No. Obs.	Av. Sys. Dev.	Prob. Error
Seares	79	$+0^{m}.025$	$\pm 0^{m}.071$
Haynes	155	-0.017	± 0.055
All	234	-0.003	± 0.060

The systematic differences between the observers, derived from the measures of $c-b$ and from observations on the variable during changing light, are, respectively, $S-H=-0^{m}.032$ and $S-H=+0^{m}.042$. These are of the same order of magnitude as the values usually obtained for such differences. The change in sign is accounted for by the fact that, while $c-b$ is a negative quantity, the positive sign predominates largely in the values for $v-b$. Both values therefore indicate that the difference in brightness of two stars was measured systematically larger by Seares than by Haynes. These differences are small, especially for the variable, as the amplitude of variation is considerable and the observations are difficult.

Columbia, Missouri, 1908, July 9. *E. S. Haynes.*

PRELIMINARY ANNOUNCEMENTS CONCERNING VARIABLE STARS

RS Boötis (14.1907)

BD. $+32°2489$, $9^m.3$ 1855.0 $14^h27^m21^s$ $+32°\ 23'.4$
1900.0 29 16 11.4

Variability discovered by Mrs. Fleming from a photograph made at Arequipa 1906, May 24.[1] An examination of 274 Harvard plates covering the interval 1890, July 8 – 1906. July 9, gave for the period $0^d.49931$. The range shown by the photographs is $8^m.9 - 10^m.0$.

The star has been under observation here since March, 1907. It was followed throughout the night by Mr. Haynes upon several occasions during 1907, but, owing to the intervals separating the dates of observation, no very definite information could be derived from these measures, excepting the impossibility of reconciling the observed variation with the published value of the period.

TABLE XI

Observed Maxima of RS Boötis

E	Date	Obs. Geoc. Maxima	O – C	Observer
0	1908, June 22	8115.626	$0^d.000$	S & H
6	24	8117.889	0.000	S & H
8	25	8118.636	— 0.008	S
43	July 8	8131.846	— 0.001	S & H
64	16	8139.768	0.000	S & H

Recent observations in June and July have given the values of several maxima and the general form of the light-curve. The Julian dates of the observed epochs of geocentric maxima are in column three of Table XI, the decimals being in G.M.T. A comparison of the second and third maxima indicates that the period must be $0^d.747$ or a submultiple of this quantity. A consideration of the remainder of the data together with the form of the curve shows that the approximate period is one-half the above amount. The following are the preliminary elements derived from the data:

$$\text{Max.} = \text{J. D. } 2418115.626 + 0^d.37722E, \quad \text{G.M.T.}$$

The representation of the observations by these elements is shown by the residuals in column four.

The earlier observations by Mr. Haynes are not completely reduced, so that a detailed comparison of these measures with the above elements is not yet possible. A preliminary examination indicates that the general agreement is satisfactory.

[1] H. C. No. 124; *Astronomische Nachrichten*, v. 174, p. 105; v. 176, p. 188.

The variable appears to be of the Antalgol type. Of the interval of $9^h\ 3^m$ corresponding to the period, $1^h\ 21^m$ are required for the ascent from minimum to maximum. The decline covers about $4^h\ 50^m$. Throughout the remaining $2^h\ 52^m$, the light is sensibly constant, the variation during this interval amounting, at most, to $0^m.05$ or $0^m.06$. At maximum the variable is equal to BD. $+32°2481$, $9^m.2$. The amplitude is one magnitude.

43.1907 Draconis

BD. $+68°652$, $9^m.5$ 1855.0 $11^h\ 29^m\ 36^s$ $+68°\ 8'.1$
1900.0 $32\ 14$ $67\ 53.2$

Variability discovered by Miss Leavitt from plates of the Harvard Map of the sky.[1] The photographs indicated a very short period with a range of $9^m.0 - 9^m.7$.

A few observations were made here in 1907 and early in 1908, but the star was not systematically followed until May of this year. The recent measures give maxima for the epochs shown

TABLE XII

Observed Maxima of 43.1907 Draconis

E	Date	Obs. Geoc. Maxima	O − C	Observer
0	1908, May 11	8073.70	$0^d.00$	S & H
47	June 11	8104.76	+ 0.02	S & H
68	25	8118.63	+ 0.02	S
74	29	8122.57	0.00	S
88	July 8	8131.82	0.00	S & H
91	10	8138.80	0.00	H

in the third column of Table XII, the decimals of the Julian date being in G.M.T. The resulting preliminary elementary elements are

$$\text{Max.} = \text{J. D. } 2418073.70 + 0^d.6604E, \quad \text{G.M.T.}$$

The representation of the observations by these elements is shown by the residuals in column four.

The variation approaches closely the Antalgol type. Three hours are required for the ascent from minimum to maximum. Seven hours after maximum the star is within a few hundredths of a magnitude of minimum brightness, the variation during the remaining $5^h\ 50^m$ of the period amounting, at most, to $0^m.06$ or $0^m.08$. The range shown by the observations is $0^m.7$, the maximum being $0^m.5$ brighter than BD. $+68°657$, $9^m.4$.

[1] H. C. No. 127; *Astronomische Nachrichten*, v. 175, p. 91.

44.1907 Ursae Majoris

BD. + 52°1579, 9m.4 1855.0 11h 32m 56s + 52° 48'.8
1900.0 35 24 33.9

Variability discovered by Miss Leavitt from plates of the Harvard Map of the sky.[1] The photographs indicated a range of 9m.5 – 10m.5 with a variation apparently of the Algol type.

The star has been under observation here since August, 1907. No variation was detected until 1908, March 11. On this date Mr. Haynes secured a series of observations covering a part of the ascending branch of the curve. During the interval 20h 36m — 23h 12m, G.M.T., the variable increased from 0m.90 fainter than *b* = BD. + 52°1580, 9m.1, to 0m.38 fainter than *b*, normal brightness being about 0m.2 fainter than the comparison star. It is possible that the light increase began about 20h. In addition, I have found the variable in minimum on June 14 and 29, and July 21. Upon the first of these dates haze prevented measures being made. At 19h G.M.T. the variable was barely visible, being about 1m fainter than normal brightness. Although it soon disappeared as a result of thickening haze, *b* was seen until 20h with sufficient clearness to render improbable any increase in the light during this interval. On June 29 the star was followed from 14h 37m to 19h 40m G.M.T. At the beginning it was about 1m fainter than normal light, and so remained, apparently, until about 18h 30m. On July 21 the observations extended from 15h 40m to 17h 15m G.M.T., further measures being prevented by clouds. Throughout this interval the variable was 1m fainter than normal.

These observations leave the question of Algol variability beyond doubt. They are satisfied by a period of 7d.33. Measures upon other dates seem to preclude the possibility of a smaller value. It is probable that the variable remains of constant minimum brightness for a period of at least eight hours, and that three or four hours are required both for the increase and the decrease, making a total light change of at least 14 or 15 hours. The following may be accepted as preliminary elements:

Min. = J. D. 2418012.67 + 7d.33E, G.M.T.

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[1] H. C. No. 127; *Astronomische Nachrichten*, v. 175, p. 91.

Columbia, Missouri, 1908, July 25. *F. H. Seares.*

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BULLETIN NO. 16

THE LONG-PERIOD ALGOL VARIABLE RZ OPHIUCHI (103.1905)[1]

BD. + 7°3832, 9m4 1855.0 18h38m44s.9 + 7° 3′.8
1900.0 40 55.8 6.4

The variable RZ Ophiuchi, remarkable on account of the length of its period, has been under observation here for about two years. During this interval three minima have occurred, one in January, 1907, one in September, 1907, and a third in June, 1908. The first of these could not be observed as the star was then lost in the rays of the sun. The second and third were followed, but under unfavorable atmospheric conditions. During changing light 92 observations were secured. In addition, 110 observations were made during constant light. The instruments employed were the equalizing Zeiss-Wedge and the Zöllner-Müller photometers, both used in connection with the 7½-inch equatorial. The details of these measures are shown in Tables I and II. Table I gives for each observation during constant light the Julian date with decimals expressed in G.M.T., the comparison star and the initial of the observer—S for Seares and H for Haynes,—the means of the readings of the wedge and of the intensity circle, the comparison star differences derived from each series of settings, the magnitude difference between the variable and the comparison star, and, finally, the residual, O – C, referred to the calculated normal brightness of the variable. The arrangement of Table II, which contains the observations during changing light, is similar, except that the calendar date, the G.M.T., and the phase referred to the nearest minimum are also included. In this table the residuals are referred to the smooth curve best representing the observations made with the wedge photometer. In both tables the readings are in each case the mean of four settings, the first number referring to the comparison star, the second to the variable. The magnitude differences between the variable and the comparison star, and of the comparison stars themselves, were derived by means of Table V, Bulletin No. 7, and Table I, Bulletin No. 13.

[1] The observations in this bulletin antedating June 1, 1907, were made, in part, with the assistance of a grant from the Gould Fund of the National Academy of Sciences. *Laws Observatory Bulletin*, No. 10, p. 148.

TABLE I

OBSERVATIONS ON RZ OPHIUCHI DURING CONSTANT LIGHT

Julian Date	Obs. Star	Readings		Comp. Star	Δm	O − C	Julian Date	Obs. Star	Readings		Comp. Star	Δm	O − C
7416.7	S, *a*	29.7	29.6	+0m41	−0m01	− 1	7460.6	H, *a*	31.1	31.7	−0m43	+0m07	+ 7
	S, *b*	33.4	29.9		− 0.39	+ 15		H, *c*	27.2	31.7		+ 0.50	□
7417.6	H, *a*	30.6	30.3	+ 0.43	− 0.04	− 4	7461.7	S, *a*	27.7	29.4		+ 0.18	+ 18
	H, *b*	34.5	30.3		− 0.47	+ 9		H, *a*	30.2	31.4		+ 0.14	+ 14
7422.8	H, *a*	30.4	30.5	+ 0.45	+ 0.02	+ 2	7462.7	S, *a*	27.2	28.4		+ 0.13	+ 13
	H, *b*	34.4	30.4		− 0.45	+ 11	7466.7	S, *a*	32.7	32.7		0.00	0
7434.8	S, *a*	31.5	33.1	− 0.64	+ 0.18	+ 18	7471.6	H, *a*	30.0	29.2		− 0.09	− 9
	S, *c*	25.7	31.9		+ 0.69	+ 22	7472.6	S, *a*	30.9	31.4		+ 0.06	+ 6
7436.8	H, *a*	33.9	34.8	− 0.44	+ 0.10	+ 10	7473.6	S, *a*	29.7	30.2		+ 0.05	+ 5
	H, *c*	30.0	34.9		+ 0.55	+ 5	7474.7	S, *a*	26.4	26.9		+ 0.06	+ 6
7437.8	S, *a*	31.4	33.3	− 0.15	+ 0.21	+ 21	7475.6	H, *a*	29.5	29.0		− 0.05	− 5
	S, *c*	30.1	33.0		+ 0.33	− 14	7476.7	H, *a*	31.7	32.0		+ 0.04	+ 4
7439.7	H, *a*	30.1	30.4	− 0.61	+ 0.03	+ 3	7478.6	H, *a*	33.0	32.7		− 0.03	− 3
	H, *c*	24.5	30.5		+ 0.66	+ 16	7479.7	S, *a*	31.3	32.3		+ 0.11	+ 11
	S, *a*	26.8	28.0	− 0.48	+ 0.13	+ 13	7480.6	S, *a*	34.3	34.1		− 0.02	− 2
	S, *c*	22.3	29.4		+ 0.76	+ 29	7485.6	S, *a*	30.0	29.8		− 0.02	− 2
7440.7	S, *a*	27.5	29.2	− 0.47	+ 0.19	+ 19	7488.6	S, *a*	29.9	29.7		− 0.02	− 2
	S, *c*	23.0	28.1		+ 0.54	+ 7	7490.7	S, *a*	33.2	34.2		+ 0.11	+ 11
7441.7	H, *a*	32.1	32.2	− 0.66	+ 0.01	+ 1	7491.6	H, *a*	35.8	34.8		− 0.11	− 11
	H, *c*	26.2	33.1		+ 0.77	+ 27	7493.6	H, *a*	33.0	32.6		− 0.04	− 4
7443.8	S, *a*	26.6	29.1	− 0.51	+ 0.27	+ 27	7495.6	H, *a*	34.0	33.5		− 0.05	− 5
	S, *c*	21.8	27.5		+ 0.60	+ 13	7496.6	H, *a*	30.8	30.8		0.00	0
7445.6	H, *a*	25.4	27.2	− 0.28	+ 0.20	+ 20	7502.6	H, *a*	34.6	34.0		− 0.07	− 7
	H, *c*	22.6	27.9		+ 0.56	+ 6	7503.6	H, *a*	31.5	31.8		+ 0.04	+ 4
7446.8	S, *a*	24.6	27.9	− 0.44	+ 0.36	+ 36	7510.6	H, *a*	34.8	34.8		0.00	□
	S, *c*	20.4	26.8		+ 0.68	+ 21	7511.6	H, *a*	32.1	33.0		+ 0.10	+ 10
7447.7	H, *a*	25.7	28.9	− 0.42	+ 0.35	+ 35	7515.6	H, *a*	34.9	34.2		− 0.08	− 8
	H, *c*	21.7	29.2		+ 0.80	+ 30	7516.6	H, *a*	32.4	32.3		− 0.01	− 1
7449.6	S, *a*	27.8	29.9	− 0.47	+ 0.23	+ 23	7521.6	H, *a*	34.7	34.1		− 0.07	− 7
	S, *c*	23.4	29.9		+ 0.70	+ 23	7522.6	H, *a*	39.2	38.8		− 0.04	− 4
7450.6	S, *a*	27.6	29.1	− 0.47	+ 0.16	+ 16	7523.5	H, *a*	26.8	26.9		+ 0.01	+ 1
	S, *c*	23.2	29.0		+ 0.62	+ 15	7524.6	S, *a*	34.2	33.7		− 0.05	− 5
7451.7	H, *a*	27.3	28.0	− 0.47	+ 0.08	+ 8	7525.6	H, *a*	30.5	30.2		− 0.04	− 4
	H, *c*	22.8	27.9		+ 0.54	+ 4	7527.6	H, *a*	32.3	31.8		− 0.05	− 5
7452.7	S, *a*	27.1	29.2	− 0.57	+ 0.23	+ 23	7536.5	H, *a*	33.6	32.7		− 0.10	− 10
	S, *c*	21.7	28.7		+ 0.75	+ 28	7541.6	H, *a*	31.8	31.7		− 0.01	− 1
7453.7	H, *a*	36.1	36.0	− 0.65	− 0.01	− 1	7542.5	H, *a*	29.3	29.3		0.00	0
	H, *c*	30.3	36.2		+ 0.66	+ 16	7543.5	H, *a*	30.4	31.1		+ 0.08	+ 8
7459.6	H, *a*	29.5	31.0	− 0.55	+ 0.17	+ 17	7548.6	H, *a*	42.8	41.9		− 0.11	− 11
	H, *c*	24.4	30.6		+ 0.68	+ 18	7550.6	H, *a*	39.3	39.7		+ 0.05	+ 5

Julian Date	Obs. Star	Readings		Comp. Star	Δm	O−C	Julian Date	Obs. Star	Readings		Comp. Star	Δm	O−C
7551.5	H, *a*	39.9	39.6		− 0.03	− 3	7813.7	H, *a*	33.8	35.4		− 0.08	− 8
7553.5	H, *a*	29.4	29.1		− 0.03	− 3	7830.6	H, *a*	36.7	40.9		− 0.20	− 20
7718.7	H, *a*	35.4	36.3		+ 0.10	+ 10	7831.7	H, *a*	51.2	51.4		0.00	0
7790.8	H, *a*	30.2	31.3	+ 0m08	− 0.07	− 7	7865.6	H, *a*	28.3	28.8		+ 0.06	+ 6
	H, *b*	29.0	34.0		− 0.31	+ 7	7872.6	H, *a*	26.8	26.5		− 0.03	− 3
7797.7	H, *a*	32.0	33.9	+ 0.46	− 0.11	− 11	7993.9	H, *a*	28.1	29.0	+ 0.46	+ 0.10	+ 10
	H, *b*	25.4	32.6		− 0.50	− 12		H, *b*	32.2	29.5		− 0.31	+ 25
7798.7	H, *a*	30.6	36.8	+ 0.23	− 0.35	− 35	8010.9	H, *a*	25.4	25.1		− 0.03	− 3
	H, *b*	27.4	37.0		− 0.59	− 21	8080.8	H, *a*	27.3	27.7		+ 0.05	+ 5
7800.7	H, *a*	26.8	28.6	+ 0.46	+ 0.20	+ 20		S, *a*	33.4	34.4		+ 0.11	+ 11
	H, *b*	31.0	28.5		− 0.27	− 11	8081.7	S, *a*	34.5	35.4		+ 0.10	+ 10
7802.8	H, *a*	30.3	32.2	+ 0.73	+ 0.22	+ 22	8082.7	S, *a*	43.7	43.6		− 0.01	− 1
	H, *b*	36.8	31.9		− 0.54	+ 2	8087.6	S, *a*	36.2	36.4		+ 0.02	+ 2
7806.8	H, *a*	39.7	39.5		0.00	0	8091.8	S, *a*	31.1	31.0		− 0.01	− 1
7812.7	H, *a*	33.6	38.6		− 0.52	− 25	8092.8	S, *a*	32.0	32.3		+ 0.03	+ 3

Table III shows the results for the comparison star differences derived from the data in columns four and ten of Table I and column seven of Table II. Columns two and four give the means for S and H, respectively, and column six, those corresponding to all the measures. The number of separate determinations included in each mean is given in columns three, five and seven. Eight and nine show the probable errors, and ten, the instrument employed.

The brightness of star *a* was determined by a reference to four bright stars designated by *p*, *q*, *r* and *s*, whose magnitudes are contained in the catalogues of both Potsdam and Harvard. The results are shown in Table IV.

The combination of the magnitudes for *a* from Table IV with the differences in Table III gives for *b* and *c*:

	Potsdam	Harvard	Instrument
b	10m30	10m09	W
b	10.17	9 96	Z
c	9 26	9.05	W

The magnitudes of the variable referred to either system can be derived by combining the values of Δm in Tables I and II with the results for the brightness of the comparison stars.

For the determination of the character of the variation all of the measures referred to stars *b* and *c* during both constant and changing light were reduced to star *a* by means of the comparison star differences in Table III. In order to eliminate, in so far as possible, the effect of systematic error, this reduction was made with the means in columns two and four.

In calculating the normal brightness of the variable the observations in Table I were divided into groups, the results for each group being shown in Table V. With the exception of

TABLE II

OBSERVATIONS ON RZ OPHIUCHI DURING CHANGING LIGHT

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	$b-a$	Δm	O − C
1907, Sept. 16	$15^h\ 34^m$	2417835.65	− 9.05	H, *a*	48.7 48.2		+ 0.01	+ 1
17	15 48	7836.66	− 8.04	H, *a*	45.4 46.7		− 0.05	− 7
18	15 58	7837.67	− 7.03	H, *a*	35.3 32.8		+ 0.14	+ 4
23	16 16	7842.68	− 2.02	H, *a*	32.7 27.2	+ 0.32	+ 0.36	− 48
				H, *b*	27.8 26.2		+ 0.12	− 34
24	16 6	7843.67	− 1.03	H, *a*	36.6 26.3	+ 0.40	+ 0.65	− 19
				H, *b*	29.7 25.0		+ 0.35	− 11
28	13 6	7847.55	+ 2.85	H, *a*	48.7 32.9	+ 0.75	+ 0.71	− 13
				H, *b*	32.2 28.2		+ 0.26	− 20
	14 0	.58	+ 2.88	S, *a*	60.8 37.1	+ 0.39	+ 0.81	− 3
				S, *b*	46.9 38.0		+ 0.37	+ 5
	13	.59	+ 2.89	H, *a*	37.4 26.4	+ 0.26	+ 0.68	− 16
				H, *b*	32.7 29.1		+ 0.22	− 24
30	13 23	7849.56	+ 4.86	S, *a*	29.7 16.5	+ 0.66	[+ 1.21]	+ 44
				S, *b*	21.5 18.9		+ 0.27	+ 3
	36	.57	+ 4.87	H, *a*	36.4 26.9	+ 0.29	+ 0.60	− 17
				H, *b*	31.4 26.7		+ 0.32	− 7
Oct. 1	13 27	7850.56	+ 5.86	S, *a*	26.7 19.4		+ 0.65	+ 34
	36	.57	+ 5.87	H, *a*	27.3 25.3	+ 0.47	+ 0.15	− 16
				H, *b*	21.6 ——		——	——
	47	.57	+ 5.87	H, *a*	25.3 23.6	+ 0.23	+ 0.14	− 17
				H, *b*	22.5 25.2		− 0.23	− 16
	51	.58	+ 5.88	H, *a*	26.5 25.2		+ 0.09	− 22
				H, *a*	29.3 24.6	+ 0.37	+ 0.35	+ 4
				H, *b*	24.4 ——		——	——
	14 0	.58	+ 5.88	H, *a*	27.2 22.2	+ 0.25	+ 0.41	+ 10
				H, *b*	24.0 22.0		+ 0.18	+ 25
2	13 46	7851.57	+ 6.87	S, *a*	22.4 20.8	+ 0.37	+ 0.16	+ 5
				S, *b*	18.8 21.4		− 0.27	+ 14
	52	.58	+ 6.88	S, *a*	21.7 21.3	+ 0.56	+ 0.04	− 7
				S, *b*	16.6 20.4		− 0.43	− 2
4	14 7	7853.59	+ 8.89	S, *a*	69.2 51.0	+ 0.62	[+ 0.40]	+ 40
				S, *b*	44.6 56.5		− 0 37	+ 15
	13	.59	+ 8.89	H, *a*	33.4 32.7	+ 0.46	+ 0.04	+ 4
				H, *b*	26.4 34.1		− 0.50	− 12
	18	.60	+ 8.90	S, *a*	41.7 41.4	[−0.02]	+ 0.02	+ 2
				S, *b*	42.2 40.6		[+ 0.08]	+ 60
5	13 32	7854.56	+ 9.86	H, *a*	48.6 48.5	+ 0.37	0.00	0
				H, *b*	39.4 51.1		− 0.44	− 6

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	$b-a$	Δm	O−C
1908, June 5	15h 38m	2418098.65	−8d05	S, *a*	34.2 33.6		−0m06	− 8
6	15 42	8099.65	−7.05	S, *a*	35.6 37.8		+0.23	+12
	44	.66	−7.04	S, *a*	36.7 38.5		+0.18	+ 7
7	17 18	8100.72	−5.98	S, *a*	32.6 35.3		+0.29	0
8	15 36	8101.65	−5.05	S, *a*	34.1 42.0		+0.86	+13
	40	.65	−5.05	S, *a*	35.8 43.2		+0.81	+ 8
	44	.66	−5.04	S, *b*	41.6 41.6		0.00	−19
	48			S, *b*	40.6 41.7		+0.12	− 7
	21 18	.89	−4.81	S, *a*	27.9 35.9	+0.69	+0.89	+11
				S, *b*	34.1 35.8		+0.19	− 5
	28	.89	−4.81	S, *b*	33.8 36.3		+0.27	+ 3
	32	.90	−4.80	S, *b*	33.4 35.2		+0.20	− 4
9	15 21	8102 64	−4.06	S, *a*	33.0 41.2	+0.58	+0.88	+ 5
				S, *b*	38.4 41.3		+0.32	+ 3
	27	.64	−4.06	H, *b*	36.8 39.3		+0.25	− 2
	30	.65	−4.05	H, *b*	35.5 37.7		+0.23	− 4
10	18 10	8103.76	−2.94	S, *a*	26.2 34.1	+0.65	+0.89	+ 5
				S, *b*	32.0 33.6		+0.18	−12
	16	.76	−2.94	H, *a*	29.4 35.5	+0.54	+0.69	−15
				H, *b*	34.2 35.2		+0.11	−17
11	17 41	8104.74	−1.96	S, *a*	32.9 41.0	+0.46	+0.87	+ 3
				S, *b*	37.1 40.5		+0.35	+ 5
	48	.74	−1.96	H, *a*	30.7 37.1	+0.51	+0.71	−13
				H, *b*	35.3 36.7		+0.16	−12
14	18 27	8107.77	+1.07	S, *a*	31.6 39.5	+0.52	+0.85	+ 1
				S, *b*	36.3 40.1		+0.40	−10
	36	.78	+1.08	S, *b*	36.7 39.6		+0.30	0
15	16 56	8108.71	+2.01	S, *b*	40.2 42.9		+0.32	+ 2
	17 0	.71	+2.01	H, *b*	37.6 42.0		+0.48	+20
17	14 50	8110.62	+3.92	S, *b*	40.8 43.2		+0.28	0
	53	.62	+3.92	S, *b*	40.0 42.5		+0.29	+ 1
	57	.62	+3.92	H, *b*	36.9 39.4		+0.25	− 1
	15 0	.62	+3.92	H, *b*	39.4 41.8		+0.28	+ 2
18	15 18	8111.64	+4.94	S, *b*	36.3 38.5		+0.23	0
	21	.64	+4.94	S, *b*	35.7 37.8		+0.22	− 1
	26	.64	+4.94	H, *b*	34.3 36.3		+0.22	+ 1
	28	.64	+4.94	H, *b*	34.4 36.3		+0.21	0
19	14 52	8112.62	+5.92	S, *a*	28.8 33.0	+0.68	+0.47	+ 8
				S, *b*	34.9 33.0		−0.21	− 4
	15 2	.63	+5.93	H, *a*	27.3 32.1	[+0.73]	+0.54	+15
	8	.63	+5.93	H, *b*	33.8 31.5		−0.26	−11

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	$b-a$	Δm	O − C
1908, June 19	18h 25m	2418112.77	+ 6d07	S, *a*	21.7 23.8	[+0.41]	+ 0m22	— 6
	28	.77	+ 6.07	S, *b*	25.6 22.6		— 0.31	— 5
	19 1	.79	+ 6.09	H, *a*	20.4 23.7	[+0.51]	+ 0.35	+ 8
	9	.80	+ 6.10	H, *b*	25.3 21.5		— 0.40	— 1
20	15 30	8113.65	+ 6.95	S, *a*	24.8 26.1	+ 0.51	+ 0.14	+ 2
				S, *b*	29.5 26.5		— 0.32	+ 10
	57	.66	+ 6.96	H, *a*	35.7 36.7	+ 0.60	+ 0.11	— 1
				H, *b*	41.3 36.4		— 0.53	— 9
21	16 28	8114.69	+ 7.99	S, *a*	22.9 24.1	+ 0.44	+ 0.12	+ 8
				S, *b*	27.1 24.2		— 0.31	+ 19
	56	.71	+ 8.01	H, *a*	29.5 30.2	+ 0.75	+ 0.08	+ 4
				H, *b*	36.2 30.5		— 0.63	— 11
22	19 24	8115.81	+ 9.11	S, *a*	32.8 31.6		— 0.14	— 14
	50	.83	+ 9.13	H, *a*	29.8 30.5		+ 0.08	+ 8

REMARKS

The observations of 1907, August 2–10, and Aug. 18–Oct. 5, inclusive, were made with the Zöllner-Müller photometer. All others, with the Zeiss-Wedge equalizing photometer.

1906, Sept. 26, Oct. 7, 27, 31, Nov. 1; 1907, Aug. 24, 25, Sept. 16, 17, 18, Oct. 16; 1908, June 11—Sky white.

1906, Nov. 7, Dec. 3, 5; 1907, Aug. 14, Sept. 28, Oct. 1, 2; 1908, June 5, 6, 17—Haze.

1906, Oct. 9, Nov. 28; 1907, Aug. 2, Oct. 5—Definition bad.

1908, May 20, June 7, 15—Varying transparency.

1908, May 25, June 7—Stopped by clouds. The observation of June 7 consists of two settings, only, on each star.

It is possible that the last observation on 1907, Oct. 4, was referred to *a* instead of *b*, as indicated by the record. The corresponding residual would be + 0m08.

During the minimum of June, 1908, the observations were generally very difficult on account of the moon.

Bracketed values were excluded in forming the mean comparison star differences and the curve shown in Plate III.

column two, inserted to show the limiting dates of each group, the arrangement of the table is similar to that of Table III. Groups 2 and 4 fall approximately midway between minima. The others are all well clear of midway points. A comparison of the results for Nos. 2 and 4 with those of the remaining groups should therefore give information as to the possibility of an intermediate minimum. No. 4 is of no service, however, as it contains but a single observation; but No. 2, including 39 observations divided nearly equally between the two observers, is well adapted to the purpose. The mean value of $v-a$ for Group 2 is + 0m16. The means for Groups 1, 3, 5, and 6 range from — 0m05 to + 0m05, with an average for the four of 0m00. The probable error of this mean is ± 0m007, and of a single observation, ± 0m062. The mean for No. 2 therefore exceeds that for all the measures during what was certainly constant light by 0m16, a quantity fifteen times larger than the probable error of the mean of the second group,

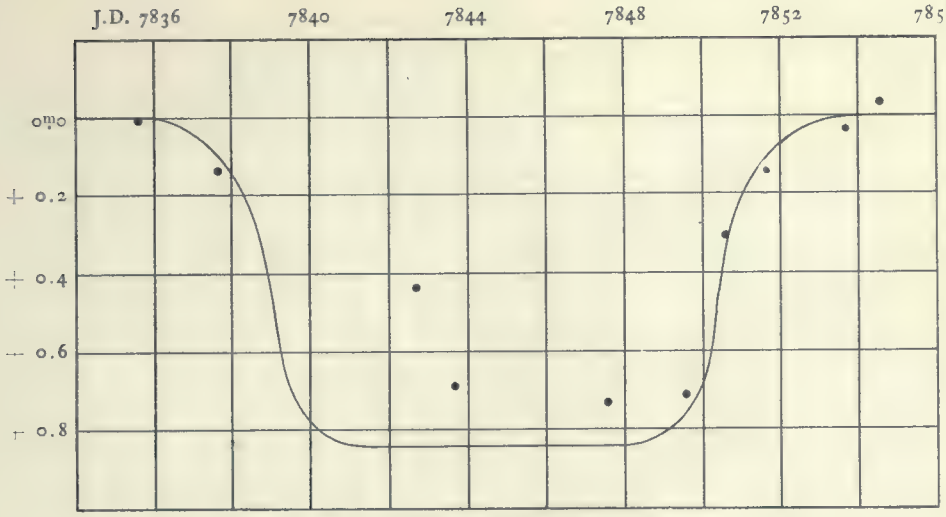

Fig. 1—Minimum of September, 1907

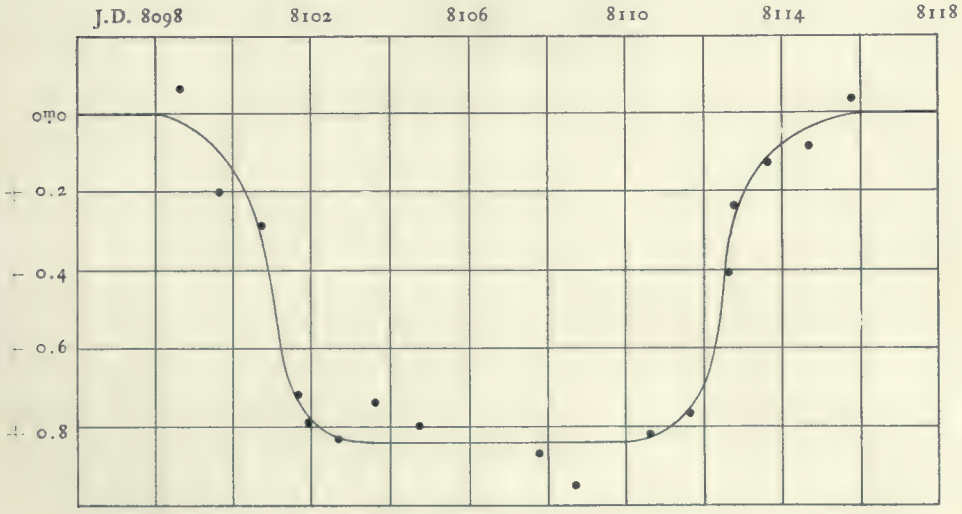

Fig. 2—Minimum of June, 1908

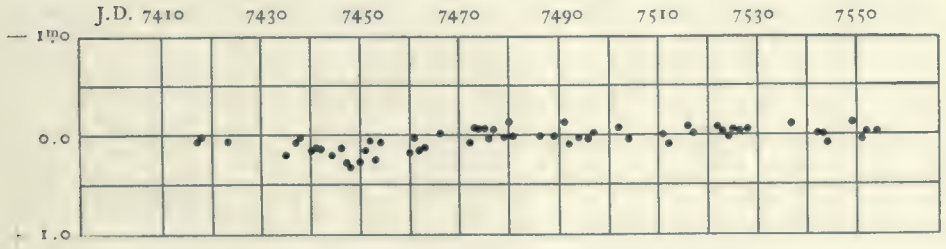

Fig. 3—Observations during constant light

PLATE III—The Long-Period Algol Variable RZ Ophiuchi

TABLE III

COMPARISON STAR DIFFERENCES

Diff.	Seares		Haynes		All Observations				Inst.
	Δm	No.	Δm	No.	Δm	No.	P. E. Mean	P. E. 1 Obs.	
$c-a$	$-0^m.47$	10	$-0^m.50$	9	$-0^m.49$	19	$\pm 0^m.018$	$\pm 0^m.077$	W
$b-a$	+0.54	10	+0.56	8	+0.55	18	0.020	0.083	W
$b-a$	+0.52	5	+0.38	11	+0.42	16	±0.027	±0.108	Z
$c-b$	+1.00	1	———	—	+1.00	1	———	———	W

TABLE IV

MAGNITUDE OF COMPARISON STAR a

Star	D. M.	PD.		HD. Mag.	Δm			Mag. of a		No. Obs.	No. Nts.
		No.	Mag.		S	H	Mean	P.	H.		
p	+6°3874	10009	7.37	7.07	$+2^m.46$	$+2^m.34$	$+2^m.40$	$9^m.77$	$9^m.47$	3, 3	3
q	6 3859	9977	7.24	7.29	2.53	2.25	2.39	9.63	9.68	3, 3	3
r	5 3846	9994	6.88	6.62	2.95	2.80	2.88	9.76	9.50	3, 3	3
s	5 3831	9980	8.34	8.00	+1.53	+1.50	+1.52	9.86	9.52	3, 3	3
							Mean	9.75	9.54		

TABLE V

MEAN VALUES OF $v-a$ DURING CONSTANT LIGHT

No.	Limiting Julian Dates	Seares		Haynes		All Observations				Inst.
		$v-a$	No.	$v-a$	No.	$v-a$	No.	P. E. Mean	P. E. 1 Obs.	
1	7416—7422	$+0^m.07$	2	$+0^m.04$	4	$+0^m.05$	6	$\pm 0^m.024$	$\pm 0^m.059$	W
2	7434—7462	+0.19	20	+0.12	19	+0 16	39	0.011	0.070	W
3	7466—7553	+0.03	10	−0.03	27	−0.01	37	0.007	0.041	W
4	7718	———	—	+0.10	1	+0 10	1	———	———	W
5	7790—7831	———	—	−0.05	15	−0.05	15	0.030	0.114	Z & W
6	7865—8092	+0.04	5	+0.07	6	+0.05	12	± 0.015	± 0.052	W

Mean, excluding Nos. 2 and 4, 0.00

and more than twice as large as the probable error of a single observation derived from the measures of the four groups. The data for the two observers, considered separately, reveals a similar result. The phenomenon is also illustrated graphically by Fig. 3 in Plate III, which gives the plot of the mean $v-a$ for each night from the beginning of the series to J.D. 7553. This latter indicates that the maximum deviation of the value of $v-a$ from the mean of Groups 1, 3, 5, and 6 occurred about J.D. 7448. The date midway between the two adjacent minima is J.D. 7452.

Considered collectively these results are suggestive of the existence of a shallow intermediate minimum having an amplitude of about $0^m.2$. Although the general consistency of the measures and the internal agreement of the data is such as to indicate that a change in the value of $v-a$ really occurred during the interval indicated, it should be noted that this variation may have been due to a fluctuation in the comparison star a. The fact that the measures during this interval were also referred to star c does not make it possible to eliminate this alternative, for it happens that these were the only measures referred to c. Consequently, the values of $c-v$ and $c-a$ resulting from these observations afford no test of the constancy of a.

The matter requires further investigation, especially in view of the fact that the observations of Graff[1] covering midway points give no trace of an intermediate minimum. Results by Jordan[2] have also been published, but throw no light upon the question, as his measures do not extend beyond the limits of a possible minimum. It may be remarked, however, that in this particular case the existence of an intermediate minimum is theoretically probable, although it is not certain that the corresponding amplitude is large enough to be revealed with certainty by observations of the precision now attainable. The matter will be recurred to after the discussion of the observations in Table II.

The measures in Table II covering the first of the two minima, that of September, 1907, were made with the Zöllner-Müller photometer. As will be seen by a reference to the column of residuals, they are extremely irregular, and, as compared with the wedge photometer observations of the minimum of June, 1908, present a pronounced systematic difference. Their unsatisfactory character is probably due, partly to the fact that these measures were among the first made with the Zöllner-Müller photometer, and partly to the fact that the eye-piece of this instrument when attached to the 7½-inch equatorial gives a magnification greater than should be used, in view of the inferior quality of the objective of this telescope.

The observations during changing light, reduced to star a as indicated above, were combined into means, the results being shown graphically by Figs. 1 and 2 of Plate III. The curve defined by the wedge photometer observations of the minimum of June, 1908, is the more reliable and has been superposed upon the plot of the results obtained with the Zöllner-Müller photometer for the minimum of September, 1907, in such a way as to represent these measures as closely as possible.

[1] *Astronomische Nachrichten*, v. 176, p. 79.
[2] *Astronomische Nachrichten*, v. 172, p. 385.

An examination of the plots yields the following data:

	1907, Sept.	1908, June.
Beginning of light change	J.D. 7836.0	8098.0
Beginning of constant minimum light	7841.5	8103.5
End of constant minimum light	7848.0	8110.0
End of light change	7854.0	8116.0
Epoch of minimum	7844.7	8106.7

It is evident from the plate that these dates are uncertain by $0^d.5$, or perhaps even more; but, accepting them for the present, we find

Interval of decreasing light	$5^d.5$
" " constant minimum light	6.5
" " increasing light	6.0
" " light change	18.0

These figures indicate a slight asymmetry of the curve, but for the present this may be disregarded.

Further,

		Harvard	Potsdam
For normal brightness	$v-a = 0^m.00$,	$M = 9^m.75$,	$9^m.54$,
For minimum brightness	$v-a = +0.84$,	$m = 10.59$,	10.38.
	Amplitude $= 0^m.84$.		

A comparison of the corresponding dates for the two minima gives for the interval separating them $262^d.0$. The epochs for which $v-a = +0^m.4$ on the ascending branch give $262^d.1$. Both are in good agreement with $261^d.8$, the value for the period given by Blajko.

The elements are

$$\text{Min.} = \text{J.D. } 2417844.7 + 261^d.8E, \quad \text{G.M.T.}$$

The average deviation from the curve of a single observation made with the wedge photometer during the minimum of June, 1908, is $+ 0^m.069$.

Returning now to the question of an intermediate minimum, it will be noted that the curve shown by the plate is of the type associated with the eclipse of a relatively small bright body by a larger and fainter object. Even though the exact form of the curve is at present uncertain, this conclusion is apparently well founded, for the essential factor upon which it is based, namely, a relatively long interval of constant minimum brightness coupled with a considerable amplitude of variation, appears as a marked characteristic of all of the results thus far published.[1]

[1] Blajko, A. N., v. 172, p. 58; Graff, A. N., v. 176, p. 79; Hartwig, A. N., v. 177, p. 69.

Indeed, the ratio of constant minimum light to the interval of total light change derived from the measures of other observers is considerably greater than resulting from observations made here, while, at the same time, the amplitude observed here appears to be less than that found elsewhere.

Assuming, therefore, that the system of RZ Ophiuchi consists of two luminous bodies, unequal in size, the smaller being the brighter, and, further, that the minima covered by the observations in Table II correspond to the eclipse of the smaller by the larger, there is, as stated above, a certain intrinsic probability for the existence of an intermediate minimum, for obviously the eclipse of the larger body by the smaller must also produce a light fluctuation.

Admitting certain more or less plausible assumptions, it is possible to calculate the maximum value of the amplitude of the intermediate minimum. For example, let it be assumed that the two bodies are spherical in form, that their surfaces are uniformly illuminated, and that the light variation is due solely to the eclipses resulting from the revolution of one body about the other in a circular orbit. Further, assume for a moment that the plane of the orbit coincides with the line of sight, in other words, that the eclipses occur centrally. If now we let

k = ratio of the radius of the smaller body to that of the larger,
I_1 = total interval of light change,
I_2 = interval of constant minimum light,

we shall have, under the conditions assumed above,

$$k\,(I_1 + I_2) = I_1 - I_2 \tag{1}$$

Further, we have the relation

$$h_1 k^2 = 1 - h_2, \tag{2}$$

where h_1 and h_2 are connected with the amplitudes of the two minima by the relations

$$\log h_1 = \log 1 - 0.4\,\Delta m_1, \tag{3}$$
$$\log h_2 = \log 1 - 0.4\,\Delta m_2. \tag{4}$$

The notation presupposes that h_1 and Δm_1 correspond to the eclipse of the smaller and brighter body by the larger and fainter, in other words, to the principal minimum.

Equations (1) and (3) give k and h_1 in terms of observed data. The substitution of their numerical values into (2) will give the value of h_2, whence Δm_2 can be determined from (4).

That the resulting value of Δm_2 will be a close approximation to the maximum possible corresponding to any given values of I_1, I_2, and Δm_1 may be seen as follows: Of the various assumptions underlying the above relations, that relating to the inclination of the orbit is perhaps the most likely to affect the accuracy of the result. If the orbit does not coincide with the line of sight, the value of I_2/I_1 derived from the curve will be smaller than that corresponding to cen-

tral eclipse; the calculated value of k will be too large; h_2 will be too small, and Δm_2 too large. In other words the actual value of Δm_2 will always be less than that based upon the value of k derived from equation (1), unless the assumption concerning the inclination which underlies this equation corresponds to the actual position of the orbit in space. Deviations from the other special conditions presupposed may modify to some extent this result, but unless they are extreme, it is not likely that the numerical value resulting for the limit of Δm_2 will be seriously in error.

For RZ Ophiuchi the values of I_1 and I_2 given above are $18^d.0$ and $6^d.5$, respectively. Further, the value of Δm_1 is $0^m.84$. We thus find $k = 0.469$, $h_1 = 0.461$, $h_2 = 0.899$ and $\Delta m_2 = 0^m.12$. In the present case, however, this upper limit for the amplitude of the intermediate minimum is uncertain on account of the unreliability of the values used for I_1 and I_2. If we assume that the instants of external and internal contact for the eclipse are in error by $1^d.0$ and $0^d.5$, respectively, we find as extreme limits for the maximum amplitude of the secondary minimum $0^m.07$ and $0^m.18$. The larger of these, corresponding to $I_2/I_1 = 0.275$, is in agreement with the abnormal deviation in the value of $v - a$ shown by Fig. 2. So far as the present series of observations is concerned there is nothing impossible in the above value of the ratio I_2/I_1, although the conclusions which result from its acceptance must for the present be regarded as possessing only a certain degree of plausibility.

In conclusion, it may be added that the value of k corresponding to this assumption is 0.569. By well known formulæ it is found, further, that the ratio of brightness per unit area of the smaller to the larger of the bodies is 3.6, and that the radius of the relative orbit is 6.6 times that of the larger body. The latter quantity, based as it is upon the times of contact, is unreliable. A more accurate value could have been obtained by a consideration of points on the ascending and descending branches of the curve for which the rate of light change is rapid; but the value given above is sufficient to indicate that, in spite of the unusual period of RZ Ophiuchi, the *relative* dimensions of the system exhibit nothing abnormal as compared with those ordinarily occurring in other binary systems giving rise to Algol variation.

PRELIMINARY ANNOUNCEMENT CONCERNING SW ANDROMEDAE (5.1907)

BD. + 28°54, 9m0, 1855.0 0h16m 8s + 28°35′.8
1900.0 18 29 50.8

Variability discovered by Miss Cannon[1] from an examination of chart plates of the Harvard College Observatory. Four hundred and sixty-one plates covering the interval 1889, Nov. 14–1906, Nov. 16, showed a range of variation of 8m.7 to 9m.9, approximately. Period indicated, 0d.49932.

TABLE VI

MAXIMA OF SW ANDROMEDAE

E	Date	Obs. Geoc. Maxima	O − C	Observer
0	1908, July 9	8132.805	0d000	H
2	10	8133.688	— 0.001	H
34	24	8147.829	+ 0.001	S & H
36	25	8148.704	— 0.008	S & H
41	27	8150.922	+ 0.001	H
43	28	8151.811	+ 0.006	S & H

The star has been under observation here since June, 1907, but the character of its variation was not recognized until July of the present year. Observations on six nights give maxima whose geocentric epochs are in column three of Table VI, the decimals being in G.M.T. The preliminary elements resulting from these data are

$$\text{Max.} = \text{J.D. } 2418132.805 + 0^d.44185E, \quad \text{G.M.T.}$$

The variable appears to be of the Antalgol type. The ascent from minimum to maximum occupies 1h 15m, while five hours are required for the decline. During the remaining 4h 20m of the period the variation is, at most, but a few hundredths of a magnitude.

The amplitude is about 0m.85, the maximum being 0m.15 fainter than BD. + 28°53, 9m0.

---

[1] H. C. No. 124; *Astronomische Nachrichten*, v. 174, p. 103; v. 176, p. 185.

Columbia, Missouri, 1908, August 3. *F. H. Seares.*

END OF VOLUME I

JUL 21 1910

ERRATA

Page	Line	
7,		Observations of Comet 1903 I for Feb. 8 and 16: replace the printed values by

Columbia M.T.	log p	Δ
$6^h\ 52^m\ 20^s$	9.637	0.711
7 30 26	9.665	0.723

17,	7, column 1,	*for* measurs *read* measures.
34,	last,	*for PAS read PAZ.*
35,	12,	*for PZS read* PZS_0.
41,	last,	*for* $\tan\varphi_0$ *read* $\tan\varphi$.
81,	12,	*for* the Polaris *read* Polaris.
83,		in first column of the calculation, the value of S: *for* 179° 7′.8 *read* 179° 7′.3.
102,	4, from bottom,	*for* Peliades *read* Pleiades.
150,	4,	*for* Wilkins *read* Wilkens.
189,	2, from bottom,	*for* obssrvatory *read* observatory.

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THE UNIVERSITY OF MISSOURI BULLETIN

ASTRONOMICAL SERIES

LAWS OBSERVATORY BULLETIN NO. 17

THE ANTALGOL VARIABLE ST OPHIUCHI (52.1907)

1900.0 R. A. $17^h\ 28^m\ 50^s$ Dec.—1° 0'.4[1]

The variability of ST Ophiuchi was discovered at Harvard from an examination of 31 photographs taken during the interval 1893-1904. Professor Pickering announced[2] that the light variation was of short duration, evidently about a day or a fraction thereof, and that the rise from minimum to maximum brightness through a range of at least one magnitude was very rapid. The photometric measures of the star made at the Laws Observatory on August 1, 5, and 14, 1908, indicated that it belonged to the antalgol class of variables, and that its period was approximately $0^d.45$. The resulting light curve showed a very steep ascending branch. These early results were announced by Professor Seares at the ninth annual meeting of the Astronomical and Astrophysical Society of America at Put-in-Bay, Ohio, August 25-28, 1908, and an abstract of his paper has been published.[3] The star was observed at Berlin during 1908, from May 26 to August 31, by Guthnick, who found the variation was of the antalgol type, and from the observations of four complete and two fragmentary maxima derived the elements:

$$\text{Max.} = \text{J. D. } 2418088.5066 + 0^d.450363E, \quad \text{G.M.T.}[4]$$

Guthnick considers the star a very abnormal case. From his observations he found an irregularity in the light curve near maximum and accounts for it by supposing a shallow algol minimum superposed upon a normal antalgol maximum. Assuming, then, that this anomalous condition is proved to exist, he offers suggestions concerning the theory of this class of variables. The observations made at the Laws Observatory throughout an interval of more than two years do not verify the existence of any unusual deviation from the ordinary type of antalgol variation.

On the 29 nights that the star has been measured at this observatory, a total of 293 observations have been made with the equalizing wedge photometer attached to the 7½-inch equatorial

[1] Position from A. N. vol. 175, p. 169.

[2] Harvard College Observatory Circ. No. 129; A. N. vol. 175, p. 169.

[3] Publications of the Astronomical and Astrophysical Society of America, vol. 1, p. 310.

[4] A. N. vol. 179, p. 181.

TABLE I

OBSERVATIONS OF ST OPHIUCHI

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	$b-a$	Δm	Mag.	O − C
1908, Aug. 1	15h 50m	2418155.660	+ 0d.056	S, *a*	31.6 36.9	+0m.50	+0m.59	11m.21	− 14
				S, *b*	36.1 37.3		+ 0.13	11.32	− 3
	58	.665	+ 0.061	H, *a*	25.1 29.3	0.48	+ 0.46	11.08	− 31
				H, *b*	29.5 29.5		0.00	11.19	− 20
	16 6	.671	+ 0.067	S, *a*	33.3 37.6	0.43	+ 0.47	11.09	− 34
				S, *b*	37.2 38.6		+ 0.14	11.33	− 10
	17 16	.719	+ 0.115	S, *a*	31.8 43.5	0.53	+ 1.29	11.91	+ 23
				S, *b*	36.6 42.1		+ 0.59	11.78	+ 10
	26	.726	+ 0.122	H, *a*	31.2 39.3	0.42	+ 0.88	11.50	− 22
				H, *b*	34.9 39.0		+ 0.43	11.62	− 10
	37	.734	+ 0.130	S, *a*	32.3 42.8	0.53	+ 1.16	11.78	+ 2
				S, *b*	37.1 42.2		+ 0.55	11.74	− 2
	18 24	.767	+ 0.163	S, *a*	33.8 46.8	0.41	+ 1.43	12.05	+ 8
				S, *b*	37.6 45.9		+ 0.92	12.11	+ 14
	32	.772	+ 0.168	H, *a*	30.9 40.4	0.47	+ 1.03	11.65	− 35
				H, *b*	35.1 40.7		+ 0.60	11.79	− 21
	42	.779	+ 0.175	S, *a*	32.3 44.6	0.58	+ 1.36	11.98	− 7
				S, *b*	37.6 44.4		+ 0.75	11.94	− 11
4	18 5	158.753	− 0.004	H, *a*	29.2 32.3	0.63	+ 0.35	10.97	+ 7
				H, *b*	34.8 32.3		− 0.28	10.91	+ 1
5	15 20	159.639	− 0.019	S, *b*	39.3 41.2		+ 0.21	11.40	+ 9
	24	.642	− 0.016	S, *b*	38.4 40.0		+ 0.17	11.36	+ 15
	28	.644	− 0.014	H, *b*	30.7 30.4		− 0.04	11.15	+ 1
	31	.646	− 0.012	H, *b*	30.3 28.9		− 0.15	11.04	− 5
	37	.651	− 0.007	S, *b*	38.1 36.5		− 0.17	11.02	+ 6
	42	.654	− 0.004	S, *b*	36.9 33.9		− 0.33	10.86	− 4
	48	.658	0.000	H, *a*	27.2 29.1		+ 0.21	10.83	+ 2
				H, *b*	31.7 29.7	0.50	− 0.22	10.97	− 12
	54	.662	+ 0.004	S, *a*	30.0 33.1		+ 0.35	10.97	+ 7
				S, *b*	36.1 33.3	0.68	− 0.31	10.88	− 2
	16 2	.668	+ 0.010	H, *a*	26.4 29.7		+ 0.37	10.99	+ 2
				H, *b*	31.8 30.3	0.61	− 0.18	11.01	+ 4
	10	.674	+ 0.016	S, *a*	31.5 35.3		+ 0.42	11.04	+ 1
				S, *b*	36.8 34.2	0.59	− 0.29	10.90	− 13
	17	.678	+ 0.020	H, *a*	26.9 30.6		+ 0.41	11.03	− 4
				H, *b*	31.6 30.9	0.52	− 0.08	11.11	+ 4
	24	.683	+ 0.025	S, *a*	30.9 35.9		+ 0.56	11.18	+ 7
				S, *b*	37.0 36.2	0.68	− 0.09	11.10	− 1

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	$b-a$	Δm	Mag.	O−C
1908, Aug. 5	16h 46m	2418159.699	+0d041	H, *a*	26.0 31.1	+0m46	+0m56	11m18	− 7
				H, *b*	30.2 31.8		+ 0.19	11.38	+ 13
	17 41	.737	+ 0.079	H, *a*	28.5 35.5	0.47	+ 0.78	11.40	− 10
				H, *b*	32.7 35.1		+ 0.26	11.45	− 5
	52	.744	+ 0.086	S, *b*	39.2 42.4		+ 0.37	11.56	+ 2
14	14 46	168.615	− 0.050	H, *a*	23.7 41.8	0.57	+ 1.98	12.60	+ 20
				H, *b*	29.0 41.0		+ 1.31	12.50	+ 10
	15 15	.635	− 0.030	H, *a*	25.5 37.2	0.52	+ 1.30	11.92	+ 8
				H, *b*	30.2 36.9		+ 0.75	11.94	+ 10
	36	.650	− 0.015	H, *a*	22.9 29.9	0.55	+ 0.75	11.37	+ 17
				H, *b*	28.1 29.6		+ 0.17	11.36	+ 16
	44	.656	− 0.009	H, *a*	25.6 27.8	0.45	+ 0.24	10.86	− 14
				H, *b*	29.7 27.5		− 0.25	10.94	− 6
	52	.661	− 0.004	H, *a*	23.4 25.7	0.61	+ 0.24	10.86	− 4
				H, *b*	29.1 25.2		− 0.43	10.76	− 14
	16 2	.668	+ 0.003	H, *a*	23.8 25.7	0.63	+ 0.20	10.82	− 6
				H, *b*	29.6 25.8		− 0.42	10.77	− 11
	8	.672	+ 0.007	H, *a*	23.1 26.2	0.58	+ 0.32	10.94	0
				H, *b*	28.5 26.0		− 0.28	10.91	− 3
	38	.693	+ 0.028	H, *a*	22.1 27.2	0.47	+ 0.54	11.16	+ 3
				H, *b*	26.5 26.4		− 0.02	11.17	+ 4
	17 18	.721	+ 0.056	H, *a*	26.1 33.9	0.61	+ 0.87	11.49	+ 14
				H, *b*	31.6 34.1		+ 0.28	11.47	+ 12
15	15 32	169.647	+ 0.081	H, *a*	27.6 35.1	0.57	+ 0.83	11.45	− 7
				H, *b*	32.7 34.9		+ 0.24	11.43	− 9
16	15 58	170.665	+ 0.199	H, *b*	31.3 40.2		+ 0.96	12.15	− 8
	16 40	.694	+ 0.228	H, *b*	32.9 43.9		+ 1.21	12.40	0
17	16 12	171.675	− 0.143	H, *a*	29.1 47.1	0.65	+ 1.99	12.61	+ 21
				H, *b*	34.9 47.1		+ 1.34	12.53	+ 13
22	14 32	176.606	− 0.167	H, *a*	26.0 41.9	0.47	+ 1.75	12.37	− 3
				H, *b*	30.3 42.0		+ 1.29	12.48	+ 8
	52	.619	− 0.154	H, *a*	35.8 53.0	0.46	+ 1.96	12.58	+ 18
				H, *b*	40.2 53.2		+ 1.53	12.72	+ 32
	15 31	.647	− 0.126	H, *a*	35.2 53.1	0.52	+ 2.04	12.66	+ 26
				H, *b*	40.1 52.8		+ 1.49	12.68	+ 28
	47	.658	− 0.115	H, *a*	37.6 55.1	0.53	+ 2.03	12.65	+ 25
				H, *b*	42.4 55.5		+ 1.55	12.74	+ 34
	16 3	.669	− 0.104	H, *a*	38.3 55.3	0.49	+ 1.99	12.61	+ 21
				H, *b*	42.7 55.1		+ 1.47	12.66	+ 26
	25	.684	− 0.089	H, *a*	35.6 53.0	0.48	+ 1.98	12.60	+ 20
				H, *b*	40.2 52.6		+ 1.45	12.64	+ 24

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings		$b-a$	Δm	Mag.	O − C
1908, Aug. 22	16h 42m	2418176.696	−0d077	H, *a*	31.7	50.7	+0m63	+2m13	12m75	+35
				H, *b*	37.4	50.9		+1.53	12.72	+32
	58	.707	−0.066	H, *a*	32.0	51.1	0.71	+2.14	12.76	+36
				H, *b*	38.6	51.8		+1.52	12.71	+31
24	15 39	178.652	+0.078	H, *a*	33.7	39.4	0.46	+0.60	11.22	−28
				H, *b*	38.0	39.5		+0.16	11.35	−15
	50	.660	+0.086	H, *a*	32.7	41.2	0.58	+0.91	11.53	−1
				H, *b*	38.1	40.8		+0.29	11.48	−6
27	14 19	181.597	−0.130	H, *a*	27.4	43.1	0.70	+1.74	12.36	−4
				H, *b*	33.6	42.5		+0.97	12.16	−24
	33	.606	−0.121	H, *a*	27.9	44.3	0.68	+1.81	12.43	+3
				H, *b*	34.0	44.3		+1.13	12.32	−8
	15 0	.625	−0.102	H, *a*	26.8	43.8	0.69	+1.88	12.50	+10
				H, *b*	33.0	44.1		+1.22	12.41	+1
	49	.659	−0.068	H, *a*	29.4	45.3	0.55	+1.76	12.38	−2
				H, *b*	34.3	45.2		+1.20	12.39	−1
30	14 34	184.607	+0.177	H, *a*	27.2	42.5	0.68	+1.69	12.31	+24
				H, *b*	33.3	42.5		+1.01	12.20	+13
	43	.613	+0.183	H, *a*	26.5	43.1	0.67	+1.83	12.45	+34
				H, *b*	32.5	42.8		+1.13	12.32	+21
	15 16	.636	+0.206	H, *a*	30.3	46.1	0.54	+1.75	12.37	+9
				H, *b*	35.1	45.8		+1.18	12.37	+9
	43	.655	+0.225	H, *a*	28.8	44.8	0.60	+1.77	12.39	−1
				H, *b*	34.2	45.1		+1.20	12.39	−1
Sept. 1	13 56	186.581	−0.101	H, *a*	25.5	42.8	0.62	+1.92	12.54	+14
				H, *b*	31.1	42.5		+1.26	12.45	+5
	14 18	.596	−0.086	H, *a*	26.8	42.2	0.58	+1.69	12.31	−9
				H, *b*	32.0	42.7		+1.17	12.36	−4
	41	.612	−0.070	H, *a*	27.7	43.1	0.56	+1.70	12.32	−8
				H, *b*	32.7	43.0		+1.13	12.32	−8
	15 4	.628	−0.054	H, *a*	28.9	44.5	0.57	+1.72	12.34	−6
				H, *b*	34.0	44.4		+1.14	12.33	−7
	31	.647	−0.035	H, *a*	28.2	44.0	0.64	+1.75	12.37	+22
				H, *b*	33.9	43.5		+1.06	12.25	+10
	58	.665	−0.017	H, *a*	31.2	37.4	0.58	+0.69	11.31	+6
				H, *b*	36.4	37.2		+0.09	11.28	+3
	16 4	.669	−0.013	H, *a*	30.8	35.0	0.55	+0.47	11.09	−2
				H, *b*	35.7	35 5		−0.02	11.17	+6
	7	.672	−0.010	H, *a*	29.7	33.1	0.52	+0.38	11.00	−5
				H, *b*	34.4	33.3		−0.12	11.07	+2

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings		$b-a$	Δm	Mag.	O−C
1908, Sept. 1	16h 10m	2418186.674	−0d.008	H, *a*	30.0	33.0	+0m.58	+0m.34	10m.96	− 3
				H, *b*	35.2	32.8		−0.26	10.93	− 6
	14	.676	−0.006	H, *a*	30.7	32.6	0.51	+0.22	10.84	−10
				H, *b*	35.3	32.6		−0.29	10.90	− 4
	19	.680	−0.002	H, *a*	30.6	32.2	0.49	+0.18	10.80	− 8
				H, *b*	35.0	31.7		−0.37	10.82	− 6
	23	.683	+0.001	H, *a*	30.1	32.5	0.54	+0.27	10.89	+ 3
				H, *b*	34.9	32.4		−0.28	10.91	+ 5
	28	.686	+0.004	H, *a*	29.9	33.0	0.60	+0.35	10.97	+ 7
				H, *b*	35.3	32.6		−0.29	10.90	0
	34	.690	+0.008	H, *a*	31.4	33.8	0.46	+0.27	10.89	− 6
				H, *b*	35.5	33.4		−0.24	10.95	0
	58	.707	+0.025	H, *a*	32.1	35.8	0.54	+0.41	11.03	− 8
				H, *b*	37.0	35.8		−0.13	11.06	− 5
	17 4	.711	+0.029	H, *a*	32.9	36.6	0.54	+0.41	11.03	−11
				H, *b*	37.9	36.4		−0.16	11.03	−11
2	15 44	187.656	+0.073	Sh, *a*	26.1	36.6	0.77	+1.17	11.79	+32
				Sh, *b*	33.0	36.5		+0.38	11.57	+10
	54	.662	+0.079	Sh, *a*	26.5	37.3	0.90	+1.19	11.81	+31
				Sh, *b*	34.6	36.9		+0.25	11.44	− 6
1909, July 14	16 36	502.692	−0.139	Sh, *a*	27.5	38.7	0.69	+1.23	11.85	−55
				Sh, *b*	33.6	40.7		+0.76	11.95	−45
	46	.699	−0.132	H, *a*	18.0	35.5	0.59	+1.92	12.54	+14
				H, *b*	23.4	36.2		+1.40	12.59	+19
15	15 37	503.651	−0.080	H, *a*	14.9	30.9	0.64	+1.80	12.42	+ 2
				H, *b*	20.1	29.8		+1.04	12.23	−17
	48	.658	−0.073	H, *a*	15.9	33.1	0.66	+1.91	12.53	+13
				H, *b*	21.6	33.4		+1.28	12.47	+ 7
	16 54	.704	−0.027	H, *a*	19.8	29.2	0.52	+1.00	11.62	− 4
				H, *b*	24.8	29.0		+0.46	11.65	− 1
	17 21	.723	−0.008	H, *a*	21.5	26.3	0.44	+0.51	11.13	+14
				H, *b*	25.6	26.0		+0.04	11.23	+24
	43	.738	+0.007	H, *a*	21.4	26.0	0.48	+0.49	11.11	+17
				H, *b*	25.9	26.2		+0.03	11.22	+28
	18 19	.763	+0.032	H, *a*	22.2	28.5	0.59	+0.67	11.29	+12
				H, *b*	27.7	28.7		+0.11	11.30	+13
16	15 58	504.665	+0.033	H, *a*	36.7	40.7	0.46	+0.42	11.04	−14
				H, *b*	41.1	40.2		−0.10	11.09	− 9
	16 32	.689	+0.057	H, *a*	34.8	41.9	0.54	+0.77	11.39	+ 3
				H, *b*	39 9	41.9		+0.23	11.42	+ 6

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings		$b-a$	Δm	Mag.	O − C
1909, July 16	17h 8m	2418504.714	+0d.082	H, *a*	36.3	44.2	+0m.58	+0m.87	11m.49	− 3
				H, *b*	41.7	44.7		+ 0.35	11.54	+ 2
17	14 47	505.616	+ 0.083	H, *a*	31.2	40.4	0.64	+ 1.00	11.62	+ 10
				H, *b*	36.9	40.4		+ 0.36	11.55	+ 3
	56	.622	+ 0.089	H, *a*	30.8	38.2	0.66	+ 0.81	11.43	− 12
				H, *b*	36.7	37.7		+ 0.10	11.29	− 26
	15 11	.633	+ 0.100	H, *a*	33.4	41.1	0.60	+ 0.83	11.45	− 17
				H, *b*	39.0	41.0		+ 0.22	11.41	− 21
	21	.640	+ 0.107	H, *a*	33.5	41.6	0.50	+ 0.87	11.49	− 16
				H, *b*	38.2	41.5		+ 0.36	11.55	− 10
	19 58	.832	+ 0.299	H, *a*	38.2	52.1	0.67	+ 1.60	12.22	− 18
				H, *b*	44.2	52.2		+ 0.94	12.13	− 27
19	15 14	507.635	− 0.151	H, *a*	26.8	42.7	0.57	+ 1.75	12.37	− 3
				H, *b*	31.9	42.9		+ 1.21	12.40	0
	24	.642	− 0.144	Sh, *a*	35.9	48.8	0.36	+ 1.43	12.05	− 35
				Sh, *b*	39.5	48.0		+ 0.95	12.14	− 26
	54	.662	− 0.124	H, *a*	30.5	46.3	0.51	+ 1.74	12.36	− 4
				H, *b*	35.1	46.0		+ 1.20	12.39	− 1
	16 3	.669	− 0.117	Sh, *a*	36.6	51.5	0.44	+ 1.68	12.30	− 10
				Sh, *b*	40.8	51.2		+ 1.21	12.40	0
	31	.688	− 0.098	H, *a*	34.0	47.6	0.39	+ 1.50	12.12	− 28
				H, *b*	37.6	47.4		+ 1.08	12.27	− 13
	42	.696	− 0.090	H, *a*	32.5	46.8	0.49	+ 1.58	12.20	− 20
				H, *b*	36.9	46.8		+ 1.09	12.28	− 12
	52	.703	− 0.083	H, *a*	33.7	49.1	0.51	+ 1.70	12.32	− 8
				H, *b*	38.5	49.2		+ 1.21	12.40	0
	17 2	.710	− 0.076	H, *a*	33.1	48.1	0.55	+ 1.65	12.27	− 13
				H, *b*	38.2	48.3		+ 1.13	12.32	− 8
	26	.726	− 0.060	H, *a*	33.1	48.6	0.54	+ 1.71	12.33	− 7
				H, *b*	38.1	48.6		+ 1.17	12.36	− 4
	39	.735	− 0.051	H, *a*	31.8	48.6	0.59	+ 1.85	12.47	+ 7
				H, *b*	37.2	48.7		+ 1.27	12.46	+ 6
	49	.742	− 0.044	H, *a*	32.7	49.0	0.55	+ 1.80	12.42	+ 2
				H, *b*	37.8	48.8		+ 1.23	12 42	+ 2
20	15 6	508.629	− 0.057	H, *a*	28.6	46.4	0.55	+ 1.96	12.58	+ 18
				H, *b*	33.5	46.8		+ 1.46	12.65	+ 25
	18	.638	− 0.048	H, *a*	27.0	46.4	0.68	+ 2.14	12.76	+ 36
				H, *b*	33.1	46.6		+ 1.49	12.68	+ 28
	31	.646	− 0.040	H, *a*	28.6	44.5	0.63	+ 1.75	12.37	+ 5
				H, *b*	34.3	44.4		+ 1.11	12.30	− 2

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	$b-a$	Δm	Mag.	O − C
1909, July 20	15h 43m	2418508.655	− 0d031	H, *a*	29.1 44.0	+0m51	+ 1m65	12m27	+ 37
				H, *b*	33.6 43.6		+ 1.10	12.29	+ 39
	56	.664	− 0.022	H, *a*	30.6 38.4	0.65	+ 0.85	11.47	+ 7
				H, *b*	36.5 38.1		+ 0.17	11.36	− 4
	16 5	.670	− 0.016	H, *a*	29.3 34.1	0.65	+ 0.54	11.16	− 5
				H, *b*	35.1 34.1		− 0.11	11.08	− 13
	10	.674	− 0.012	H, *b*	34.4 34.2		− 0.02	11.17	+ 8
	14	.676	− 0.010	H, *b*	35.2 34.5		− 0.08	11.11	+ 7
Oct. 2	13 32	582.564	+ 0.013	Sh, *a*	30.6 34.7	0.65	+ 0.46	11.08	+ 8
				Sh, *b*	36.5 35.8		− 0.07	11.12	+ 12
	41	.570	+ 0.019	Sh, *a*	29.8 35.1	0.79	+ 0.59	11.21	+ 15
				Sh, *b*	36.9 35.1		− 0.20	10.99	− 7
	14 2	.585	+ 0.034	Sh, *a*	30.4 35.5	0.72	+ 0.58	11.20	+ 1
				Sh, *b*	36.8 35.6		− 0.13	11.06	− 13
3	13 33	583.565	+ 0.113	Sh, *a*	31.6 42.6	0.57	+ 1.21	11.83	+ 15
				Sh, *b*	36.7 43.0		+ 0.69	11.88	+ 20
1910, July 26	18 12	879.758	− 0.025	Sh, *b*	44.2 46.4		+ 0.24	11.43	− 7
	18	.762	− 0.021	Sh, *b*	42.5 44.1		+ 0.18	11.37	+ 1
	25	.767	− 0.016	Sh, *b*	44.1 43.7		− 0.04	11.15	− 6
	29	.770	− 0.013	Sh, *b*	41.7 41.2		− 0.06	11.13	+ 3
	34	.774	− 0.009	Sh, *b*	42.4 41.0		− 0.17	11.02	+ 2
	44	.781	− 0.001	Sh, *b*	44.4 41.9		− 0.28	10.91	+ 5
	53	.787	+ 0.004	Sh, *b*	43.7 39.9		− 0.44	10.75	− 15
	58	.790	+ 0.007	Sh, *b*	43.6 39.8		− 0.44	10.75	− 19
	19 12	.800	+ 0.017	Sh, *b*	44.8 43.7		− 0.12	11.07	+ 3
	25	.809	+ 0.026	Sh, *b*	46.0 45.6		− 0.04	11.05	− 7
27	15 41	880.653	− 0.031	Sh, *b*	43.4 45.5		+ 0.24	11.43	− 47
	48	.658	− 0.026	Sh, *b*	43.0 44.6		+ 0 18	11.37	− 22
	59	.666	− 0.018	Sh, *b*	43.8 44.1		+ 0.03	11.22	+ 5
	16 5	.670	− 0.014	Sh, *b*	43.4 42.7		− 0.08	11.11	− 3
	10	.674	− 0.010	Sh, *b*	43.7 42.2		− 0.18	11.01	− 3
	19	.680	− 0.004	Sh, *b*	44.7 42.3		− 0.27	10.92	+ 2
	26	.685	+ 0.001	Sh, *b*	48.3 44.8		− 0.39	10.80	− 6
	31	.688	+ 0.004	Sh, *b*	49.8 46.7		− 0.36	10.83	− 7
	40	.694	+ 0.010	Sh, *b*	46.4 44.0		− 0.26	10.93	− 4
	17 27	.727	+ 0.043	Sh, *b*	45.5 46.5		+ 0.10	11.29	+ 2
Sept. 11	15 28	926.644	+ 0.019	Sh, *a*	42.8 45.9		+ 0.34	10.96	− 10
16	13 49	931.576	− 0.003	Sh, *a*	43.0 45.4		+ 0.26	10.88	0
	52	.578	− 0.001	Sh, *a*	43.6 45.7		+ 0.23	10.85	− 1
	57	.581	+ 0.002	Sh, *a*	42.7 46.4	0.81	+ 0.41	11.03	+ 16
				Sh, *b*	49.8 46.4		− 0.40	10.79	− 8

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	$b-a$	Δm	Mag.	O − C
1910, Sept. 16	$14^h\ 1^m$	2418931.584	$+0^d.005$	Sh, *a*	43.3 47.2		$+0^m.43$	$11^m.05$	+ 14
	5	.587	+ 0.008	Sh, *b*	50.3 47.7		− 0.32	10.87	− 8
	10	.590	+ 0.011	Sh, *a*	44.6 48.2	$+0^m.73$	+ 0.39	11.01	+ 3
				Sh, *b*	50.9 48.4		− 0.31	10.88	− 10
	17	.595	+ 0.016	Sh, *a*	43.1 48.0		+ 0.54	11.16	+ 13
	23	.599	+ 0.020	Sh, *a*	43.7 48.6	0.68	+ 0.54	11.16	+ 11
				Sh, *b*	49.7 48.4		− 0.16	11.03	− 4
	28	.603	+ 0.024	Sh, *a*	44.5 50.1		+ 0.64	11.26	+ 16
	47	.616	+ 0.037	Sh, *a*	43.9 51.5	0.87	+ 0.88	11.50	+ 28
				Sh, *b*	51.4 51.8		+ 0.05	11.24	+ 2
	15 36	.650	+ 0.071	Sh, *a*	44.8 55.7		+ 1.30	11.92	+ 46
17	13 19	932.555	+ 0.075	Sh, *a*	44.6 55.3		+ 1.28	11.90	+ 42
27	13 5	942.545	− 0.294	Sh, *a*	40.2 52.5	0.56	+ 1.44	12.06	+ 13
				Sh, *b*	45.1 52.3		+ 0.86	12.05	+ 12
	14	.551	− 0.288	Sh, *b*	46.2 55.1		+ 1.08	12.27	+ 30
	24	.558	− 0.281	Sh, *a*	42.4 52.6	0.48	+ 1.19	11.81	− 20
				Sh, *b*	46.7 52.8		+ 0.74	11.93	− 8
	32	.564	− 0.275	Sh, *b*	47.6 53.4		+ 0.71	11.90	− 15
	56	.581	− 0.258	Sh, *a*	43.7 54 7	0.47	+ 1.30	11.92	− 25
				Sh, *b*	48.0 54.8		+ 0.85	12.04	− 13
	14 27	.602	− 0.237	Sh, *b*	48.2 54.9		+ 0.84	12.03	+ 31
29	13 8	944.547	− 0.093	Sh, *a*	42.1 56.4	0.54	+ 1.70	12.32	− 8
				Sh, *b*	46.9 56.0		+ 1.11	12.30	− 10
	25	.559	− 0.081	Sh, *a*	42.7 54.4	0.47	+ 1.39	12.01	− 39
				Sh, *b*	46.9 54.9		+ 0.98	12.17	− 23
	34	.565	− 0.075	Sh, *a*	41.8 54.7	0.66	+ 1.52	12.14	− 26
				Sh, *b*	47.7 55.3		+ 0.94	12.13	− 27
	43	.572	− 0.068	Sh, *b*	48.1 54.9		+ 0.85	12.04	− 36
	48	.575	− 0.065	Sh, *b*	48.8 55.4		+ 0.82	12.01	− 39
	14 0	.583	− 0.057	Sh, *b*	49.0 54.2		+ 0.65	11.84	− 56
	4	.586	− 0.054	Sh, *b*	47.9 54.4		+ 0.81	12.00	− 40
	21	.598	− 0.042	Sh, *b*	48.1 55.3		+ 0.90	12.09	− 26
	37	.609	− 0.031	Sh, *b*	48.8 53.8		+ 0.62	11.81	− 9
	41	.612	− 0.028	Sh, *b*	48.2 52.2		+ 0.50	11.69	− 3
	44	.614	− 0 026	Sh, *b*	48.0 50.7		+ 0.33	11.52	− 5
	49	.617	− 0.023	Sh, *b*	48.8 51.1		+ 0.28	11.47	+ 2
	53	.620	− 0.020	Sh, *b*	46.9 47.5		+ 0.07	11.26	− 8
	15 0	.625	− 0.015	Sh, *b*	47.5 46.9		− 0.07	11.12	− 4
	6	.629	− 0.011	Sh, *b*	48.1 46.8		− 0.14	11.05	− 2
	12	.633	− 0.007	Sh, *b*	48.6 45.8		− 0.31	10.88	− 8
	15	.635	− 0.005	Sh, *b*	48.4 45.1		− 0.37	10.82	− 9

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	$b-a$	Δm	Mag.	O − C
1910, Sept. 30	$12^h\ 52^m$	2418945.536	−0.005	Sh, *a*	41.1 45.3		+0.48	11.10	+ 19
	56	.539	−0.002	Sh, *a*	39.9 43.4		+0.40	11.02	+ 14
	13 0	.542	+0.001	Sh, *b*	47.0 44.6		−0.26	10.93	+ 7
	4	.544	+0.003	Sh, *b*	46.4 44.5		−0.21	10.98	+ 9
	7	.547	+0.006	Sh, *a*	41.9 44.6		+0.31	10.93	0
	14	.551	+0.010	Sh, *a*	41.5 44.6		+0.36	10.98	0
				Sh, *b*	46.3 44.9	+0.54	−0.15	11.04	+ 6
	20	.556	+0.015	Sh, *b*	48.1 46.9		−0.13	11.06	+ 4
	27	.560	+0.019	Sh, *a*	43.0 47.0		+0.44	11.06	0
				Sh, *b*	46.9 47.2	0.43	+0.03	11.22	+ 16
	15 2	.626	+0.085	Sh, *a*	40.8 50.0		+1.06	11.68	+ 15
				Sh, *b*	45.8 50.2	0.57	+0.51	11.70	+ 17

each observation consisting of four settings upon the variable and four upon the comparison star so arranged as to minimize the effects of changes in the transparency of the atmosphere and in the brightness of the artificial star. In addition, on five nights the star was observed when no measures were made because the condition of the sky was such that the variable, then in minimum, was too faint for satisfactory measurements. When in minimum it is but little more than visible under favorable conditions, and on six of the 29 nights was not measurable some of the times the field was looked up. It is fainter than the twelfth magnitude during one-half of the period and the measures then made are affected by much larger errors than those made during the brighter part of the interval.

The measures were made by three different observers. Professor Seares secured 23 observations on the nights of August 1, and 5, 1908. Observations made by Mr. Haynes extend from August 1, 1908, to July 20, 1909, and are 174 in number. The remainder were obtained by me, nearly all of which were made after October 2, 1909. The entire observing interval extends from August 1, 1908, to September 30, 1910.

The observational data and certain parts of the reduction are in Table I. The general arrangement of this table follows closely that of tables of observations in former Bulletins. The phase values refer to the nearest geocentric maximum given by the final elements, and are positive for decreasing light. The initial of the observer and the comparison star used are in column five. The measured magnitude of the variable at the time of every observation is in column nine and is secured by applying to the adopted magnitudes of the comparison stars the Δm of column eight which is derived from the wedge readings.

The comparison star *a* is BD.$-0°3313$, $9^m.5$. Its magnitude was determined by reference to a star of the *Potsdam Durchmusterung*, *s* = BD.$+0°3709$, $6^m.8$. The Potsdam magnitude of the

star *s* is $6^m.88$. The mean of six concordant observations on three different nights gave $3^m.74$ for the difference *a*—*s*, and, as the relative positions of the two stars at the times of observation made corrections for differential absorption unnecessary, the value $10^m.62$ was adopted as the

TABLE II

EPOCHS OF OBSERVED MAXIMA

E	Obs. Geoc. Epochs	Red. to Sun	Obs. Helioc. Epochs	O – C
0	8159^d.658	+ 0^d.003	8159^d.661	0^d.000
20	8168.666	+ 0.003	8168.669	+0.001
60	8186.681	+ 0.001	8186.682	—0.001
764	8503.736	+ 0.005	8503.741	+0.004
775	8508.688	+ 0.004	8508.692	+0.001
1599	8879.785	+ 0.004	8879.789	—0.002
1601	8880.684	+ 0.004	8880.688	0.000
1714	8931.575	0.000	8931.575	—0.004
1743	8944.639	— 0.001	8944.638	—0.001
1745	8945.544	— 0.001	8945.543	+0.003

TABLE III

REDUCTION TO THE SUN

Date	Red.	Date	Red.	Date	Red.
Jan. 0	— 0^d.0050	Apr. 30	+ 0^d.0040	Aug. 28	+ 0^d.0016
10	46	May 10	45	Sep. 7	+ 0.0007
20	41	20	50	17	— 0.0002
30	35	30	53	27	11
Feb. 9	28	June 9	54	Oct. 7	20
19	19	19	54	17	28
Mar. 1	10	29	52	27	35
11	— 0.0001	July 9	49	Nov. 6	42
21	+ 0.0008	19	45	16	46
31	17	29	39	26	50
Apr. 10	25	Aug. 8	32	Dec. 6	52
20	33	18	24	16	52
30	+ 0.0040	28	+ 0.0016	26	— 0.0051

magnitude of *a* to be used in the reductions. The similar comparison of *a* with two BD. stars which are included in the *Revised Harvard Photometry* gave the values $10^m.63$ and $10^m.77$, respectively. Guthnick also used *a* for his comparisons and by reference to several Harvard stars found its

brightness to be $10^m.68$. The comparison star *b* follows the variable about $0^m.8$, and is 8′ north. The observed values of the magnitude difference *b*—*a* are in column seven of Table I. The mean of the 115 values there given is $+0^m.57$, with a probable error of $\pm 0^m.006$. The probable error of a single observation on *b*—*a* is $\pm 0^m.066$. In order to detect if possible any variation in the magnitudes of either of the comparison stars, the residuals referred to the mean value were plotted

TABLE IV

NORMAL PLACES

No.	Phase	Mag.	No. Obs.	O−C	No.	Phase	Mag.	No. Obs.	O−C
1	−0.0081	12.26	6	−14	25	+0.0022	11.09	7	0
2	−0.076	12.39	6	− 1	26	+0.027	11.11	6	− 1
3	−0.070	12.40	6	0	27	+0.034	11.22	8	+ 2
4	−0.066	12.38	4	− 2	28	+0.042	11.28	3	+ 3
5	−0.058	12.29	6	−11	29	+0.057	11.32	8	− 4
6	−0.052	12.45	6	+ 5	30	+0.068	11.45	3	+ 1
7	−0.046	12.44	7	+ 4	31	+0.076	11.52	7	+ 4
8	−0.033	12.30	4	+22	32	+0.081	11.54	8	+ 3
9	−0.030	11.84	4	0	33	+0.087	11.52	7	− 3
10	−0.026	11.51	6	− 6	34	+0.107	11.60	6	− 4
11	−0.022	11.38	5	− 2	35	+0.118	11.70	4	0
12	−0.017	11.28	6	+ 3	36	+0.130	11.76	2	0
13	−0.014	11.19	8	+ 4	37	+0.163	12.00	7	+ 3
14	−0.013	11.10	6	− 1	38	+0.174	12.01	7	− 3
15	−0.009	11.02	8	0	39	+0.188	12.18	4	+ 4
16	−0.006	10.92	6	− 1	40	+0.206	12.23	4	− 5
17	−0.004	10.93	11	+ 3	41	+0.226	12.39	3	− 1
18	−0.001	10.86	6	0	42	+0.294	12.41	8	+ 1
19	+0.002	10.86	8	− 1	43	+0.310	12.28	8	−12
20	+0.004	10.94	8	+ 4	44	+0.322	12.46	4	+ 6
21	+0.007	10.94	8	0	45	+0.331	12.45	8	+ 5
22	+0.010	10.99	5	+ 2	46	+0.349	12.45	8	+ 5
23	+0.014	11.03	7	+ 1	47	+0.360	12.38	8	− 2
24	+0.019	11.08	8	+ 2					

with respect to the time, but no unusual deviation from constancy was found. In like manner the residuals for the individual observers were examined and, although the average residual was rather large, no systematic corrections were considered necessary or advisable. Therefore, the adopted value of the brightness of *b* for the entire series is $11^m.19$.

The period was determined by a consideration of ten maxima. The magnitudes of the variable taken from Table I in the neighborhood of these maxima were plotted and curves were

drawn that seemed best to represent the light change. The geocentric times of maxima were read from the curves and are in the second column of Table II. On account of the relatively low latitude of the star, correction for the equation of light was necessary. The reduction to the sun in column three of Table II was taken from Table III which was computed for this star by means of the formula:

$$Helioc.\ Time - Geoc.\ Time = -497^{s}8\ R\ cos\ B\ cos\ (\lambda_{\odot} - \lambda),$$

where B and λ are the latitude and longitude of the variable, $\lambda_{\odot}$ is the longitude of the sun, and R is

TABLE V

ORDINATES OF THE MEAN LIGHT-CURVE. PHASE FROM MAXIMUM

Phase	Mag.	Phase	Mag.	Phase	Mag.	Phase	Mag.
— 0d048	12m40	+ 0d020	11m07	+ 0d088	11m55	+ 0d156	11m92
0.044	12.40	0.024	11.10	0.092	11.57	0.160	11.95
0.040	12.32	0.028	11.13	0.096	11.59	0.164	11.98
0.036	12.20	0.032	11.17	0.100	11.62	0.168	12.00
0.032	11.95	0.036	11.21	0.104	11.64	0.172	12.03
0.028	11.72	0.040	11.24	0.108	11.65	0.176	12.06
0.024	11.49	0.044	11.27	0.112	11.67	0.180	12.08
0.020	11.34	0.048	11.30	0.116	11.69	0.184	12.12
0.016	11.21	0.052	11.33	0.120	11.71	0.188	12.14
0.012	11.09	0.056	11.35	0.124	11.73	0.192	12.17
0.008	10.99	0.060	11.38	0.128	11.75	0.196	12.20
— 0.004	10.90	0.064	11.41	0.132	11.77	0.200	12.24
0.000	10.85	0.068	11.44	0.136	11.80	0.204	12.27
+ 0.004	10.90	0.072	11.47	0.140	11.82	0.208	12.30
0.008	10.95	0.076	11.49	0.144	11.85	0.212	12.34
0.012	10.99	0.080	11.51	0.148	11.87	0.216	12.37
+ 0.016	11.03	+ 0.084	11.53	+ 0.152	11.90	+ 0.220	12.40

the radius vector of the earth's orbit. The heliocentric times of observed maxima thus derived were given weights ranging from unity to eight, according to the completeness and consistency of the corresponding observations, and a least square solution of the ten equations resulted in the following heliocentric elements:

$$Max. = J.\ D.\ 2418159.661 + 0^{d}4503604E,\quad G.\ M.\ T.$$

The probable error of the period is ±3 in the last decimal place as written above. The observations of Guthnick were not used in deriving these elements, as the slight extension of the interval,

which would be obtained by including his work in this investigation, would not offset the uncertainty arising from a combination of the two different systems of observing. Neither could the Harvard photographs be used to improve the period, in this case, however, because the interval separating the dates of the photographs and of the photometric observations was too long to permit a definite determination of the epoch. The Julian date and Greenwich mean time of the maximum corresponding to the initial epoch of Guthnick's published elements was computed from the above formula and found to be J. D. 2418088.504, G. M. T. The difference of $0^d.003$ is consistent with the residuals in the last column of Table II. The range of variation determined at the Laws Observatory is $0^m.3$ greater than that observed at Berlin.

To construct the mean light curve the final elements were used to compute epochs of maxima for the entire interval of observations. These epochs were reduced to the center of the earth and

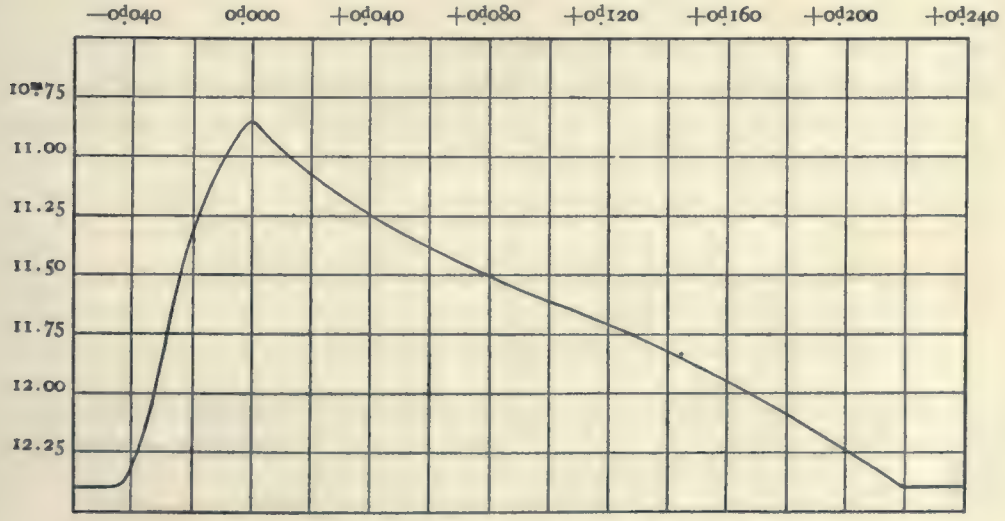

Figure 1. Light Curve for ST Ophiuchi

the phase values in column four of Table I were formed. The observations during changing light were then arranged in order of phase and were combined to form the 47 normal places of Table IV. The mean magnitudes corresponding to the mean phases of these normal places are in columns three and eight, and the number of observations of the respective groups is in columns four and nine of this table. The normal places were plotted and the mean light curve drawn. The residuals in columns five and ten, expressed in hundredths of a magnitude, are referred to this curve. The character of the variation is shown by Figure 1, and in the ordinates of the mean light curve given in Table V for every four-thousandth of a day during changing light. The range is from $12^m.40$ to $10^m.85$, the star at maximum being two tenths of a magnitude fainter than *a*. The ascent from minimum to maximum takes less than sixty-five minutes. For the descent a little more than five hours is required. The light is in constant minimum brightness for $0^d.186$, about two-fifths of the

period. The curve is quite sharp at maximum, but an examination of the residuals of the descending branch in the table of normal places failed to reveal any trace of the irregularity that Guthnick claims to have discovered.

The residuals in the last column of Table I are referred to the mean light curve and are expressed in hundredths of a magnitude. The investigation of them, made for the purpose of showing any differences between odd and even maxima, gave negative results. The average residuals for the different parts of the curve are

Light.	No. Obs'ns.	Av. Resid.
Increasing	72	$0^m.09$
Decreasing	132	0.11
Constant	89	0.17
Changing	204	0.10

The relative sizes during changing and constant light result from the difficulty of making measures during the extremely faint minima. A single abnormal series of measures made during minimum light on the night of August 22, 1908, contributes largely to the magnitude of the average residuals.

The average systematic deviations for the individual observers are

	No. Obs'ns.	Av. Syst. Dev.
Seares	23	$+0^m.001$
Haynes	174	+ 0.021
Shapley	96	— 0.030
All	293	+ 0.003

All the observations on the variable late in the year when the field was low and difficult were made by me. This accounts for my large systematic deviation, for, by excluding the observations made when the field was extremely low, the resulting errors are normal.

Columbia, Missouri, 1911, April 2. *Harlow Shapley.*

NEW ELEMENTS FOR RW CAMELOPARDALIS

BD.+58°663,8m8, 1855.0 R.A.$3^h42^m31^s.5$ Dec.+58° 13′.2
1900.0 46 9 21.6

The variability of this star was detected by Miss Leavitt at the Harvard College Observatory through the inspection of a plate made at Cambridge, February 8, 1904.[1] By making a comparison of the plate with four others of the same field, the period was estimated to be short and the photographic range to be from the ninth to the tenth magnitude. From observations made at Dombaas, Norway, in 1907, Enebo concluded that the period was about 16^d and that the range was at least $1^m.2$.[2] In 1908 he published 57 observations of the star in the second volume of his *Beobachtungen Veränderlicher Sterne* and derived the following approximate elements:

$$\text{Max.} = 1907 \text{ Oct. } 8 = 2417857^d + 16^d.4E, \text{ G.M.T.}$$

The range of light variation indicated by these observations was less than half a magnitude $8^m.48 - 8^m.93$.

The star has been observed intermittently at the Laws Observatory since the announcement of its discovery in 1907. Nearly 150 observations have been made, the earlier ones by Mr. Haynes and the later ones by me. The Zöllner-Müller photometer was used by Mr. Haynes for the first two sets of measures attributed to him in the accompanying table. The wedge photometer was used for all other measures. The comparison stars are a = BD. + 58°672, $8^m.2$, and b = BD. + 58°676, $8^m.6$. The magnitude of a in the A.G. catalogue is 8.5. It is f in the series of comparison stars used at Dombaas, and its A.G. magnitude was adopted by Enebo in determining the limits of variation.

For the investigation of the value of the period, all the observed magnitudes were plotted with respect to the time. Very few maxima or minima were sufficiently observed for a satisfactory reduction, but it was found that a certain part of the descending branch of the light curve was well represented for a number of widely separated epochs. The times during decreasing light for which $v - a = +0^m.35$ were read off from the graphs and are entered in column two of Table VI. Four of the times were obtained from graphs made of Enebo's observations. The first and last epochs observed here were used for a first determination of the period. The resulting preliminary elements are:

$$\text{Desc. Br. } v - a = +0^m.35 = \text{J.D. } 2417862.7 + 16^d.409E, \text{ G.M.T.}$$

To improve the values of initial epoch and period, the weighted residuals in column four of the

[1] Harvard College Observatory Circ. No. 127; A. N. vol. 175, p. 93.
[2] A. N. vol. 176, p. 374.

table, which are referred to the preliminary elements, were used to form eleven equations of condition, the solution of which gave for the revised elements:

$$\text{Desc. Br. } v - a = +0^{m}.35 = \text{J.D. } 2417863.3 + 16^{d}.402E, \text{ G.M.T.}$$

The sum of the squares of the residuals were reduced by this treatment from 6.04 to 2.92. The new set of residuals is in column five. The probable error of the initial epoch is $\pm 0^{d}.1$, and of the period, $\pm 0^{d}.004$. By plotting all the observations, referred to a common phase value, a mean

TABLE VI

EPOCHS FOR WHICH $v - a = +0^{m}.35$, DECREASING LIGHT

Epoch	Observed G.M.T.	Wt.	O − C	O − C_1	Observer
0	2417862^{d}.2	1	−0^{d}.5	−1^{d}.1	Enebo
0	7862.7	1	0.0	−0.6	Haynes
3	7912.6	1	+0.7	+0.1	Enebo
5	7945.4	1	+0.7	+0.1	"
5	7945.8	1	+1.1	+0.5	Haynes
8	7994.9	1	+0.9	+0.4	"
9	8011.0	2	+0.6	+0.1	"
10	8027.8	1	+1.0	+0.5	Enebo
20	8192.1	1	+1.2	+0.7	Haynes
32	8387.8	1	0.0	−0.4	Shapley
76	9109.8	2	0.0	−0.1	"

light curve was obtained that indicates the maximum precedes by $5^{d}.9$ the time for which $v - a = +0^{m}.35$ on the descending branch. Applying this value to the above elements, we have finally:

$$\text{Max.} = \text{J.D. } 2417857.4 + 16^{d}.402E, \text{ G.M.T.}$$

Except in the matter of depth, the curve given by Enebo accurately represents the light change.[1] The variation is continuous, and maxima and minima are about equally sharp. For increasing light $6^{d}.8$ are required. At maximum the variable is $0^{m}.04$ brighter than a, and at minimum is $0^{m}.66$ fainter.

[1] Beobachtungen Veränderlicher Sterne, vol. 2, p. 56.

Columbia, Missouri, 1911, April 24. *Harlow Shapley.*

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THE UNIVERSITY OF MISSOURI BULLETIN

ASTRONOMICAL SERIES

LAWS OBSERVATORY BULLETIN NO. 18

THE ALGOL VARIABLE RX DRACONIS (121.1906)

1855.0	R.A. $19^h0^m25^s$	Dec.	$+58°31'.8$[1]
1900.0	1 8		35.2

This star was announced as a variable of the Algol type in A.N. No. 4128 by Ceraski of Moscow. Its variability was discovered in September, 1905, by Madame Ceraski from plates by Blajko and confirmed visually by the latter, who found it in minimum on the night of October 18, 1906. The following provisional elements, based upon observations made here in December, 1906, were published by Seares in Laws Observatory Bulletin No. 9:

$$\text{Min.} = \text{J.D. } 2417502.400 + 1^d.894E, \quad \text{G.M.T.}$$

This paper presents a revision of these elements and a mean light curve, together with a detailed discussion of the observations from which they were derived.

The observations are 313 in number and were made on 39 nights. Thirty of them were secured by Prof. F. H. Seares, 208 by the writer, and 75 by Mr. Harlow Shapley. Each observation consists of eight settings, four upon the comparison star, and four upon the variable, arranged to eliminate the effect of any progressive change in the observing conditions. The instrument used was the equalizing Zeiss-wedge photometer attached to the 7½-inch equatorial.

The observations themselves are printed in Tables I and II. Those made during constant light, 128 in number, are entered in Table I. Columns three and nine of this table contain the means of the wedge readings, the first number referring to settings on the comparison star, and the second to settings on the variable. The observers are indicated, as in previous Bulletins, by their initials, Seares and Shapley being distinguished by the use of an italic for the latter. The columns headed Δm give the measured differences in brightness, variable minus comparison star. These values were derived from the wedge readings by means of Table V, Laws Observatory Bulletin No. 7. The measures of the magnitude difference between the comparison stars, derived in the same way, are under the caption $b-a$. Table II is devoted to the observations during

[1] Position by Ceraski, *Astronomische Nachrichten*, v. 172, p. 391.

TABLE I

OBSERVATIONS ON RX DRACONIS DURING CONSTANT LIGHT

J. D.	Obs. Star	Readings		Δm	$b-a$	O−C	J. D.	Obs. Star	Readings		Δm	$b-a$	O−C
7541.59	H, a	28.9	33.4	+0.50		+ 4	7911.61	H, a	23.9	27.9	+0.43		+ 8
	H, b	35.9	33.0	−0.32	+0.78	+ 1		H, b	29.7	28.0	−0.19	+0.63	+ 14
7542.56	H, a	27.7	31.5	+0.42		− 4	.64	H, a	24.5	27.8	+0.36		+ 1
	H, b	34.9	31.7	−0.36	0.80	− 3		H, b	29.8	27.6	−0.24	0.58	+ 9
7548.56	H, a	37.1	40.6	+0.37		− 9	.67	H, a	27.2	29.3	+0.23		− 12
7550.56	H, a	38.7	41.9	+0.36		− 9		H, b	31.6	29.2	−0.27	0.49	+ 6
7552.56	H, a	46.5	49.4	+0.34		− 11	7914.58	H, a	30.0	32.9	+0.33		− 2
7556.61	H, a	51.3	56.7	+0.44		− 1		H, b	35.3	33.0	−0.25	0.59	+ 8
7557.58	H, a	45.5	48.4	+0.32		− 13	8035.89	H, a	27.1	29.6	+0.28		− 5
.59	H, a	39.5	43.4	+0.44		− 1		H, b	32.0	29.9	−0.24	0.55	+ 9
	H, a	40.3	44.8	+0.52		+ 7	8037.83	H, a	27.5	29.2	+0.17		− 16
.60	H, a	46.2	49.3	+0.36		− 9	8043.95	H, a	29.3	32.6	+0.38		+ 5
7562.69	H, a	29.9	33.7	+0.43		− 2	8132.64	H, a	26.5	30.4	+0.42		+ 10
7563.52	H, a	21.1	23.6	+0.27		− 18		H, b	32.8	30.3	−0.29	0.70	+ 4
.65	H, a	39.8	43.3	+0.40	0.70	− 5	.67	H, a	26.5	29.9	+0.37		+ 5
	H, b	46.0	43.4	−0.29		+ 4		H, b	32.7	29.9	−0.32	0.69	+ 1
7567.50	S, a	28.8	31.8	+0.34		− 11	.72	H, a	25.3	28.9	+0.40		+ 8
7580.56	H, a	32.1	35.4	+0.36		− 8		H, b	30.6	29.3	−0.15	0.59	+ 18
	H, b	39.7	35.6	−0.43	0.82	− 10	8153.79	H, a	23.0	25.2	+0.22		− 9
7597.57	H, a	34.3	38.6	+0.46		+ 2		H, b	29.5	25.5	−0.44	0.69	− 11
	H, b	42.0	38.0	−0.44	0.84	− 11	.84	H, a	23.0	25.1	+0.21		− 10
7618.91	H, a	32.4	36.4	+0.44		+ 1		H, b	29.1	25.0	−0.45	0.65	− 12
	H, b	39.7	35.9	−0.40	0.79	− 7	8477.65	H, a	23.2	27.5	+0.46		+ 18
7621.99	H, a	27.2	29.9	+0.30		− 13		H, b	28.3	27.1	−0.13	0.54	+ 20
	H, b	33.0	29.9	−0.35	0.65	− 2	.67	H, a	26.7	31.4	+0.52		+ 24
7882.63	H, a	23.2	26.0	+0.29		− 6		H, b	33.1	31.6	−0.17	0.71	+ 16
	H, b	30.2	25.3	−0.54	0.75	− 21	.69	H, a	25.0	30.1	+0.56		+ 28
.66	H, a	24.8	28.6	+0.42		+ 7		H, b	31.5	30.3	−0.14	0.72	+ 19
.68	H, a	23.4	27.9	+0.48		+ 13	.71	S, a	29.6	33.9	+0.48		+ 20
	H, b	31.5	28.0	−0.39	0.88	− 6		S, b	35.7	34.6	−0.12	0.68	+ 21
.72	H, a	24.2	29.2	+0.54		+ 19	.73	H, a	27.7	30.0	+0.25		− 3
	H, b	32.1	29.1	−0.34	0.87	− 1		H, b	32.3	30.2	−0.24	0.51	+ 9
7911.55	H, a	26.6	29.0	+0.26		− 9	.74	S, a	30.9	35.5	+0.52		+ 24
	H, b	32.4	29.0	−0.38	0.64	− 5		S, b	37.7	34.3	−0.37	0.75	− 4
.56	H, a	26.1	28.5	+0.27		− 8	8478.66	H, a	29.8	32.1	+0.26		− 2
	H, b	32.5	29.0	−0.39	0.71	− 6		H, b	35.7	32.7	−0.33	0.66	
.58	H, a	22.2	25.9	+0.39		+ 4		S, a	32.4	35.8	+0.38		+ 10
	H, b	27.9	25.9	−0.22	0.61	+ 11		S, b	40.2	36.5	−0.38	0.84	− 5
.59	H, a	24.5	27.6	+0.34		− 1		H, a	26.6	30.6	+0.44		+ 16
	H, b	29.8	27.8	−0.22	+0.58	+ 11		H, b	32.6	30.4	−0.26	+0.67	+ 7

J. D.	Obs. Star	Readings		Δm	$b-a$	O-C	J. D.	Obs. Star	Readings		Δm	$b-a$	O-C
8478.68	S, a	30.7	34.8	+0m46	+0m76	+ 18	8946.76	H, a	28.1	30.6	+0m28		+ 5
	S, b	37.6	35.4	−0.24		+ 9		H, a	29.1	31.3	+0.25		+ 2
.69	H, a	26.1	29.4	+0.36	0.64	+ 8	8952.71	S, a	33.7	34.1	+0.04	+0m51	−19
	H, b	31.8	29.6	−0.25		+ 8		S, b	38.5	34.2	−0.46		−13
.77	H, a	28.2	30.1	+0.21	0.51	− 7	8967.53	S, a	34.2	35.5	+0.15	0.58	− 8
	H, b	32.7	30.7	−0.23		+ 10		S, b	39.6	35.7	−0.41		− 8
8924.61	S, a	30.0	32.2	+0.25	0.58	+ 2	.54	H, a	23.8	23.7	−0.01	0.59	−24
	S, b	35.2	32.4	−0.31		+ 2		H, b	29.3	24.0	−0.57		−24
8925.87	S, a	25.7	25.1	−0.07	+0.68	−30	.55	S, a	29.2	31.4	+0.25	0.55	+ 2
8926.60	S, a	31.8	34.0	+0.24		+ 1		S, b	34.1	30.4	−0.42		− 9
	S, b	38.1	34.0	−0.44		−11	.56	S, a	30.8	32.4	+0.18	0.51	− 3
69	S, a	27.0	29.4	+0.26		+ 3		S, b	35.4	32.5	−0.32		− 8
.75	S, a	39.8	42.1	+0.26		+ 3		H, a	22.6	24.6	+0.20	0.57	− 5
.80	S, a	29.8	31.7	+0.21		− 2		H, b	28.0	24.2	−0.41		+ 1
8929.57	S, a	33.1	33.8	+0.08		−15	.57	H, a	24.0	25.8	+0.19	0.56	− 4
.68	S, a	37.4	39.6	+0.23		0		H, b	29.2	25.5	−0.41		− 8
.76	S, a	39.6	41.7	+0.23		0		S, a	29.3	32.3	+0.34	0.48	+11
8931.57	S, a	32.5	34.2	+0.19		− 4		S, b	33.5	32.2	−0.15		+18
.88	S, a	38.8	41.1	+0.25		+ 2	.58	H, a	23.3	26.1	+0.29	0.60	+ 6
.92	S, a	39.6	40.8	+0.13		−10		H, b	28.9	26.2	−0.30		+ 3
8932.60	S, a	37.5	38.6	+0.11		−12	.60	H, a	24.0	26.2	+0.23	0.57	0
.61	H, a	33.4	35.2	+0.20		− 3		H, b	29.3	26.1	−0.35		− 2
8946.75	H, a	27.1	28.7	+0.18		− 5	.64	H, a	28.8	31.2	+0.26	+0.61	+ 3
	H, a	27.4	29.4	+0.22		− 1		H, b	34.3	31.6	−0.30		+ 3

TABLE II

OBSERVATIONS ON RX DRACONIS DURING CHANGING LIGHT

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings		$b-a$	Δm	Mag.	O-C
1906, Dec. 6	13h 38m	2417551.568	−0d066	H, a	34.4	41.8		+0m81	10m56	+ 21
	14 8	.589	−0.045	H, a	34.7	42.8		+0.89	10.64	+ 15
	15 21	.640	+0.006	H, a	33.6	45.1	+0m75	+1.26	11.01	+ 31
				H, b	40.6	44.7		+0.47	11.00	+ 30
	16 9	.673	+0.039	H, a	35.1	44.7		+1.06	10.81	+ 28
	17 2	.710	+0.076	H, a	35.0	39.9		+0.52	10.27	− 1
8	11 36	7553.483	−0.044	H, a	29.5	35.3		+0.65	10.40	− 10
	56	.497	−0.030	H, a	29.1	36.2		+0.79	10.54	− 5
	12 9	.506	−0.021	H, a	29.0	37.8		+0.97	10.72	+ 7
	40	.528	+0.001	H, a	30.2	40.1		+1.08	10.83	+ 12
25	12 1	7570.500	−0.066	S, a	30.8	36.2		+0.60	10.35	0

Date	G.M.T	Julian Day	Phase	Obs. Star	Readings	$b-a$	Δm	Mag.	O − C
1906, Dec. 25	12h 6m	2417570.504	− 0d.062	S, *a*	31.1 36.9		+ 0m.65	10m.40	+ 2
	12	.508	− 0.058	S, *a*	30.8 36.4		+ 0.62	10.37	− 3
	20	.514	− 0.052	S, *a*	31.2 37.2		+ 0.67	10.42	− 3
	27	.519	− 0.047	S, *a*	30.9 36.0	+ 0m.68	+ 0.57	10.32	− 16
	32	.522	− 0.044	S, *b*	37.0 36.7		− 0.03	10.50	0
	36	.525	− 0.041	S, *b*	35.8 36.4	0.74	+ 0.06	10 59	+ 7
	41	.528	− 0.038	S, *a*	29.2 36.8		+ 0.85	10.60	+ 6
	50	.535	− 0.031	S, *a*	31.5 38.7	0.68	+ 0.78	10.53	− 5
	56	.539	− 0.027	S, *b*	37.7 39.5		+ 0.19	10.72	+ 10
	13 4	.544	− 0.022	S, *b*	37.4 40.6	0.74	+ 0.34	10.87	+ 23
	7	.546	− 0.020	S, *a*	30.7 40.2		+ 1.03	10.78	+ 13
	16	.553	− 0.013	S, *b*	38.0 40.6		+ 0.28	10.81	+ 13
	23	.558	− 0.008	S, *b*	37.8 41.1		+ 0.35	10.88	+ 18
	35	.566	0.000	S, *b*	36.9 41.7		+ 0.51	11.04	+ 33
	53	.578	+ 0.012	S, *b*	38.2 42.5		+ 0.48	11.01	+ 32
	14 4	.586	+ 0.020	S, *b*	40.9 43.1		+ 0.27	10.80	+ 15
1907, Feb. 8	23 2	7615.960	− 0.042	H, *a*	24.7 35.1	0.91	+ 1.15	10.92	+ 40
				H, *b*	32.9 34.7		+ 0.20	10.73	+ 21
	22	.974	− 0.028	H, *b*	35.8 37.6		+ 0.19	10.72	+ 11
	54	.996	− 0.006	H, *b*	33.6 34.4		+ 0.08	10.61	− 9
Apr. 23	19 40	7689.819	− 0.016	H, *a*	27.2 34.7	0.81	+ 0.84	10.63	− 4
				H, *b*	34.5 35.4		+ 0.10	10.63	− 4
	20 21	.848	+ 0.013	H, *a*	24.1 32.5	0.85	+ 0.92	10.71	+ 3
				H, *b*	31.8 32.5		+ 0.08	10.61	− 7
	21 28	.894	+ 0.059	H, *b*	49.9 51.1		+ 0.14	10.67	+ 27
1908, Apr. 11	19 48	8043.825	− 0.035	H, *a*	30.1 36.2		+ 0.68	10.55	− 1
	20 6	.837	− 0.023	H, *a*	29.1 36.8	0.78	+ 0.86	10.73	+ 9
				H, *b*	36.1 36.4		+ 0.03	10.58	− 8
	20 30	.854	− 0.006	H, *a*	28.2 35.2	0.74	+ 0.78	10.65	− 5
				H, *b*	34.8 35.5		+ 0.08	10.61	− 9
	56	.872	+ 0.012	H, *a*	28.7 36.4	0.66	+ 0.85	10.72	+ 3
				H, *b*	34.6 36.1		+ 0.16	10.69	0
	21 32	.897	+ 0.037	H, *a*	28.3 33.5	0.70	+ 0.59	10.46	− 9
				H, *b*	34.5 33.2		− 0.15	10.38	− 17
	57	.915	+ 0.055	H, *a*	28.4 32.2	0.55	+ 0.43	10.30	− 13
				H, *b*	33.3 32.3		− 0 11	10.42	− 1
	22 23	.933	+ 0.073	H, *a*	27.1 31.5	0.70	+ 0.49	10.36	+ 6
				H, *b*	33.4 31.5		− 0.21	10.32	+ 2
July 9	18 31	8132.772	− 0.065	H, *a*	26.3 32.5	0.78	+ 0.69	10.57	+ 21
				H, *b*	33.3 32.3		− 0.11	10.42	+ 6
	19 44	.822	− 0.015	H, *a*	24.9 31.3	+0.64	+ 0.71	10.59	− 8
				H, *b*	30.7 30.8		+ 0.01	10.54	− 13

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	$b-a$	Δm	Mag.	O−C
1908, July 11	16h 1m	2418134.667	−0d.063	H, *a*	23.6 26.9	+0m.55	+0m.35	10m.23	−14
				H, *b*	28.7 27.0		−0.19	10.34	−3
13	15 14	8136.635	+0.011	H, *b*	27.0 28.2		+0.13	10.66	−3
	17	.637	+0.013	H, *b*	27.7 28.6		+0.10	10.63	−5
	25	.642	+0.018	S, *a*	24.6 29.6	0.57	+0.55	10.43	−23
				S, *b*	29.8 29.0		−0.09	10.44	−22
	33	.648	+0.024	H, *a*	21.9 27.8	0.72	+0.63	10.51	−12
				H, *b*	28.6 28.3		−0.04	10.49	−14
	40	.653	+0.029	S, *b*	30.3 29.5		−0.09	10.44	−16
	42	.654	+0.030	S, *b*	29.8 29.5		−0.04	10.49	−10
28	17 10	8151.715	−0.054	H, *a*	20.6 23.4	0.33	+0.30	10.19	−24
				H, *b*	23.7 23.7		0.00	10.53	+10
30	15 23	8153.641	−0.021	S, *a*	25.3 33.8	0.75	+0.95	10.84	+19
				S, *b*	32.0 32.5		+0.05	10.58	−7
	34	.649	−0.013	H, *a*	21.3 29.4	0.69	+0.87	10.76	+8
				H, *b*	27.7 29.4		+0.18	10.71	+3
	42	.654	−0.008	S, *a*	26.2 33.0	0.59	+0.76	10.65	−5
				S, *b*	31.5 33.2		+0.19	10.72	+2
	50	.660	−0.002	H, *a*	20.4 27.7	0.61	+0.78	10.67	−4
				H, *b*	26.2 27.8		+0.18	10.71	0
	16 40	.694	+0.032	S, *a*	26.4 32.1	0.65	+0.64	10.53	−5
				S, *b*	32.2 31.8		−0.04	10.49	−9
	47	.699	+0.037	H, *a*	21.3 26.6	0.70	+0.57	10.46	−9
				H, *b*	27.8 26.8		−0.11	10.42	−13
	17 50	.743	+0.081	S, *a*	26.5 27.8	0.51	+0.14	10.03	−22
				S, *b*	31.2 28.2		−0.33	10.20	−5
	58	.749	+0.087	H, *a*	22.2 25.2	0.65	+0.31	10.20	−1
				H, *b*	28.3 25 6		−0.29	10.24	+3
1909, July 19	14 56	8507.622	−0.064	H, *a*	29.6 33.5	0.53	+0.44	10.36	0
				H, *b*	34.4 33.4		−0.11	10.42	+6
	15 4	.628	−0.058	S, *a*	31.7 34.4	0.37	+0.30	10.22	−18
				S, *b*	35.0 33.7		−0.14	10.39	−1
	35	.649	−0.037	H, *a*	24.0 28.2	0.55	+0.45	10.37	−18
				H, *b*	29.1 28.5		−0.06	10.47	−8
	44	.656	−0.030	S, *a*	30.2 33.0	0.40	+0.32	10.24	−35
				S, *b*	33.7 33.2		−0.06	10.47	−12
1910, Sept. 16	17 25	8931.726	−0.031	S, *a*	33.6 39.2		+0.59	10.56	−3
	31	.730	−0.027	S, *a*	33.4 39.9	0.66	+0.70	10.67	+6
				S, *b*	39.5 40.3		+0.08	10.61	0
	45	.740	−0.017	S, *a*	36.3 42.5		+0.68	10.65	−1
	19 7	.797	+0.040	S, *a*	35.9 43.1	+0.59	+0.79	10.76	+23
				S, *b*	41.4 42.7		+0.15	10.68	+15

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	$b-a$	Δm	Mag.	O − C
1910, Sept. 16	19h 51m	2418931.827	+ 0d070	S, a	36.5 40.6		+ 0m44	10m41	+ 9
	20 2	.835	+ 0.078	S, a	37.0 40.7		+ 0.39	10.36	+ 9
	6	.837	+ 0.080	S, a	36.8 40.5		+ 0.38	10.35	+ 9
	15	.844	+ 0.087	S, a	37.4 40.2		+ 0.29	10.26	+ 5
Oct. 1	19 52	8946.828	— 0.075	H, a	31.8 35.2		+ 0.37	10.34	+ 5
	59	.833	— 0.070	H, a	32.6 35.4		+ 0.30	10.27	— 5
	20 5	837	— 0.066	H, a	33.0 35.9		+ 0.32	10.29	— 6
	10	.840	— 0.063	H, a	32.2 35.2		+ 0.33	10.30	— 7
	20	.847	— 0.056	H, a	32.8 35.1		+ 0.25	10.22	— 20
				H, b	37.2 35.3	+ 0m48	— 0.21	10.32	— 10
	30	.854	— 0.049	H, a	33.8 37.1		+ 0.36	10.33	— 14
				H, b	38.8 36.6	0.53	— 0.22	10.31	— 16
	41	.862	— 0.041	H, a	34.0 38.5		+ 0.48	10.45	— 7
				H, b	39.8 38.8	0.62	— 0.11	10.42	— 10
	53	.870	— 0.033	H, a	35.2 40.3		+ 0.54	10.51	— 6
				H, b	41.2 40.6	0.64	— 0.06	10.47	— 10
	21 2	.876	— 0.027	H, a	35.1 41.0		+ 0.63	10.60	— 1
				H, b	41.0 40.8	0.63	— 0.02	10.51	— 10
7	12 56	8952.539	— 0.044	S, a	33.6 37.6		+ 0.43	10.40	— 10
				S, b	39.1 37.1	0.58	— 0.20	10.33	— 17
	13 4	.544	— 0.039	H, a	27.3 33.0		+ 0.64	10.61	+ 8
				H, b	32.9 32.7	0.63	— 0.02	10.51	— 2
	12	.550	— 0.033	S, a	31.8 36.3		+ 0.49	10.46	— 11
				S, b	36.9 37.0	0.56	+ 0.01	10.54	— 3
	19	.555	— 0.028	H, a	25.5 30.1		+ 0.51	10.48	— 13
				H, b	30.3 30.7	0.53	+ 0.05	10.58	— 3
	26	.560	— 0.023	S, a	33.5 38.5		+ .053	10.50	— 13
				S, b	37.0 38.1	0.38	+ 0.11	10.64	+ 1
	34	.565	— 0.018	H, a	25.1 31.6		+ 0.72	10.69	+ 3
				H, b	30.7 32.0	0.62	+ 0.15	10.68	+ 2
	42	.571	— 0.012	S, a	32.9 38.0		+ 0.55	10.52	— 17
				S, b	37.7 38.4	0.52	+ 0.07	10.60	— 9
	51	.577	— 0.006	H, a	25.3 31.8		+ 0.73	10.70	0
				H, b	30.1 32.2	0.53	+ 0.24	10.77	+ 7
	14 0	.583	0.000	S, a	31.8 38.2		+ 0.69	10.66	— 5
				S, b	36.4 37.7	0.50	+ 0.14	10.67	— 4
	7	.588	+ 0.005	H, a	27.0 32.4		+ 0.60	10.57	— 13
				H, b	31.6 32.3	0.51	+ 0.08	10.61	— 9
	15	.594	+ 0.011	S, a	32.6 38.1		+ 0.59	10.56	— 13
				S, b	37.3 38.6	0.51	+ 0.13	10.66	— 3
	24	.600	+ 0.017	H, a	26.3 31.7		+ 0.60	10.57	— 9
				H, b	31.2 31.8	+0.54	+ 0.08	10.61	— 5

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	$b-a$	Δm	Mag.	O − C
1910, Oct. 7	$14^h\ 32^m$	2418952.606	$+0^d.023$	*S*, *a*	32.7 37.6	$+0^m.52$	$+0^m.53$	$10^m.50$	− 13
				S, *b*	37.5 37.6		+ 0.01	10.54	− 9
	39	.610	+ 0.207	H, *a*	27.2 31.8	0.49	+ 0.52	10.49	− 12
				H, *b*	31.6 31.4		− 0.02	10.51	− 10
	46	.615	+ 0.032	*S*, *a*	33.0 37.3	0.53	+ 0.47	10.44	− 14
				S, *b*	37.9 36.9		− 0.10	10.43	− 15
	54	.621	+ 0.038	H, *a*	27.8 32.0	0.52	+ 0.47	10.44	− 10
				H, *b*	32.5 32.3		− 0.02	10.51	− 3
	15 20	.639	+ 0.056	*S*, *a*	33.1 36.5	0.47	+ 0.37	10.34	− 8
				S, *b*	37.4 35.7		− 0.18	10.35	− 7
22	15 22	8967.640	− 0.088	*S*, *a*	30.8 33.4	0.56	+ 0.29	10.26	+ 6
				S, *b*	35.8 33.7		− 0.23	10.30	+ 10
	45	.656	− 0.072	H, *a*	27.1 30.0	0.58	+ 0.32	10.29	− 2
				H, *b*	32.3 30.0		− 0.26	10.27	− 4
	53	.662	− 0.066	*S*, *a*	32.4 35.6	0.45	+ 0.36	10.33	− 2
				S, *b*	36.5 35.7		− 0.08	10.45	+ 10
	16 0	.667	− 0.061	H, *a*	26.2 30.9	0.58	+ 0.52	10.49	+ 10
				H, *b*	31.4 31.2		− 0.03	10.50	+ 11
	7	.672	− 0.056	*S*, *a*	30.7 35.6	0.62	+ 0.55	10.52	+ 10
				S, *b*	36.3 36.4		+ 0.01	10.54	+ 12
	14	.676	− 0.052	H, *a*	25.9 31.4	0.61	+ 0.61	10.58	+ 13
				H, *b*	31.4 31.6		+ 0.02	10.55	+ 10
	20	.680	− 0.048	*S*, *a*	29.5 34.4	0.55	+ 0.55	10.52	+ 4
				S, *b*	34.4 34.2		− 0.02	10.51	+ 3
	28	.686	− 0.042	H, *a*	25.4 31.8	0.61	+ 0.72	10.69	+ 17
				H, *b*	30.9 31.7		+ 0.10	10.63	+ 11
	17 23	.724	− 0.004	H, *a*	28.9 36.8	0.69	+ 0.88	10.85	+ 14
				H, *b*	35.1 37.1		+ 0.22	10.75	+ 4
	30	.729	+ 0.001	*S*, *a*	34.1 40.2	0.42	+ 0.65	10.62	− 9
				S, *b*	38.0 39.9		+ 0.20	10.73	+ 2
	42	.738	+ 0.010	H, *a*	29.6 36.9	0.58	+ 0.81	10.78	+ 9
				H, *b*	34.8 36.7		+ 0.21	10.74	+ 5
	52	.744	+ 0.016	H, *a*	30.9 38.2	0.56	+ 0.80	10.77	+ 10
				H, *b*	35.9 38.2		+ 0.24	10.77	+ 10
	18 10	.757	+ 0.029	H, *a*	31.3 37.9	0.57	+ 0.72	10.69	+ 9
				H, *b*	36.5 38.1		+ 0.17	10.70	+ 10
	20	.764	+ 0.036	H, *a*	30.9 37.5	0.58	+ 0.73	10.70	+ 14
				H, *b*	36.1 37.2		+ 0.12	10.65	+ 9
	34	.774	+ 0.046	H, *a*	31.2 36.7	0.63	+ 0.62	10.59	+ 10
				H, *b*	36.8 36.7		− 0.01	10.52	+ 3
	42	.779	+ 0.051	H, *a*	30.8 35.6	$+0.62$	+ 0.54	10.51	+ 6
				H, *b*	36.4 35.7		− 0.07	10.46	+ 1

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings		$b-a$	Δm	Mag.	O-C
1910, Oct. 22	$18^h\ 56^m$	2418967.789	+0.061	H, a	32.7	36.2		+0.38	10.35	− 3
				H, b	37.7	36.3	+0.54	−0.15	10.38	0
	19 8	.797	+0.069	H, a	32.6	35.9		+0.36	10.33	0
				H, b	37.9	36.1	0.57	−0.19	10.34	+ 1
	23	.808	+0.080	H, a	34.9	36.9		+0.22	10.19	− 6
				H, b	40.3	37.0	+0.57	−0.34	10.19	− 6

REMARKS

1906, Dec. 6—Moon.
11—Thick haze.
22—Thin haze, star low.
25—Thin haze, star low.
1907, Jan. 21—Thin haze.
Apr. 23—Sky very white, occasional clouds, moon.
Dec. 1—Sky white.
4—Sky white.
1908, Apr. 5—Occasional light haze.
1908, Apr. 13—Full moon.
July 15—Haze.
30—Occasional haze.
1910, Sept. 9—Current unsteady.
10—Current unsteady.
11—Haze.
14—Occasional clouds, moon.
16—Sky white, moon.
17—Sky white, moon.
Oct. 22—Moon.

TABLE III

COMPARISON STAR DIFFERENCES

Limiting Julian Days	Mean J. D.	Mean Δm	P. E. Mean	P. E. 1 Obsn.	No. Obsns.
7541—7689	7593	+0.77	±0.013	±0.049	15
7882—7914	7904	0.67	0.025	0.084	11
8035—8043	8043	0.67	0.023	0.060	7
8132—8153	8145	0.63	0.016	0.068	19
8477—8507	8487	0.62	0.021	0.088	17
8924—8967	8958	+0.56	±0.006	±0.044	56

changing light, 185 in number. The column headings identical with those used in Table I, have the same significance as before. The additional columns either require no explanation or will be discussed later.

The comparison stars used are a = BD. +58°1858, $9^m.5$, and b = BD. +58°1863, $9^m.5$. Tables I and II furnish 125 measures of their difference in brightness. These measures were collected into six groups with respect to the time. The means of these groups, the probable errors of the means, and the probable errors of a single observation are displayed in Table III, columns three, four, and five, respectively. An examination of these means reveals a slight but

progressive decrease in the observed values of the quantity $b-a$. Although slight, the decrease is considerably larger than the probable errors of the means.

In order to determine the cause of this variation, the measures of $v-a$ made during normal light were combined into groups and the means of the groups plotted, but the errors of observation, incident to these measures, were found to be so great that it was impossible to decide whether the irregularity noted above was due to actual change in the brightness of a, or not. A similar investigation of the observed values of $v-b$, secured during normal light, was also made and gave conclusive evidence that this quantity had remained sensibly constant.

TABLE IV

COMPARISON STAR CURVE

J. D.	$b-a$	J. D.	$b-a$
7500	+0m80	8300	+0m63
7600	0.77	8400	0.62
7700	0.73	8500	0.61
7800	0.70	8600	0.60
7900	0.68	8700	0.59
8000	0.66	8800	0.58
8100	0.65	8900	0.56
8200	+0.64	9000	+0.55

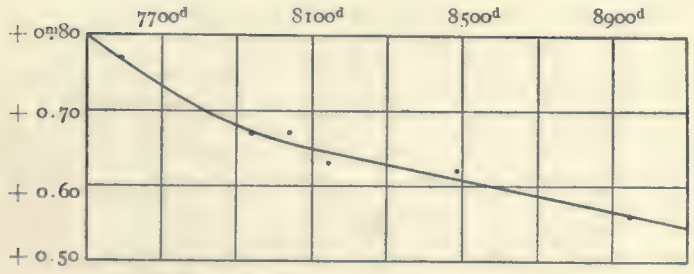

Fig. 1. COMPARISON STAR CURVE

The preceding attempt to secure an explanation of the anomalous series of comparison star differences having failed, the means from Table III were used as normal places for a curve, the coördinates of which are given in Table IV. The curve itself is shown in Fig. 1. This curve was accepted as defining the magnitude difference $b-a$. The probable error of a single observation referred to it is $\pm 0^m059$. By means of its ordinates, all measures of the quantity $v-b$ were reduced to a, and all measures of the quantity $v-a$ were reduced to b. The former were combined with the direct comparisons of the variable and a, the latter with the direct comparisons of the variable and b, thus furnishing two separate series of observational data. Each of these

series was made the basis of a complete reduction of all observations. The probable errors of single observations as calculated in the two reductions are shown below:

	Referred to *a*	Referred to *b*
Constant light	± 0m.086	± 0m.072
Changing light	± 0.111	± 0.081

Since the probable errors affecting the first series of data are larger than those affecting the second, the reduction presented in the following pages is the one secured by referring all observations to *b*.

TABLE V

MAGNITUDE OF COMPARISON STAR *b*

Star	BD. No.	PD. No.	PD. Mag.	Mean Δm	Mag. *b*	No. Obs.
q	+59°1947	III 2905	7.86	+2.66	10.52	4
r	58 1872	2919	7.90	2.61	10.51	4
s	57 1947	2910	8.02	2.64	10.66	4
t	+58 1873	2920	7.22	+3.22	10.44	4
				Mean	10.53	

TABLE VI

REDUCTION TO THE SUN

Day	Red.	Day	Red.
Jan. 0	— 0d.0008	July 0	+ 0d.0008
Feb. 0	— 0.0010	Aug. 0	+ 0.0011
Mar. 0	— 0.0010	Sept. 0	+ 0.0010
Apr. 0	— 0.0007	Oct. 0	+ 0.0007
May 0	— 0.0002	Nov. 0	+ 0.0002
June 0	+ 0.0004	Dec. 0	— 0.0003

The magnitude of the star *b* was determined by comparing it with four PD. stars. The particulars of the comparison are summarized in Table V. All observations upon which it depends were made by the writer. The resulting magnitude of *b* is 10.53 with a probable error of ± 0.02.

The observed magnitudes of the variable were secured from observations referred to *a* by subtracting the appropriate ordinate of the curve in Fig. 1 from 10.53 and then adding the measured value of $v-a$. In the case of observations referred to *b*, the observed difference $v-b$ was added directly to 10.53. The resulting magnitudes for the observations during changing light may be found in the ninth column of Table II. The mean of those obtained from the data in

TABLE VII

OBSERVED MINIMA AND THEIR REPRESENTATION

Date	Obs. Geoc. Min.	Red. to Sun	Obs. Helioc. Min.	Wt.	E	Comp. Helioc. Min.	O − C
1906, Oct. 18	7502d400	0d000	7502d400	a	0	7502d411	———
Dec. 6	7551.631	0.000	7551.631	4	26	7551.634	—0 003
8	7553.548	0.000	7553.548	a	27	7553.527	+0 021
25	7570.576	— 0.001	7570.575	3	36	7570.565	+0.010
1907, Feb. 8	7615.966	— 0.001	7615.965	1	60	7616.001	—0 036
Apr. 23	7689.859	0.000	7689.859	0	99	7689.835	+0.024
1908, Apr. 11	8043.858	— 0.001	8043.857	4	286	8043.859	—0.002
July 9	8132.828	+ 0.001	8132.829	0	333	8132.837	—0.008
13	8136.603	+ 0.001	8136.604	0	335	8136.625	—0.021
30	8153.650	+ 0.001	8153.651	3	344	8153.663	—0.012
1909, July 19	8507.674	+ 0.001	8507.675	0	531	8507.687	—0.012
1910, Sept. 16	8931.768	+ 0.001	8931.769	4	755	8931.758	+0.011
Oct. 1	8946.914	+ 0.001	8946.915	4	763	8946.904	+0.011
7	8952.580	+ 0.001	8952.581	7	766	8952.583	—0.002
22	8967.724	0.000	8967.724	10	774	8967.728	—0 004

TABLE VIII

NORMAL PLACES AND MEAN LIGHT CURVE

Normal Places				Mean Light Curve			
No. Obs.	Phase	Mag.	O − C	Phase	Mag.	Phase	Mag.
22	±0d004	10.74	+ 3	±0d000	10.710	±0d045	10.495
29	0.014	10.66	— 2	0.005	10.705	0.050	10.460
27	0.025	10.62	0	0.010	10.693	0.055	10.426
29	0.035	10.50	— 6	0.015	10.675	0.060	10.393
21	0.044	10.53	+ 3	0.020	10.650	0.065	10.358
19	0.055	10.42	— 1	0.025	10.623	0.070	10.325
19	0.064	10.39	+ 2	0.030	10.593	0.075	10.290
9	0.073	10.32	+ 2	0.035	10.562	0.080	10.255
10	±0 084	10.22	— 1	0 040	10.528	0.085	10.220
				±0.045	10.495	±0.090	10.200

Table I gave 10.20 as the magnitude of the variable during normal light. The probable error of this result is $\pm 0^{m}.006$, and the probable error of a single observation, $\pm 0^{m}.072$. The residuals from this mean are in the sixth and twelfth columns of Table I. These residuals were arranged in order of phase and plotted, but no change in the brightness of the variable was revealed. Unfortunately, however, only one observation of the entire series falls midway between minima. Therefore information concerning the possible existence of a secondary minimum is not available.

Four or more observations on the variable were secured during each of fourteen minima. For the determination of the period, the magnitudes in column nine of Table II corresponding to these observations were plotted; and, from these plots, epochs of geocentric minima were estimated with as much care as possible. These values, together with the initial epoch used by

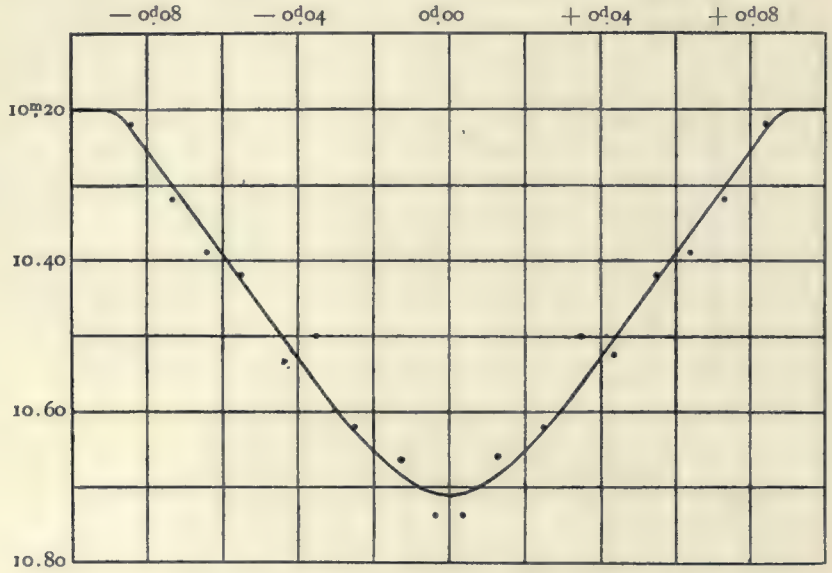

Fig. 2. Light Curve of RX Draconis

Seares and due to Blajko, are printed in the second column of Table VII. The decimals of a day are Greenwich mean time. The reduction to the sun, calculated for the zero day of each month, is shown in Table VI. It is practically negligible on account of the high latitude of the star.

A least square solution based upon the heliocentric epochs of observed minima, contained in the fourth column of Table VII, and weighted as shown in the next column, gave the following revised elements:

$$\text{Min.} = \text{J.D. } 2417502.411 + 1^{d}.893175E, \quad \text{G.M.T.}$$

The residuals of the epochs of observed minima from these elements are entered in the last column of Table VII. The probable error of the period is ± 3 in the last decimal place as printed above.

TABLE IX

PROBABLE ERRORS OF OBSERVATIONS DURING CHANGING LIGHT

Phase Limits	P. E.	No. Obs.	Phase Limits	P. E.	No. Obs.
± 0d000, ± 0d009	± 0m096	22	± 0d060, ± 0d069	± 0m063	19
± 0.010, ± 0.019	0.080	29	± 0.070, ± 0.079	0.039	9
± 0.020, ± 0.029	0.078	27	± 0.080, ± 0.089	0.065	10
± 0.030, ± 0.039	0.086	29	− 0.090, 0.000	0.077	106
± 0.040, ± 0.049	0.106	21	0.000, + 0.090	0.085	76
± 0.050, ± 0.059	±0.088	19	− 0.090, + 0.090	± 0.081	185

TABLE X

AVERAGE SYSTEMATIC DEVIATIONS OF INDIVIDUAL OBSERVED MINIMA FROM THE MEAN LIGHT CURVE

E	J. D.	Phase Limits	Sys. Dev.	Obsr.	No. Obs.
26	7551	− 0d066, + 0d076	+ 0m21	H	6
27	7553	− 0.044, + 0.001	+ 0.01	H	4
36	7570	− 0.066, + 0.020	+ 0.08	S	17
60	7615	− 0.042, − 0.006	+ 0.16	H	4
99	7689	− 0.016, + 0.059	+ 0.03	H	5
286	8043	− 0.035, + 0.073	− 0.03	H	13
333	8132	− 0.065, − 0.015	+ 0.01	H	4
335	8136	+ 0.011, + 0.030	− 0.13	S and H	8
344	8153	− 0.021, + 0.087	− 0.03	S and H	16
531	8507	− 0.064, − 0.030	− 0.11	H and *S*	8
755	8931	− 0.031, + 0.087	+ 0.07	*S*	10
763	8946	− 0.075, − 0.027	− 0.08	H	14
766	8952	− 0.044, + 0.056	− 0.07	H and *S*	34
774	8967	− 0.088, + 0.080	+ 0.06	H and *S*	38

TABLE XI

AVERAGE SYSTEMATIC DEVIATIONS AND PROBABLE ERRORS BY OBSERVERS

Measures on	Seares			Haynes			Shapley		
	Sys. Dev.	P. E.	No. Obs.	Sys. Dev.	P. E.	No. Obs.	Sys. Dev.	P. E.	No. Obs.
Comp. Stars	− 0m061	± 0m059	9	+ 0m008	± 0m055	87	− 0m015	± 0m073	29
Var. Const. L.	——	——	1	+ 0.002	± 0.069	94	− 0.003	± 0.082	33
Var. Chang. L.	+ 0.014	± 0.102	29	+ 0.002	± 0.077	114	− 0.029	± 0.078	42

For the formation of the mean light curve, the computed heliocentric epochs in the seventh column of Table VII, and also those corresponding to scattered observations during changing light, were reduced to the earth and used for the determination of phase values. These are printed in the fourth column of Table II. The magnitudes in column nine of the same table were then arranged in order of absolute phase, the algebraic sign being disregarded because the plots of individual minima did not indicate any asymmetry in the light change. The observed magnitudes thus arranged were combined to form nine normal places, displayed in the first half of Table VIII. The second half of the same table gives the coördinates of the adopted mean light curve. The curve itself is shown in Fig. 2. The period of light change is about 4^h20^m. The limits of variation are $10^m.20$ and $10^m.71$, giving a range of $0^m.51$.

The residuals from the mean light curve are in the last column of Table II. The average systematic deviation of the observations during decreasing light is $-0^m.002$; during increasing light, $-0^m.004$. These results justify the formation of the light curve without regard to the algebraic signs of the phase values. A detailed analysis of the probable errors of the measures secured during changing light is presented in Table IX. As the choice of phase limits here is the same as that made in computing the normal places, the average systematic deviations from the mean light curve for these limits are identical with the residuals in the fourth column of Table VIII. Both the deviations and the probable errors are in some cases rather large. The cause for this may be found by a reference to Table X, which gives the average systematic deviations from the mean light curve of individual observed minima. Several of these values are excessive. Reference to the epoch numbers in the first column shows that they cannot be explained by the assumption of alternately unequal minima. Reference to the phase limits in column three shows that they are not due to a poorly determined period. The preponderance of positive deviations for the early minima reminds one of the variation in the measured magnitude difference of the comparison stars. Both of these phenomena probably result from some gradual change of instrumental adjustment which affected the magnitude scale of the observations progressively.

The residuals of all observations were collected according to observers and the average systematic deviations and probable errors computed for each person. The resulting values are printed in Table XI. No particular significance should be attached to them, however, as the observing intervals were not coincident.

Columbia, Missouri, 1911, April 28. *E. S. Haynes.*

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33.

THE UNIVERSITY OF MISSOURI BULLETIN

ASTRONOMICAL SERIES

LAWS OBSERVATORY BULLETIN NO. 19

THE ALGOL VARIABLE RZ DRACONIS (26.1907)

1855.0 R.A. $18^h21^m10^s$ Dec. $+58°48'.6$[1]
1900.0 49 50.1

The variability of this star was discovered at Moscow by Madame Ceraski from plates by Blajko.[2] Visual observations by the latter established the Algol character of the variation, and the following elements, based upon four observed minima, were published by him in A. N. No. 4196:

$$\text{Min.} = \text{J. D. } 2417674.348 + 0^d.55088E, \quad \text{G.M.T.}$$

The star was observed by Ichinohe at the Yerkes Observatory from May 8 to July 6, 1907, and at Tokyo from November 10, 1907, to August 29, 1908. A preliminary note, based upon the earlier observations, appeared in A. N. No. 4188; and a complete discussion of the entire series was published in the Annals of the Tokyo Observatory.[3] In the final report Ichinohe gives the following elements, secured by combining twelve minima observed by him with the four determined by Blajko:

$$\text{Min.} = \text{J. D. } 2417673.24629 + 0^d.55088360E, \quad \text{G.M.T.},$$

for which the probable error of the epoch is $\pm 0^d.00022$, and of the period, $\pm 0^d.000038$.

RZ Draconis has been on the observing list of the Laws Observatory since May, 1907, and, up to the present date, its light has been measured 346 times on 28 nights. Nine of the measures were made by Seares, 210 by Haynes, and 127 by Shapley. Nine minima were either completely or partially determined by this series of observations. All measures were made with the Zeiss-wedge photometer in connection with the 7½-inch telescope, the method employed being the same as that of similar work discussed in previous Bulletins. These observations are printed in Table I, columns one, two, five, and six. The observers are indicated as in Bulletin No. 18. Column seven contains the magnitude differences of the comparison stars in the sense $b-a$

[1] Position by Ceraski, *Astronomische Nachrichten*, v. 174, p. 285.
[2] *Astronomische Nachrichten*, v. 174, p. 285.
[3] Vol. III, pt. 5.

TABLE I

OBSERVATIONS ON RZ DRACONIS

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	Comp. Stars	Δm	Mag.	O−C
1907, May 22	16h 18m	2417718.679	+0d257	H, *a*	37.5 38.2	−0m47	+0m07	10m01	+ 9
				H, *b*	33.2 38.2		+ 0.54	10.07	+ 15
	31	.688	+ 0.266	H, *a*	38.1 39.4		+ 0.13	10.07	+ 15
	59	.708	+ 0.286	H, *a*	38.8 40.2		+ 0.15	10.09	+ 17
	17 18	.721	+ 0.299	H, *a*	38.8 40.2		+ 0.15	10.09	+ 17
	55	.747	+ 0.325	H, *a*	37.4 38.3		+ 0.09	10.03	+ 11
	18 40	.778	+ 0.356	H, *a*	35.5 36.9		+ 0.15	10.09	+ 17
	19 41	.820	+ 0.398	H, *a*	36.2 37.6		+ 0.15	10.09	+ 17
June 14	16 20	7741.681	+ 0.122	H, *a*	31.7 31.6		− 0.01	9.93	+ 1
				H, *b*	28.4 32.0	− 0.37	+ 0.41	9.94	+ 2
15	17 12	7742.717	+ 0.056	H, *a*	32.7 33.4		+ 0.07	10.01	+ 2
16	18 36	7743.775	+ 0.013	H, *a*	33.3 38.6	− 0.44	+ 0.57	10.51	+ 17
				H, *b*	29.4 38.8		+ 1.03	10.56	+ 22
	19 16	.803	+ 0.041	H, *a*	31.4 33.5		+ 0.24	10.18	+ 10
17	21 10	7743.882	+ 0.018	H, *a*	37.8 40.0		+ 0.23	10.17	− 11
1908, Apr. 22	17 58	8054.749	+ 0.291	H, *a*	36.2 36.6	− 0.39	+ 0.05	9.99	+ 7
				H, *b*	32.6 36.6		+ 0.44	9.97	+ 5
Aug. 30	15 6	8184.629	+ 0.165	H, *a*	21.3 21.3	+ 0.31	0.00	9.94	+ 2
				H, *c*	24.2 21.0		− 0.34	9.98	+ 6
	58	.665	+ 0.201	H, *a*	22.8 22.8	+ 0.46	0.00	9.94	+ 2
				H, *c*	27.2 22.4		− 0.51	9.81	− 11
	16 48	.700	+ 0.236	H, *a*	22.1 23.1	+ 0.39	+ 0.11	10.05	+ 13
				H, *c*	25.8 23.0		− 0.29	10.03	+ 11
	17 40	.736	+ 0.272	H, *a*	22.5 23.0	+ 0.39	+ 0.05	9.99	+ 7
				H, *c*	26.3 23.0		− 0.34	9.98	+ 6
	18 30	.771	+ 0.307	H, *a*	22.4 22.8	+ 0.40	+ 0.05	9.99	+ 7
				H, *c*	26.2 22.6		− 0.37	9.95	+ 3
	19 58	.832	+ 0.368	H, *a*	22.1 21.9	+ 0.45	− 0.02	9.92	[illegible]
				H, *c*	26.4 22.0		− 0.46	9.86	− 6
	21 28	.894	+ 0.430	H, *a*	24.3 24.5	+ 0.38	+ 0.02	9.96	+ 4
				H, *c*	27.8 24.6		− 0.35	9.97	+ 5
	22 28	.936	+ 0.472	H, *a*	30.1 29.5	+ 0.33	− 0.07	9.87	− 5
				H, *c*	33.0 30.1		− 0.33	9.99	+ 7
Sept. 1	15 24	8186.642	− 0.026	H, *a*	24.6 25.7	+ 0.38	+ 0.12	10.06	− 18
				H, *c*	28.1 25.7		− 0.26	10.06	− 18
	16 42	.696	+ 0.028	H, *a*	25.5 26.0	+ 0.38	+ 0.06	10.00	− 19
				H, *c*	28.9 25.7		− 0.35	9.97	− 22
	18 40	.778	+ 0.110	H, *a*	21.8 22.1	+ 0.28	+ 0.03	9.97	+ 5
				H, *c*	24.5 22.0		− 0.26	10.06	+ 14

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	Comp. Stars	Δm	Mag.	O − C
1908, Sept. 1	19h 35m	2418186.816	+0d.148	H, a	26.6 26.1	+0m.22	−0m.06	9m.88	− 4
				H, c	28.6 25.3		−0.37	9.95	+ 3
	20 12	.842	+0.174	H, a	23.9 23.9	+0.35	0.00	9.94	+ 2
				H, c	27.2 23.8		−0.36	9.96	+ 4
	55	.872	+0.204	H, a	24.4 24.0	+0.36	−0.04	9.90	− 2
				H, c	27.7 24.6		−0.34	9.98	+ 6
	22 2	.918	+0.250	H, a	25.3 25.6	+0.44	+0.04	9.98	+ 6
				H, c	29.3 25.4		−0.43	9.89	− 3
5	13 40	8190.569	+0.045	S, a	22.0 20.2	−0.39	−0.19	9.75	− 30
				S, b	18.4 21.1		+0.29	9.82	− 23
	14 8	.589	+0.065	H, a	17.5 17.2	+0.47	−0.04	9.90	− 5
				H, c	21.7 17.9		−0.42	9.90	− 5
	15 24	.642	+0.118	H, a	18.3 18.4	+0.43	+0.01	9.95	+ 3
				H, c	22.3 18.4		−0.42	9.90	− 2
6	14 22	8191.599	−0.027	H, a	19.9 21.5	+0.35	+0.16	10.10	− 13
				H, c	23.2 21.2		−0.22	10.10	− 13
	36	.608	−0.018	H, a	21.1 23.7	+0.39	+0.28	10.22	− 10
				H, c	24.8 23.3		−0.15	10.17	− 15
	56	.622	−0.004	H, a	18.8 22.9	+0.46	+0.44	10.38	− 1
				H, c	23.1 22.9		−0.02	10.30	− 9
	15 18	.638	+0.012	H, a	19.1 22.6	+0.35	+0.38	10.32	− 3
				H, c	22.4 22.6		+0.03	10.35	0
	48	.658	+0.032	H, a	19.1 20.8	+0.38	+0.18	10.12	− 3
				H, c	22.6 20.8		−0.20	10.12	− 3
	16 42	.696	+0.070	H, a	21.5 21.2	+0.38	−0.03	9.91	− 2
				H, c	25.1 20.9		−0.44	9.88	− 5
	17 32	.731	+0.105	S, a	25.1 24.6	+0.36	−0.05	9.89	− 3
				S, c	28.4 24.1		−0.46	9.86	− 6
	18 17	.762	+0.136	H, a	20.0 19.5		−0.05	9.89	− 3
	43	.780	+0.154	H, a	22.9 22.4		−0.06	9.88	− 4
7	13 56	8192.581	+0.404	H, a	20.9 20.7	+0.38	−0.02	9.92	0
				H, c	24.5 20.7		−0.40	9.92	0
	14 39	.610	+0.433	S, a	23.4 22.5	+0.52	−0.09	9.85	− 7
				S, c	28.3 22.9		−0.57	9.75	− 17
	46	.615	+0.438	H, a	20.7 20.5	+0.38	−0.02	9.92	0
				H, c	24.3 20.2		−0.43	9.89	− 3
	15 14	.635	+0.458	S, a	25.8 25.5	+0.30	−0.03	9.91	− 1
				S, c	28.5 25.0		−0.39	9.93	+ 1
	22	.640	+0.463	H, a	23.2 21.9	+0.27	−0.14	9.80	− 12
				H, c	25.8 21.3		−0.48	9.84	− 8
	53	.662	−0.065	S, a	27.6 28.4	+0.48	+0.08	10.02	+ 10
				S, c	31.9 28.7		−0.36	9.96	+ 4

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	Comp. Stars	Δm	Mag.	O − C
1908, Sept. 7	16h 2m	2418192.668	− 0d.059	H, *a*	23.5 23.3		− 0m.02	9m.92	− 1
				H, *c*	26.3 23.2	+ 0m.29	− 0.32	10.00	+ 7
	13	.676	− 0.051	*S*, *a*	27.0 26.8		− 0.02	9.92	− 8
				S, *c*	30.0 26.6	+ 0.33	− 0.37	9.95	− 5
	22	.682	− 0.045	H, *a*	22.4 23.4		+ 0.11	10.05	0
				H, *c*	26.0 23.4	+ 0.38	− 0.27	10.05	0
	29	.687	− 0.040	*S*, *a*	29.0 29.4		+ 0.04	9.98	− 12
				S, *c*	32.4 29.8	+ 0.38	− 0.29	10.03	− 7
	37	.692	− 0.035	H, *a*	22.1 24.6		+ 0.26	10.20	+ 5
				H, *c*	26.1 24.8	+ 0.42	− 0.14	10.18	+ 3
	47	.699	− 0.028	*S*, *a*	28.4 29.6		+ 0.14	10.08	− 14
				S, *c*	31.6 29.7	+ 0.36	− 0.21	10.11	− 11
	55	.705	− 0.022	H, *a*	22.4 25.5		+ 0.32	10.26	− 2
				H, *c*	26.4 25.4	+ 0.42	− 0.11	10.21	− 7
	17 4	.711	− 0.016	*S*, *a*	27.1 29.4		+ 0.25	10.19	− 15
				S, *c*	30.2 29.2	+ 0.34	− 0.11	10.21	− 13
	12	.717	− 0.010	H, *a*	24.2 28.0		+ 0.41	10.35	− 4
				H, *c*	27.7 28.0	+ 0.38	+ 0.03	10.35	− 4
	22	.724	− 0.003	*S*, *a*	28.3 31.0		+ 0.30	10.24	− 15
				S, *c*	31.5 31.6	+ 0.36	+ 0.01	10.33	− 6
	30	.729	+ 0.002	H, *a*	24.3 28.9		+ 0.50	10.44	+ 5
				H, *c*	28.3 28.4	+ 0.43	+ 0.01	10.33	− 6
	40	.736	+ 0.009	*S*, *a*	28.9 32.6		+ 0.42	10.36	− 3
				S, *c*	31.7 32.3	+ 0.31	+ 0.07	10.39	0
	48	.742	+ 0.015	H, *a*	23.5 26.8		+ 0.35	10.29	− 2
				H, *c*	26.8 26.4	+ 0.35	− 0.05	10.27	− 4
	56	.747	+ 0.020	*S*, *a*	27.7 30.8		+ 0.34	10.28	+ 1
				S, *c*	31.0 31.3	+ 0.36	+ 0.04	10.36	+ 9
	18 4	.753	+ 0.026	H, *a*	22.4 24.9		+ 0.26	10.20	0
				H, *c*	25.5 24.9	+ 0.32	− 0.06	10.26	+ 6
	12	.758	+ 0.031	*S*, *a*	28.3 31.6		+ 0.37	10.31	+ 15
				S, *c*	32.1 31.0	+ 0.43	− 0.13	10.19	+ 3
	18	.762	+ 0.035	H, *a*	22.1 23.8		+ 0.18	10.12	− 1
				H, *c*	25.3 23.8	+ 0.33	− 0.15	10.17	+ 4
	27	.769	+ 0.042	*S*, *a*	27.8 29.8		+ 0.22	10.16	+ 9
				S, *c*	31.0 29.8	+ 0.35	− 0.13	10.19	+ 12
	33	.773	+ 0.046	H, *a*	22.5 23.6		+ 0.11	10.05	0
				H, *c*	25.7 23.7	+ 0.33	− 0.21	10.11	+ 6
	40	.778	+ 0.051	*S*, *a*	28.7 29.9		+ 0.13	10.07	+ 5
				S, *c*	31.7 29.8	+ 0.33	− 0.21	10.11	+ 9
	48	.783	+ 0.056	H, *a*	22.2 23.3		+ 0.12	10.06	+ 7
				H, *c*	26.0 23.0	+ 0.40	− 0.31	10.01	+ 2

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	Comp. Stars	Δm	Mag.	O − C
1908, Sept. 7	18h 56m	2418192.789	+ 0d062	S, a	28.0 28.8		+ 0m09	10m03	+ 7
				S, c	31.3 28.5	+ 0m37	− 0.31	10.01	+ 5
	19 3	.794	+ 0.067	H, a	23.0 22.9		− 0.01	9.93	− 1
				H, c	26.4 22.8	+ 0.35	− 0.37	9.95	+ 1
	11	.799	+ 0.072	S, a	30.3 30.0		− 0.03	9.91	− 2
				S, c	33.4 30.3	+ 0.35	− 0.35	9.97	+ 4
	18	.804	+ 0.077	H, a	23.6 23.1		− 0.05	9.89	− 3
				H, c	27.0 23.6	+ 0.36	− 0.36	9.96	+ 4
	48	.825	+ 0.098	S, a	29.0 28.3		− 0.08	9.86	− 6
				S, c	32.1 28.7	+ 0.35	− 0.38	9.94	+ 2
	54	.829	+ 0.102	H, a	24.9 24.2		− 0.07	9.87	− 5
				H, c	28.3 24.2	+ 0.37	− 0.44	9.88	− 4
13	13 32	8198.564	+ 0.328	S, a	26.0 25.7		− 0.03	9.91	− 1
				S, c	28.6 25.2	+ 0.29	− 0.38	9.94	+ 2
Oct. 3	14 22	8218.599	− 0.020	H, a	21.9 24.6		+ 0.28	10.22	− 8
	28	.603	− 0.016	S, a	26.3 29.8		+ 0.39	10.33	− 1
	58	.624	+ 0.005	S, a	26.1 30.1		+ 0.44	10.38	− 1
				S, c	30.0 29.7	+ 0.43	+ 0.03	10.35	− 4
	15 8	.631	+ 0.012	H, a	21.1 23.8		+ 0.29	10.23	− 12
				H, c	24.1 23.6	+ 0.32	− 0.05	10.27	− 8
	16 4	.669	+ 0.050	S, a	27.4 27.6		+ 0.03	9.97	− 5
				S, c	32.3 28.1	+ 0.55	− 0.47	9.85	− 17
	17 25	.726	+ 0.107	S, a	29.4 27.8		− 0.17	9.77	− 15
				S, c	34.5 28.9	+ 0.57	− 0.62	9.70	− 22
	34	.732	+ 0.113	H, a	23.9 23.5		− 0.04	9.90	− 2
				H, c	26.9 23.3	+ 0.32	− 0.38	9.94	+ 2
8	14 40	8223.611	+ 0.034	S, a	26.6 28.0		+ 0.15	10.09	− 4
				S, c	30.3 27.7	+ 0.40	− 0.28	10.04	− 9
	47	.616	+ 0.039	H, a	18.8 19.2		+ 0.04	9.98	− 12
				H, c	21.5 18.9	+ 0.28	− 0.27	10.05	− 5
	15 58	.665	+ 0.088	H, a	19.1 18.9		− 0.02	9.92	0
				H, c	23.0 19.0	+ 0 42	− 0.43	9.89	− 3
14	15 51	8229.660	+ 0.023	S, a	37.4 36.5		− 0.10	9.84	− 39
				S, c	39.5 36.8	+ 0.22	− 0.28	10.04	− 19
1909, June 20	15 2	8478.626	− 0.006	H, a	24.3 27.8		+ 0.38	10.32	− 7
				H, c	27.0 27.8	+ 0.29	+ 0.09	10.41	+ 2
21	16 34	8479.690	− 0.044	H, a	29.7 31.3		+ 0.18	10.12	+ 6
				H, c	31.8 31.1	+ 0.24	− 0.09	10.23	+ 17
	42	.696	− 0.038	H, a	28.7 30.8		+ 0.23	10.17	+ 5
				H, c	31.5 31.0	+ 0.31	− 0.06	10.26	+ 14
	17 14	.718	− 0.016	H, a	28.4 31.8		+ 0.39	10.33	− 3
				H, c	31.7 32.2	+ 0.37	+ 0.06	10.38	+ 4

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	Comp. Stars	Δm	Mag.	O − C
1909, June 21	$17^h\ 23^m$	2418479.724	−0.010	H, *a*	27.5 32.1			10.36	− 3
				H, *c*	31.3 31.8	+0.43	+0.06	10.38	− 1
	54	.746	+0.012	H, *a*	30.0 33.7		+0.42	10.36	+ 1
				H, *c*	33.4 33.8	+0.38	+0.05	10.37	+ 2
	18 2	.751	+0.017	H, *a*	29.3 33.6		+0.49	10.43	+13
				H, *c*	33.0 33.3	+0.42	+0.03	10.35	+ 5
	34	.774	+0.040	H, *a*	30.1 32.3		+0.25	10.19	+10
				H, *c*	33.4 32.2	+0.37	−0.13	10.19	+10
	42	.779	+0.045	H, *a*	30.0 31.7		+0.19	10.13	+ 8
				H, *c*	33.0 31.8	+0.34	−0.13	10.19	+14
	19 22	.807	+0.073	H, *a*	35.8 35.1		−0.08	9.86	− 6
				H, *c*	39.3 35.6	+0.36	−0.38	9.94	+ 2
July 13	16 34	8501.690	+0.472	H, *a*	22.4 22.7		+0.04	9.98	+ 6
				H, *c*	24.8 22.8	+0.25	−0.20	10.12	+20
	42	.696	+0.478	H, *a*	22.4 22.6		+0.03	9.97	+ 5
				H, *c*	24.6 22.5	+0.23	−0.21	10.11	+19
	17 20	.722	−0.047	H, *a*	21.3 22.5		+0.14	10.08	+ 5
				H, *c*	24.3 22.2	+0.32	−0.22	10.10	+ 7
17	14 27	8505.602	−0.023	H, *a*	30.5 34.2		+0.42	10.36	+ 9
				H, *c*	33.9 34.0	+0.39	+0.01	10.33	+ 6
	35	.608	−0.017	H, *a*	31.2 34.2		+0.34	10.28	− 5
				H, *c*	34.4 34.5	+0.36	+0.01	10.23	
	21 52	.911	+0.286	H, *a*	29.5 29.5		0.00	9.94	+ 2
				H, *c*	32.2 29.3	+0.31	−0.33	9.99	+ 7
19	18 4	8507.753	+0.475	H, *a*	27.3 26.2		−0.12	9.82	−10
				H, *c*	30.2 25.9	+0.32	−0.47	9.85	− 7
	15	.760	+0.482	H, *a*	27.8 27.9		+0.01	9.95	+ 3
				H, *c*	30.8 27.0	+0.33	−0.42	9.90	− 2
	27	.769	−0.060	H, *a*	27.4 27.4		0.00	9.94	+ 1
				H, *c*	30.1 27.2	+0.30	−0.32	10.00	+ 7
	37	.776	−0.053	H, *a*	27.6 28.1		+0.05	9.99	+ 1
				H, *c*	30.8 28.3	+0.35	−0.28	10.04	+ 6
	48	.783	−0.046	H, *a*	27.7 28.7		+0.11	10.05	+ 1
				H, *c*	30.8 29.1	+0.34	−0.19	10.13	+ 9
	57	.790	−0.039	H, *a*	26.8 28.7		+0.21	10.15	+ 4
				H, *c*	30.5 29.1	+0.40	−0.15	10.17	+ 6
	19 10	.799	−0.030	H, *a*	28.2 29.8		+0.18	10.12	− 8
				H, *c*	30.9 29.9	+0.30	−0.11	10.21	+ 1
	21	.806	−0.023	H, *a*	29.0 31.6		+0.29	10.23	− 4
				H, *c*	32.8 31.9	+0.43	−0.10	10.22	− 5
	31	.813	−0.016	H, *a*	29.3 32.7		+0.39	10.33	− 1
				H, *c*	32.9 32.6	+0.41	−0.03	10.29	− 5

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	Comp. Stars	Δm	Mag.	O − C
1909, July 19	19h 39m	2418507.819	−0.010	H, *a*	29.6 33.0		+0m.38	10m.32	− 7
				H, *c*	32.8 33.2	+0m.36	+ 0.04	10.36	− 3
	49	.826	− 0.003	H, *a*	29.0 33.3		+ 0.48	10.42	+ 3
				H, *c*	33.2 33.4	+ 0.47	+ 0.02	10.34	− 5
	20 0	.833	+ 0.004	H, *a*	29.5 33.3		+ 0.43	10.37	− 2
				H, *c*	33.0 33.4	+ 0.40	+ 0.04	10.36	− 3
	11	.841	+ 0.012	H, *a*	28.2 32.1		+ 0.44	10.38	+ 3
				H, *c*	31.5 32.0	+ 0.37	+ 0.06	10.38	+ 3
	22	.849	+ 0.020	H, *a*	29.8 33.8		+ 0.45	10.39	+ 12
				H, *c*	33.5 33.8	+ 0.42	+ 0.03	10.35	+ 8
	36	.858	+ 0.029	H, *a*	30.0 32.1		+ 0.24	10.18	0
				H, *c*	32.7 31.9	+ 0.31	− 0.09	10.23	+ 5
	44	.864	+ 0.035	H, *a*	28.8 30.9		+ 0.23	10.17	+ 4
				H, *c*	31.8 30.7	+ 0.34	− 0.13	10.19	+ 6
	52	.869	+ 0.040	H, *a*	29.7 31.4		+ 0.19	10.13	+ 4
				H, *c*	32.8 31.3	+ 0.35	− 0.17	10.15	+ 6
	21 10	.882	+ 0.053	H, *a*	30.5 31.5		+ 0.11	10.05	+ 4
				H, *c*	33.9 31.5	+ 0.38	− 0.27	10.05	+ 4
	20	.889	+ 0.060	H, *a*	30.2 31.2		+ 0.11	10.05	+ 8
				H, *c*	33.7 31.1	+ 0.40	− 0.30	10.02	+ 5
	28	.894	+ 0.065	H, *a*	30.6 30.4		− 0.03	9.91	− 3
				H, *c*	33.9 30.2	+ 0.37	− 0.42	9.90	− 4
	48	.908	+ 0.079	H, *a*	29.5 29.4		− 0.02	9.92	0
				H, *c*	32.8 29.3	+ 0.37	− 0.40	9.92	0
	54	.912	+ 0.083	H, *a*	29.5 29.2		− 0.03	9.91	− 1
				H, *c*	32.6 29.2	+ 0.36	− 0.39	9.93	+ 1
Sept. 26	15 2	8576.626	− 0.063	*S*, *a*	22.2 22.0		− 0.02	9.92	0
				S, *c*	28.5 21.8	+ 0.68	− 0.72	9.61	− 32
Oct. 2	14 17	8582.595	+ 0.397	*S*, *a*	21.3 20.8		− 0.05	9.89	− 3
				S, *c*	26.7 20.7	+ 0.58	− 0.64	9.68	− 24
	15 38	.651	+ 0.453	*S*, *a*	25.3 24.7		− 0.06	9.88	− 4
				S, *c*	29.1 24.5	+ 0.42	− 0.50	9.82	− 10
	16 58	.707	− 0.042	*S*, *a*	27.8 28.5		+ 0.08	10.02	− 6
	17 2	.710	− 0.039	*S*, *a*	28.3 29.1		+ 0.09	10.03	− 8
	12	.717	− 0.032	*S*, *a*	28.5 29.9		+ 0.15	10.09	− 9
	17	.720	− 0.029	*S*, *a*	29.3 31.4		+ 0.24	10.18	− 3
	25	.726	− 0.023	*S*, *a*	28.8 31.7		+ 0.32	10.26	− 1
				S, *c*	33.2 32.0	+ 0.49	− 0.13	10.19	− 8
	34	.732	− 0.017	*S*, *a*	28.5 32.4		+ 0.43	10.37	+ 4
	41	.737	− 0.012	*S*, *a*	30.3 35.1		+ 0.54	10.48	+ 10
	54	.746	− 0.003	*S*, *a*	28.6 34.8		+ 0.69	10.63	+ 24
	58	.749	0.000	*S*, *a*	29.6 35.6		+ 0.67	10.61	+ 22

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	Comp. Stars	Δm	Mag.	O − C
1909, Oct. 2	18h 34m	2418582.774	+0d025	S, a	31.5 34.7		+0m36	10m30	+ 8
	19 1	.792	+ 0.043	S, a	30.2 31.1		+ 0.10	10.04	— 2
	7	.797	+ 0.048	S, a	31.5 32.1		+ 0.07	10.01	— 2
	32	.814	+ 0.065	S, a	32.4 32.3		— 0.01	9.93	— 1
1910, June 13	14 51	8836.619	+ 0.467	S, a	37.0 37.1	+0m24	+ 0.01	9.95	+ 3
				S, c	39.4 36.7		— 0.27	10.05	+ 13
	15 0	.625	+ 0.473	S, a	39.3 38.4	+ 0.26	— 0.09	9.85	— 7
				S, c	41.6 38.4		— 0.35	9.97	+ 5
	16 10	.674	— 0.029	S, a	38.8 41.1	+ 0.28	+ 0.25	10.19	— 2
				S, c	41.3 41.0		— 0.04	10.28	+ 7
	19	.680	— 0.023	S, c	41.5 41.6		+ 0.01	10.33	+ 6
	23	.683	— 0.020	S, c	41.6 42.2		+ 0.07	10.39	+ 9
	26	.685	— 0.018	S, c	42.2 42.9		+ 0.09	10.41	+ 9
	42	.696	— 0.007	S, c	42.5 43.5		+ 0.12	10.44	+ 5
	50	.701	— 0.002	S, a	38.8 42.8	+ 0.37	+ 0.46	10.40	+ 1
				S, c	42.1 43.0		+ 0.11	10.43	+ 4
	17 1	.709	+ 0.006	S, c	42.1 43.1		+ 0.12	10.44	+ 5
	12	.717	+ 0.014	S, c	41.3 42.4		+ 0.13	10.45	+ 12
	24	.725	+ 0.022	S, c	42.0 40.6		— 0.16	10.16	— 8
	27	.727	+ 0.024	S, c	42.2 40.7		— 0.17	10.15	— 7
	34	.732	+ 0.029	S, a	38.8 40.1	+ 0.37	+ 0.14	10.08	— 9
				S, c	42.1 39.7		— 0.27	10.05	— 12
	43	.738	+ 0.035	S, a	37.6 38.3		+ 0.07	10.01	— 12
	18 0	.750	+ 0.047	S, a	35.1 34.8		— 0.03	9.91	— 13
1911, April 21	15 26	9148.643	+ 0.143	S, a	37.0 37.5	+ 0.53	+ 0.05	9.99	+ 7
				S, c	41.9 37.1		— 0.52	9.80	— 12
23	14 13	9150.592	+ 0.440	S, a	23.4 22.3		— 0.12	9.82	— 10
	20	.597	+ 0.445	S, a	25.5 25.4	+ 0.61	— 0.01	9.93	+ 1
				S, c	31.0 25.8		— 0.57	9.75	— 17
	55	.622	+ 0.470	S, a	25.9 26.6		+ 0.08	10.02	+ 10
	15 0	.625	+ 0.473	S, a	25.6 26.0		+ 0.04	9.98	+ 6
	44	.656	— 0.047	S, a	25.0 26.4		+ 0.15	10.09	+ 5
	51	.660	— 0.043	S, a	24.5 26.6	+ 0.52	+ 0.23	10.17	+ 10
				S, c	29.3 26.1		— 0.35	9.97	— 10
	58	.665	— 0.038	S, a	24.6 27.0		+ 0.26	10.20	+ 8
	16 35	.691	— 0.012	S, a	21.6 26.8		+ 0.55	10.49	+ 11
30	15 7	9157.630	+ 0.316	S, a	9.8 9.6		— 0.02	9.92	0
	14	.635	+ 0.321	S, a	10.8 10.8	+ 0.66	0.00	9.94	+ 2
				S, c	15.7 11.0		— 0.63	9.69	— 23
	30	.646	+ 0.332	S, a	10.6 10.8		+ 0.03	9.97	+ 5
May 2	15 46	9159.657	+ 0.140	S, a	45.6 45.8		+ 0.02	9.96	+ 4

Date	G.M.T.	Julian Day	Phase	Obs. Star	Readings	Comp. Stars	Δm	Mag.	O − C
1911, May 4	$14^h\ 36^m$	2419161.608	$+0^d.438$	*S*, *a*	49.2 48.7	$+0^m.57$	$-0^m.07$	$9^m.87$	− 5
				S, *c*	53.8 48.1		− 0.71	9.61	− 31
	46	.615	+ 0.445	H, *a*	40.0 39.9	+ 0.41	− 0.01	9.93	+ 1
				H, *c*	43.5 39.8		− 0.43	9.89	− 3
	15 20	.639	+ 0.469	H, *a*	40.3 40.1	+ 0.43	− 0.02	9.92	0
				H, *c*	44.0 40.3		− 0.43	9.89	− 3
	28	.644	+ 0.474	*S*, *a*	46.1 46.0	+ 0.47	− 0.01	9.93	+ 1
				S, *c*	50.1 46.6		− 0.41	9.91	− 1
	45	.656	− 0.065	H, *a*	38.9 38.9	+ 0.37	0.00	9.94	+ 2
				H, *c*	42.2 38.9		− 0.37	9.95	+ 3
	16 5	.670	− 0.051	*S*, *a*	46.2 46.4	+ 0.42	+ 0.02	9.96	− 4
				S, *c*	49.8 46.3		− 0.41	9.91	− 9
	13	.676	− 0.045	H, *a*	40.0 40.6	+ 0.37	+ 0.07	10.01	− 4
				H, *c*	43.2 40.0		− 0.37	9.95	− 10
	22	.682	− 0.039	*S*, *a*	46.2 48.□	+ 0.44	+ 0.20	10.14	+ 3
				S, *c*	49.9 47.5		− 0.29	10.03	− 8
	31	.688	− 0.033	H, *a*	39.5 41.7	+ 0.39	+ 0.24	10.18	+ 1
				H, *c*	42.9 42.4		− 0.06	10.26	+ 9
	40	.694	− 0.027	*S*, *a*	44.6 48.6	+ 0.43	+ 0.44	10.38	+ 15
				S, *c*	48.5 48.5		0.00	10.32	+ 9
	47	.699	− 0.022	H, *a*	38.9 43.4	+ 0.42	+ 0.51	10.45	+ 17
				H, *c*	42.6 43.5		+ 0.11	10.43	+ 15
	54	.704	− 0.017	*S*, *a*	44.3 49.1	+ 0.43	+ 0.54	10.48	+ 15
				S, *c*	48.2 49.4		+ 0.15	10.47	+ 14
	17 2	.710	− 0.011	H, *a*	38.9 43 9	+ 0.46	+ 0.57	10.51	+ 12
				H, *c*	42.9 44.4		+ 0.16	10.48	+ 9
	11	.716	− 0.005	*S*, *a*	43.9 48.7	+ 0.49	+ 0.53	10.47	+ 8
				S, *c*	48.3 48.8		+ 0.06	10.38	− 1
	20	.722	+ 0.001	*S*, *a*	45.0 48.9	+ 0.37	+ 0.44	10.38	− 1
				S, *c*	48.3 48.8		+ 0.06	10.38	− 1
	28	.728	+ 0.007	*S*, *a*	45.2 49.2	+ 0.43	+ 0.46	10.40	+ 1
				S, *c*	49.0 48.9		− 0.01	10.31	− 8
	41	.737	+ 0.016	H, *a*	38.8 42.□	+ 0.30	+ 0.36	10.30	− 1
				H, *c*	41.5 41.4		− 0.01	10.31	□
	51	.744	+ 0.023	*S*, *a*	44.6 47.4	+ 0.37	+ 0.30	10.24	+ 1
				S, *c*	48.0 47.5		− 0.05	10.27	+ 4
	58	.749	+ 0.028	H, *a*	38.9 41.5	+ 0.39	+ 0.29	10.23	+ 5
				H, *c*	42.3 41.9		− 0.05	10.27	+ 9
	18 4	.753	+ 0.032	*S*, *a*	42.3 45.2	+ 0.51	+ 0.32	10.26	+ 11
				S, *c*	46.9 45.8		− 0.12	10.20	+ 5
	12	.758	+ 0.037	H, *a*	37.9 40.0	+ 0.38	+ 0.22	10.16	+ 5
				H, *c*	41.4 39.6		− 0.20	10.12	+ 1

Date	G.M.T	Julian Day	Phase	Obs. Star	Readings		Comp. Stars	Δm	Mag.	O − C
1911, May 4	$18^h\ 19^m$	2419161.763	$+0^d.042$	*S*, *a*	43.6	44.9	$+0^m.41$	$+0^m.14$	$10^m.08$	+ 1
				S, *c*	47.4	44.9		−0.27	10.05	− 2
	26	.768	+0.047	H, *a*	39.0	40.2	+0.36	+0.13	10.07	+ 3
				H, *c*	42.2	40.2		−0.23	10.09	+ 5
	34	.774	+0.053	*S*, *a*	41.2	41.7	+0.44	+0.06	10.00	0
				S, *c*	45.0	41.9		−0.35	9.97	− 3
	48	.783	+0.062	H, *a*	40.1	40.8	+0.41	+0.08	10.02	+ 6
				H, *c*	43.6	39.9		−0.43	9.89	− 7
	56	.789	+0.068	*S*, *a*	44.4	44.8	+0.41	+0.05	9.99	+ 5
				S, *c*	48.1	44.6		−0.38	9.94	0
	19 30	.812	+0.091	H, *a*	41.8	41.0	+0.35	−0.10	9.84	− 8
				H, *c*	44.9	41.2		−0.43	9.89	− 3
	37	.817	+0.096	*S*, *a*	43.9	43.4	+0.41	−0.06	9.88	− 4
				S, *c*	47.6	43.8		−0.42	9.90	− 2

REMARKS

1907, May 22—Moon.
June 14—Sky slightly white.
16—Sky white, air unsteady.
17—Sky white.
1908, Sept. 13—Thin haze.
Oct. 8—Sky very white, moon.
1909, Oct. 2—Moon after 16^h.
1911, Apr. 30—Clouds at end.

TABLE II

MAGNITUDE OF COMPARISON STAR *a*

Star	BD. No.	PD. No.	PD. Mag.	Mag. of *a*			No. Obs.	
				Haynes	Shapley	Mean	H	*S*
p	+58°1789	III 2715	8.53	9.88	10.12	10.00	4	4
q	57 1857	2752	7.77	9.83	9.96	9.90	4	4
r	59 1898	2778	7.26	9.88	9.97	9.93	4	4
s	+57 1882	2806	7.52	9.83	10.03	9.93	4	4
			Means	9.86	10.02	9.94		

and $c-a$, while the next column gives like values for the difference, variable minus comparison star.

The field around RZ Draconis is shown in Fig. 1. This chart, which is only approximate, was made by plotting the coördinates of the brighter stars as given in the BD. and inserting estimated positions of the variable and the faint stars in its immediate neighborhood. The variable is indicated by the letter *v*, and the comparison stars by *a*, *b*, and *c*. The only one of these stars included in the BD. is *b*, which is No.+58°1808 in that catalogue.

For the determination of the magnitude difference, $b-a$, five measures are available, four of which are by Haynes and one by Shapley. The resulting value is $-0^m.41$, the probable error of a single observation being $\pm 0^m.028$. The accepted value of the difference $c-a$ is $+0^m.38$, which is the mean of 146 observations, 4 by Seares, 95 by Haynes, and 47 by Shapley. The probable error of a single observation referred to this mean is $\pm 0^m.053$. The magnitude of the comparison stars was determined by referring a to four PD. stars. The details of this comparison are given

TABLE III

OBSERVED MINIMA AND THEIR REPRESENTATION

E	Obs. Geoc. Min.	Red. to Sun	Obs. Helioc. Min.	Wt.	No. Obs.	O – C	Observers
0	$7673^d.244$	$0^d.000$	$7673^d.244$	1	7	$-0^d.007$	Blajko
2	7674.347	0.000	7674.347	3	16	−0.005	"
53	7702.442	0.000	7702.442	4	20	−0.005	"
62	7707.402	0.000	7707.402	1	9	−0.003	"
126	———	——	7742.657	1	13	−0.004	Ichinohe
128	———	——	7743.770	1	6	+0.007	"
130	———	——	7744.862	4	18	+0.003	"
862	———	——	8148.110	4	23	+0.003	"
873	———	——	8154.167	4	18	+0.001	"
875	———	——	8155.263	3	10	−0.005	"
880	———	——	8158.020	1	9	−0.003	"
882	———	——	8159.133	1	6	+0.009	"
891	———	——	8164.086	1	4	+0.004	"
898	———	——	8167.948	0	3	+0.010	"
909	———	——	8173.996	0	1	−0.002	"
911	———	——	8175.089	1	4	−0.011	"
941	8191.632	+0.001	8191.633	3	12	+0.007	H
943	8192.734	+0.001	8192.735	8	50	+0.007	H and *S*
990	8218.615	0.000	8218.615	1	8	−0.004	S and H
1464	8479.735	+0.001	8479.736	3	18	+0.001	H
1515	8507.833	+0.001	8507.834	8	36	+0.004	H
1651	8582.752	0.000	8582.752	3	14	+0.003	*S*
2112	8836.694	+0.001	8836.695	4	16	−0.008	*S*
2682	9150.698	0.000	9150.698	1	5	−0.005	*S*
2702	9161.718	0.000	9161.718	6	44	−0.003	H and *S*

in Table II. The disagreement between the values of the magnitude of a, secured by the two observers, does not result from accidental errors; but arises from a systematic difference in the estimate of the light interval, for the probable errors of the two means are $\pm 0^m.022$ and $\pm 0^m.025$, respectively. The mean of the two determinations, 9.94, is accepted as the magnitude of a. Applying to this value the comparison star differences derived above, the resulting magnitude of b is 9.53,

and of c is 10.32. These values, when combined with the magnitude differences, variable minus comparison star, in column eight of Table I, give the magnitudes of the variable entered in the next column.

In order to obtain revised elements, the observed magnitudes of the variable were plotted and epochs of geocentric minima determined. These, together with the minima due to Blajko, are in column two of Table III. The corresponding heliocentric minima, and those of Ichinohe, are printed in column four of the same table. In assigning weights to the individual minima, the number of observations, their consistency, and the length of the observing interval were considered. A least square reduction, based upon this data, gave the following formula:

$$\text{Min.} = \text{J. D. } 2417673.2507 + 0^{d}.5508772E, \quad \text{G.M.T.}$$

The residuals of the observed minima from these elements are in column seven of Table III. The

TABLE IV

REDUCTION TO THE SUN

Day	Red.	Day	Red.
Jan. 0	$-0^{d}.0008$	July 0	$+0^{d}.0008$
Feb. 0	−0.0007	Aug. 0	+0.0008
Mar. 0	−0.0005	Sept. 0	+0.0006
Apr. 0	−0.0001	Oct. 0	+0.0002
May 0	+0.0003	Nov. 0	−0.0002
June 0	+0.0006	Dec. 0	−0.0006

TABLE V

NORMAL PLACES

Phase	Mag.	No. Obs.	O − C	Phase	Mag.	No. Obs.	O − C
$-0^{d}.062$	9.92	10	$0^{m}.00$	$+0^{d}.008$	10.38	5	$-0^{m}.01$
0.052	9.97	6	−0.02	0.012	10.37	10	+0.02
0.046	10.06	9	+0.02	0.016	10.32	8	+0.01
0.040	10.11	15	+0.01	0.022	10.22	9	−0.02
0.033	10.18	7	+0.01	0.027	10.17	8	−0.03
0.028	10.20	9	−0.02	0.031	10.15	12	−0.01
0.023	10.26	11	−0.01	0.036	10.11	9	−0.01
0.019	10.32	12	+0.01	0.042	10.08	12	+0.01
0.015	10.34	9	−0.01	0.046	10.07	8	+0.02
0.010	10.39	8	0.00	0.053	10.01	11	0.00
−0.005	10.39	10	+0.01	0.063	9.96	11	+0.01
+0.001	10.40	13	−0.01	+0.070	9.93	10	0.00

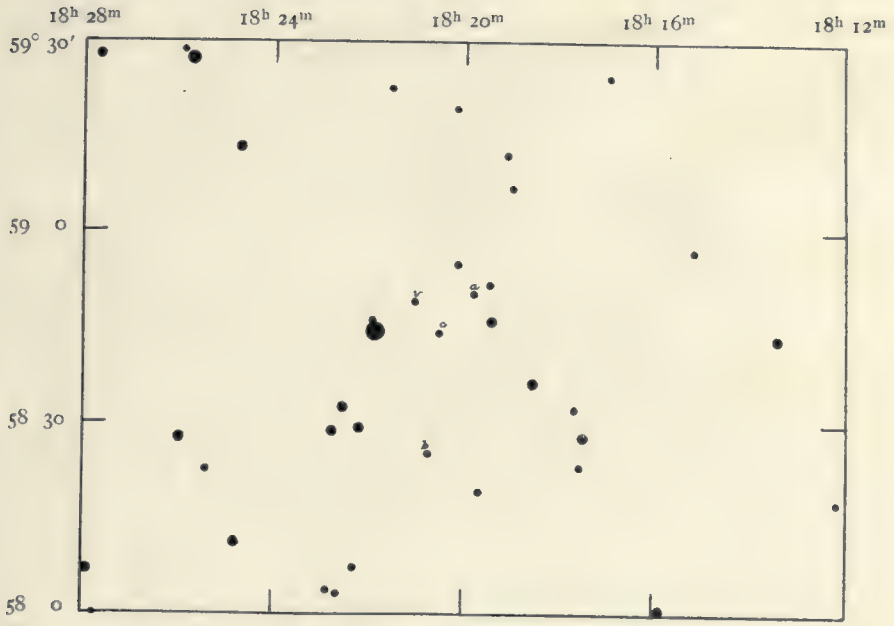


Fig. 1—Field for RZ Draconis

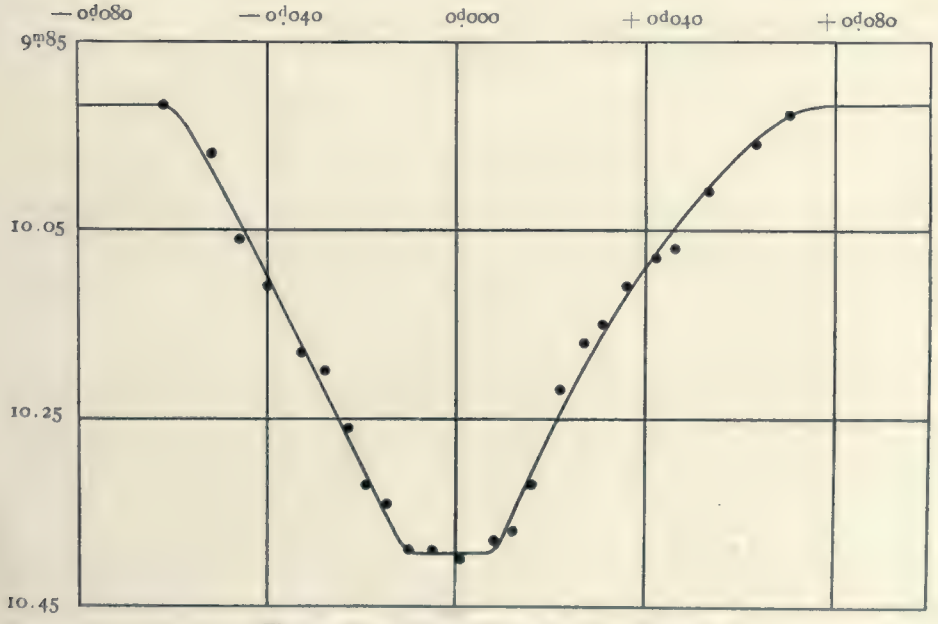


Fig. 2—Mean Light Curve for RZ Draconis

PLATE I—The Algol Variable RZ Draconis

probable error of the period, computed from the residuals, is ± 5 in the last decimal place as written above.

Geocentric epochs, corresponding to all observations, were computed from the revised elements and used to form the phase values in the fourth column of Table I. All are referred to the next preceding minimum, except those for observations during decreasing light, in which case they are referred to the minimum immediately following and a minus sign is used. For the formation of the mean light curve, the observations made during changing light were combined into 24 normal places. Table V contains these normal places and the number of observations

TABLE VI

COÖRDINATES OF THE MEAN LIGHT CURVE

Phase	Mag.	Phase	Mag.	Phase	Mag.
— 0.d065	9.920	— 0.d015	10.350	+ 0.d035	10.127
0.060	9.927	0.010	10.390	0.040	10.088
0.055	9.958	— 0.005	10.392	0.045	10.053
0.050	10.006	0.000	10.392	0.050	10.022
0.045	10.052	+ 0.005	10.392	0.055	9.996
0.040	10.101	0.010	10.371	0.060	9.969
0.035	10.152	0.015	10.315	0.065	9.946
0.030	10.200	0.020	10.266	0.070	9.930
0.025	10.251	0.025	10.216	+ 0.075	9.920
— 0.020	10.302	+ 0.030	10.168		

TABLE VII

AVERAGE SYSTEMATIC DEVIATIONS FROM THE MEAN LIGHT CURVE

For Obs. during	Average Sys. Dev.	No. Obs.	For Obs. during	Average Sys. Dev.	No. Obs.
Decreasing Light	+ 0.m001	90	Changing Light	0.m000	232
Const. Min. Light	— 0.004	32	Odd Minima	— 0.009	110
Increasing Light	+ 0.001	110	Even Minima	+ 0.008	122

upon which each depends. Their agreement with the accepted mean light curve is shown in Fig. 2, and also by the residuals in columns four and eight of the last mentioned table. The coördinates of the mean light curve are given by Table VI. The range of variation is 0.m47. The interval of light change is $3^h\ 17^m$, of which $1^h\ 14^m$ is occupied by the descent, 26 minutes by constant minimum light, and $1^h\ 37^m$ by the rise to normal brightness. The slight asymmetry of the curve is responsible for the displacement of the zero phase from the middle point of constant minimum light, because the epochs of individual minima were estimated on the assumption that the curve

was symmetrical. The exactness with which the curve represents the observations is shown in Table VII, which also contains the systematic deviations from the mean light curve for odd and even minima. Although these values are larger than those secured when all observations are taken together, they are not large enough certainly to indicate an inequality of successive minima. The mean of the 114 values for the magnitude of the variable, secured from the observations during constant light, gave 9.92 for the normal magnitude of the variable. The probable error of a single observation is ± 0.060. These magnitudes, when plotted in order of phase, seemed to indicate a slight continuous decrease in the brightness of the variable from the limiting phase values to the point half way between minima; but the range of the variation is within the errors of observation and probably is not real.

The consistency of the work of individual observers and their systematic differences are shown by the probable errors and systematic deviations in Table VIII. It is to be noted that the probable errors for observations on the comparison star difference and on the magnitude of

TABLE VIII

Systematic Deviations and Probable Errors by Observers

Observations on	Seares			Haynes			Shapley		
	Sys. Dev.	P. E.	No. Obs.	Sys. Dev.	P. E.	No. Obs.	Sys. Dev.	P. E.	No. Obs.
$c-a$	$+0^{m}.11$	$\pm 0^{m}.057$	4	$-0^{m}.01$	$\pm 0^{m}.036$	95	$+0^{m}.03$	$\pm 0^{m}.070$	47
Var. Const. L.	——	——	2	+ 0.03	± 0.049	76	— 0.04	± 0.065	36
Var. Chang. L.	— 0.06	± 0.057	7	+ 0.01	± 0.052	134	— 0.01	± 0.074	91

the variable during constant light are referred to the individual means, and are therefore independent of the corresponding systematic deviations; while, in the case of observations during changing light, the probable errors are referred to the mean light curve. The values of the probable errors are in all cases extremely satisfactory. No particular significance can be attached to the systematic deviations for Seares, as they depend upon so few observations. In the case of Haynes and Shapley, the systematic deviations for observations on the comparison stars and the variable during constant light indicate that the latter consistently measures a light interval larger than the former. This is also evident from the values secured in determining the magnitude of a, and, moreover, it is seen that the systematic difference depends directly on the extent of the interval measured. The conclusion is further supported by the fact that the deviations for observations during changing light, when the average interval is nearly zero, are practically negligible.

Columbia, Missouri, 1911, May 29.

E. S. Haynes.
Harlow Shapley.

THE UNIVERSITY OF MISSOURI BULLETIN

ASTRONOMICAL SERIES

LAWS OBSERVATORY BULLETIN NO. 20

STUDIES WITH THE POLARIZING PHOTOMETER

By Robert H. Baker

In July, 1912, a number of variable stars were selected for study at the Laws Observatory. It was decided that the polarizing photometer should be used for this purpose, although the published results with this instrument have indicated its inferiority as compared with the equalizing wedge photometer. The polarizing photometer has been described by Professor Seares[1], and only a few points concerning it require repetition here. The instrument is similar to one designed by Müller for the Astrophysical Observatory at Potsdam and in many respects is like the Zöllner photometer. The artificial star is formed in the following manner: the light from a miniature incandescent lamp, after passing through a ground glass window and an iris diaphragm, is received by a small silvered hemisphere; whence it is reflected along the lateral tube containing the colorimeter and the nicols. Finally it is brought to the eyepiece by a second reflection from a glass plate at a 45° angle.

Two advantages of this instrument over the wedge photometer of this observatory have been pointed out by Seares, namely the superior definition of the artificial star formed by the silvered hemisphere, and the registering device that permits the observer to keep his eye at the telescope during a set of comparisons and to be unaware of the results he is getting. As attached in its present form to the 7½-inch telescope the eyepiece is found to give a power too great for good definition of the real stars. What effect this has on the accuracy of the observations may be learned by making measures with the instrument attached to a telescope of shorter focus.

For the present studies the polarizing photometer has been used in connection with the 4½-inch equatorial. The iris diaphragm is set to give an aperture of 3 mm. The source of light for the artificial star is, as before, an 8-volt lamp requiring a current of 1.3 amperes. This is fed by eight storage cells arranged in two sets in parallel of four

[1]Laws Observatory Bulletin No. 13, 187, 1907.

cells in series, each cell having a 6 ampere hour capacity. The batteries have been charged frequently and the voltmeter has been read at intervals during the observations. The variations in voltage has often been greater than desirable. It is the intention to soon substitute larger storage cells. The registering device has been used throughout the observations. The colorimeter is set at readings 112 and 120, the latter setting being made except for stars of later type and brighter than are usually observed. It is probable that more experience with this instrument will result in eliminating the colorimeter as at least superfluous if not detrimental to accuracy.

The observations have thus far been made by the writer and by Mr. C. Clayton Wylie who has assisted also in the reductions. The star whose brightness is to be measured is put between the two images of the artificial star and compared always with the one that appears to the right of it. The images are not reversed. Care is taken to have the line joining the two stars parallel to the line between the eyes. A single measure consists of eight settings in this order: two settings on the comparison star, four on the real star, two on the comparison star. A single observation is defined as the mean of measures of the variable referred to each of the two comparison stars. This differs from the definition of an observation previously used in these bulletins and the purpose is to condense the tabular matter. Two comparison stars are selected for each variable studied. The attempt is to have them in the same field with the variable, but this is not always possible. Work with the polarizing photometer in the present arrangement is confined to stars between magnitudes 6.5 and 9.0. It is intended to employ this photometer, as time permits from a photographic program contemplated, for the study of variable stars whose periods are short and those whose periods are announced as intermediate between the usual values for short and long periods—values that may well be suspected of error until confirmed. An example of this case is given later in the provisional results for TU Cassiopeiae.

Three other stars are discussed in this Bulletin, namely YZ Sagittarii, found to be a variable of the short period type like Zeta Geminorum; Z Vulpeculae, an eclipsing variable found to have a continuous light curve resembling Beta Lyrae; and RZ Scuti, an Algol variable, the character of whose light curve is studied.

THE GEMINID VARIABLE YZ SAGITTARII (140.1908)

The light variation of YZ Sagittarii was detected at Moscow by Mme. L. Ceraski[1] from four photographs of this region secured by M. S. Blazko between 1899 and 1906.

[1]Astronomische Nachrichten, 179, 48, 1908.

On three of these plates the star appears of the same brightness, about $7^{m}.5$, while on the fourth the star is 0.8 magnitudes fainter. The variability was confirmed at Bamberg by Dr. Ernst Zinner[1] who from seven observations made in 1911 derived the elements:

$$\text{Max.} = \text{J.D. } 2419257 + 20^{d}.E, \text{ G.M.T. (Elements by Zinner)}$$

the brightness ranging from magnitude 7.5 to 8.2.

This star was among the first to be put on the program of the polarizing photometer because the range in magnitude is that best observed with the instrument when attached to the 4½-inch refractor. The observing data for the variable and for the two comparison stars are as follows:

Variable	= BD. −16°5041, $7^{m}.3$,	R.A.(1900)	$18^{h}43^{m}.7$,	Decl. (1900) −16°50′
Comp. star *a* =	−17°5347, $6^{m}.8$,		$18^{h}45^{m}.5$	−17°16′
Comp. star *b* =	−16°5033, $7^{m}.4$,		$18^{h}42^{m}.2$	−17°53′

The Harvard Durchmusterung[2] gives for BD. −17°5347 magnitude 6.79 and for BD. −16°-5033 magnitude 7.07.

Photometric measures of the variable referred to the two comparison stars have been made on 61 nights during 1912 at the Laws Observatory. The number of observations is 117, divided about evenly between the two observers. These are found in Table I. The second column of the table contains the initials of the observers, Mr. Wylie and the writer. The phase in column (5) is referred to maximum light according to the final elements derived below, and is reduced to the sun; this reduction, that reaches a maximum of 0.0057 days, frequently changes the phase one unit in the last decimal place. Columns (6) and (7) contain the measured differences of magnitude between the variable and the comparison stars *a* and *b*, found from the differences of the settings of the intensity circle by entering Table I of Bulletin No. 13. The measures of the variable as compared with stars *a* and *b* appear in columns (8) and (9), and the mean of the two results gives in the tenth column the magnitude of the variable star for each observation. Finally come the residuals, the basis of which will be taken up later.

A discussion of the observations indicates that the period given by Zinner is nearly a multiple of the true period. As a first approximation I have collected the observations according to the elements:

$$\text{Max.} = \text{J.D. } 2419645.7 + 9^{d}.50 \cdot E, \text{ G.M.T. (Preliminary Elements)}$$

Before improving these elements it is necessary to consider any systematic differences between the two observers. For this purpose the observations are divided into 16 groups

[1] Astronomische Nachrichten, 190, 377, 1912.

[2] Harvard College Observatory Annals, vol. 45.

TABLE I

OBSERVATIONS OF YZ SAGITTARII

No.	Obs.	Date, G.M.T.	Julian Day	Phase	$v-a$	$v-b$	$v(a)$	$v(b)$	v	O—C	O—C′
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
		1912 d. h. m.		days	mag.	mag.	mag.	mag.	mag.		
1	B	July 5 18 20	2419589.76	1.18	+0.41	+0.27	7.29	7.34	7.32	+ 3	+ 4
2	B	7 18 06	591.75	3.17	.63	.41	7.51	7.48	7.50	0	− 8
3	B	8 16 16	592.68	4.10	.95	.48	7.83	7.55	7.69	0	+ 1
4	B	8 16 37	592.69	4.11	.96	.71	7.84	7.78	7.81	+12	+13
5	B	9 17 10	593.72	5.13	.84	.89	7.72	7.96	7.84	+15	+15
6	B	12 15 58	596.67	8.08	.50	.24	7.38	7.31	7.34	+ 6	+ 5
7	B	12 16 08	596.67	8.09	.37	.30	7.25	7.37	7.31	+ 3	+ 2
8	B	14 16 20	598.68	0.55	.21	.20	7.09	7.27	7.18	− 2	− 2
9	B	19 16 57	603.71	5.57	.79	.37	7.67	7.44	7.56	−10	−10
10	B	19 17 08	603.71	5.58	.66	.51	7.54	7.58	7.56	−10	−10
11	B	20 15 00	604.62	6.49	.71	.47	7.59	7.54	7.56	− 2	− 2
12	B	20 15 12	604.63	6.50	.67	.58	7.55	7.65	7.60	+ 2	+ 2
13	B	20 15 24	604.64	6.51	.68	.46	7.56	7.53	7.54	− 4	− 4
14	B	22 15 02	606.63	8.49	.56	.30	7.44	7.37	7.40	+13	+15
15	B	22 15 09	606.63	8.50	.41	.18	7.29	7.25	7.27	0	+ 2
16	B	23 14 42	607.61	9.48	.09	+0.06	6.97	7.13	7.05	− 9	−11
17	B	23 14 50	607.62	9.48	.19	−0.14	7.07	6.93	7.00	−14	−16
18	B	25 15 14	609.63	1.95	.60	+0.36	7.48	7.43	7.46	+ 1	+ 7
19	B	25 15 24	609.64	1.96	.58	.45	7.46	7.52	7.49	+ 4	+10
20	B	26 15 10	610.63	2.95	.52	.51	7.40	7.58	7.49	+ 2	− 6
21	B	26 15 18	610.64	2.95	.71	.43	7.59	7.50	7.54	+ 7	− 1
22	B	27 14 38	611.61	3.92	.74	.68	7.62	7.75	7.68	− 1	+ 1
23	B	27 14 46	611.62	3.93	.88	.65	7.76	7.72	7.74	+ 5	+ 7
24	B	28 17 20	612.72	5.04	.90	.66	7.78	7.73	7.76	+ 7	+ 7
25	B	28 17 30	612.73	5.04	.85	.90	7.73	7.97	7.85	+16	+16
26	B	30 14 31	614.60	6.92	.72	.34	7.60	7.41	7.50	− 2	− 2
27	B	30 14 40	614.61	6.92	.77	.53	7.65	7.60	7.62	+10	+10
28	B	31 14 40	615.61	7.92	.39	.09	7.27	7.16	7.22	− 7	− 9
29	B	31 14 50	615.62	7.93	.32	.09	7.20	7.16	7.18	−11	−13
30	B	Aug. 1 15 34	616.65	8.96	.44	.30	7.32	7.37	7.34	+10	+13
31	B	1 15 42	616.65	8.97	.55	.23	7.43	7.30	7.36	+12	+15
32	W	4 14 45	619.61	2.38	.29	.47	7.29	7.52	7.40	− 7	− 5
33	B	5 14 20	620.60	3.36	.74	.52	7.62	7.59	7.60	+ 3	− 1
34	B	5 14 28	620.60	3.36	.53	.36	7.41	7.43	7.42	−15	−19
35	B	10 14 27	625.60	8.36	.32	.07	7.20	7.14	7.17	−10	− 9
36	W	11 14 48	626.62	9.38	.37	.09	7.37	7.14	7.26	+11	+10
37	W	12 15 34	627.65	0.86	.07	.05	7.07	7.10	7.08	−15	−15
38	W	12 15 43	627.65	0.86	.44	.11	7.44	7.16	7.30	+ 7	+ 7
39	B	16 14 14	631.59	4.80	.86	.56	7.74	7.63	7.68	− 2	− 2
40	B	16 14 20	631.60	4.81	.84	.88	7.72	7.95	7.84	+14	+14
41	W	18 15 08	633.63	6.84	.55	.42	7.55	7.47	7.51	− 2	− 2
42	W	22 15 32	637.65	1.30	.28	.26	7.28	7.31	7.30	− 1	0
43	W	22 15 38	637.65	1.31	.47	.27	7.47	7.32	7.40	+ 9	+10
44	W	23 15 42	638.65	2.31	.48	.42	7.48	7.47	7.48	+ 1	+ 4
45	W	26 14 14	641.59	5.25	.65	.50	7.65	7.55	7.60	− 8	− 8

TABLE I—Continued

OBSERVATIONS OF YZ SAGITTARII

No.	Obs.	Date, G.M.T.	Julian Day	Phase	$v-a$	$v-b$	$v(a)$	$v(b)$	v	O—C	O—C′
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
		1912 d. h. m.		days	mag.	mag.	mag.	mag.	mag.		
46	W	Aug. 27 14 08	2419642.59	6.24	+0.56	+0.36	7.56	7.41	7.48	−13	−13
47	W	27 14 16	642.59	6.25	.68	.57	7.68	7.62	7.65	+ 4	+ 4
48	W	30 14 42	645.61	9.27	.16	.15	7.16	7.20	7.18	0	0
49	W	30 14 48	645.62	9.27	.04	.19	7.04	7.24	7.14	− 4	− 4
50	W	31 14 09	646.59	0.69	.17	.22	7.17	7.27	7.22	+ 1	+ 1
51	W	31 14 16	646.59	0.70	.19	.22	7.19	7.27	7.23	+ 2	+ 2
52	W	Sept. 1 14 34	647.61	1.71	.37	.15	7.37	7.20	7.28	−10	− 7
53	W	1 14 40	647.61	1.71	0.37	.56	7.37	7.61	7.49	+11	+14
54	W	3 14 08	649.59	3.69	1.01	.69	8.01	7.74	7.88	+20	+23
55	W	3 14 16	649.59	3.70	0.92	.88	7.92	7.93	7.92	+24	+27
56	W	4 14 14	650.59	4.70	.76	.79	7.76	7.84	7.80	+10	+10
57	W	4 14 22	650.60	4.70	.51	.69	7.51	7.74	7.62	− 8	− 8
58	W	5 14 24	651.60	5.70	.66	.78	7.66	7.83	7.74	+ 8	+ 8
59	W	5 14 32	651.61	5.71	.81	.80	7.81	7.85	7.83	+17	+17
60	W	6 14 18	652.60	6.70	.56	.44	7.56	7.49	7.52	− 3	− 3
61	W	6 14 27	652.60	6.70	.57	.55	7.57	7.60	7.58	+ 3	+ 3
62	W	7 14 10	653.59	7.69	.25	.14	7.25	7.19	7.22	− 7	−11
63	W	7 14 18	653.60	7.70	.45	.40	7.45	7.45	7.45	+16	+12
64	W	8 14 21	654.60	8.70	.18	.24	7.18	7.29	7.24	− 2	+ 1
65	W	8 14 28	654.60	8.71	.25	.07	7.25	7.12	7.18	− 8	− 5
66	W	9 14 50	655.62	0.17	.34	.08	7.34	7.13	7.24	+ 8	+ 7
67	W	10 15 06	656.63	1.18	.21	.17	7.21	7.22	7.22	− 7	− 6
68	W	10 15 16	656.64	1.19	.20	.32	7.20	7.37	7.28	− 1	0
69	W	11 13 44	657.57	2.12	.32	.38	7.32	7.43	7.38	− 9	− 4
70	W	11 13 52	657.58	2.13	.50	.23	7.50	7.28	7.39	− 8	− 3
71	W	12 13 54	658.58	3.13	.72	.62	7.72	7.67	7.70	+20	+12
72	W	12 14 02	658.58	3.13	.47	.41	7.47	7.46	7.46	− 4	−12
73	W	17 14 00	663.58	8.13	.40	.36	7.40	7.41	7.40	+12	+12
74	B	17 14 50	663.62	8.17	.46	.26	7.34	7.33	7.34	+ 6	+ 6
75	B	18 13 49	664.58	9.12	.46	.11	7.34	7.18	7.26	+ 5	+ 7
76	W	19 14 50	665.62	0.62	.41	.13	7.41	7.18	7.30	+10	+10
77	W	21 14 29	667.60	2.60	.41	.30	7.41	7.35	7.38	− 9	−11
78	W	21 14 37	667.61	2.61	.51	.69	7.51	7.74	7.62	+15	+13
79	B	22 13 48	668.58	3.57	.63	.69	7.51	7.76	7.64	− 2	0
80	B	22 13 54	668.58	3.58	.62	.59	7.50	7.66	7.58	− 8	− 6
81	B	22 14 02	668.58	3.58	.56	.43	7.44	7.50	7.47	−20	−17
82	B	22 14 08	668.59	3.59	.81	.38	7.69	7.45	7.57	−10	− 7
83	B	23 13 17	669.55	4.55	.85	.50	7.73	7.57	7.65	− 5	− 5
84	B	23 13 25	669.56	4.56	.81	.67	7.69	7.74	7.72	+ 2	+ 2
85	W	23 13 32	669.56	4.56	.79	.47	7.79	7.52	7.66	− 4	− 4
86	W	23 13 38	669.57	4.57	.47	.53	7.47	7.58	7.52	−18	−18
87	B	26 13 46	672.57	7.57	.51	.13	7.39	7.20	7.30	[illegible]	− 6
88	B	26 13 54	672.58	7.58	.35	.36	7.23	7.43	7.33	+ 3	− 3
89	W	26 14 01	672.58	7.58	.10	.06	7.10	7.11	7.10	−20	−26
90	W	26 14 08	672.59	7.59	.18	.41	7.18	7.46	7.32	+ 2	− 4

TABLE I—Continued

OBSERVATIONS OF YZ SAGITTARII

No.	Obs.	Date, G.M.T.	Julian Day	Phase	$v-a$	$v-b$	$v(a)$	$v(b)$	v	O—C	O—C′
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
		1912 d. h. m.		days	mag.	mag.	mag.	mag.	mag.		
91	W	Sept. 27 13 58	2419673.58	8.58	+0.19	+0.30	7.19	7.35	7.27	0	+ 3
92	W	27 14 16	673.59	8.59	.20	.03	7.20	7.08	7.14	−13	−10
93	B	30 13 32	676.56	2.01	.77	.44	7.65	7.51	7.58	+12	+18
94	B	30 13 37	676.57	2.01	.52	.60	7.40	7.67	7.54	+ 8	+14
95	B	Oct. 3 13 10	679.55	4.99	.86	.51	7.74	7.58	7.66	− 3	− 3
96	B	3 13 15	679.55	5.00	.70	.68	7.58	7.75	7.66	− 3	− 3
97	W	3 13 26	679.56	5.00	.73	.66	7.73	7.71	7.72	+ 3	+ 3
98	W	3 13 34	679.57	5.01	.60	.63	7.60	7.68	7.64	− 5	− 5
99	B	4 12 58	680.54	5.98	.76	.48	7.64	7.55	7.60	− 4	− 4
100	B	4 13 02	680.54	5.99	.71	.51	7.59	7.58	7.58	− 6	− 6
101	W	5 13 30	681.56	7.01	.77	.70	7.77	7.75	7.76	+26	+26
102	W	5 13 40	681.57	7.01	.62	.28	7.62	7.33	7.48	− 2	− 2
103	B	7 13 26	683.56	9.00	.31	.09	7.19	7.16	7.18	− 6	− 2
104	B	7 13 39	683.57	9.01	.55	.15	7.43	7.22	7.32	+ 8	+12
105	B	12 12 30	688.52	4.41	.78	.55	7.66	7.62	7.64	− 6	− 6
106	W	13 13 21	689.56	5.45	.65	.46	7.65	7.51	7.58	− 9	− 9
107	W	13 13 25	689.56	5.45	.49	.56	7.49	7.61	7.55	−12	−12
108	B	14 12 29	690.52	6.41	.60	.59	7.48	7.66	7.57	− 2	− 2
109	B	14 12 38	690.53	6.42	.51	.36	7.39	7.43	7.41	−18	−18
110	W	15 13 10	691.55	7.44	.21	.36	7.21	7.41	7.31	− 2	− 6
111	W	15 13 16	691.55	7.44	.20	.34	7.20	7.39	7.30	− 3	− 7
112	W	20 12 40	696.53	2.87	.35	.54	7.35	7.59	7.47	0	− 7
113	W	20 12 46	696.53	2.87	0.32	.38	7.32	7.43	7.38	− 9	−16
114	B	23 12 20	699.51	5.85	1.04	.64	7.92	7.71	7.82	+17	+17
115	B	23 12 26	699.52	5.86	0.95	.54	7.83	7.61	7.72	+ 7	+ 7
116	B	24 12 13	700.51	6.85	.45	.41	7.33	7.48	7.40	−13	−13
117	B	24 12 18	700.51	6.85	.51	.50	7.39	7.57	7.48	− 5	− 5

according to phase, each group containing measures by both observers. Eight observations by Wylie are not here included because they occur at a phase not observed by Baker. The results of this procedure appear in Table II. The phase in column (1) is based on the above elements and is referred to maximum light; columns (2), (3) and (4) contain the means of magnitude differences measured by Baker and the number of observations by this observer included in the group. A similar treatment of Wylie's observations follows. Subtracting columns (2)-(5) and (3)-(6) we have in columns (8) and (9) the discrepancy between the measures by the two observers for the same phase of the variable star. From an inspection of these columns it appears that a systematic difference exists and that the amount varies with the comparison star used. If this difference is a function of the relative brightness of the stars compared, then it will not be a constant for either

comparison star, but will vary with the intensity of the variable star. A graphic representation of the group differences Baker-Wylie shows that when the variable is near maximum the differences v-a and v-b are greater for Baker than for Wylie, and that when the variable is near minimum the values $\triangle(v\text{-}a)$ and $\triangle(v\text{-}b)$ become algebraically smaller. That the change in these values is not simply a function of the magnitude differences is evident from the fact that for the same phase of the variable $\triangle(v\text{-}b)$ is algebraically smaller than $\triangle(v\text{-}a)$, although from the assumed magnitudes of the comparison stars the difference v-b is actually always smaller than v-a. The difference between measures by the

TABLE II

COMPARISON OF MAGNITUDE DIFFERENCES

Prelim. Phase	Baker			Wylie			Baker-Wylie			Resid. O-C	
	v-a	v-b	No. Obs.	v-a	v-b	No. Obs.	$\triangle(v\text{-}a)$	$\triangle(v\text{-}b)$	Wt.	$\triangle(v\text{-}a)$	$\triangle(v\text{-}b)$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
days	mag.	mag.		mag.	mag.		mag.	mag.		mag.	mag.
0.4	+0.21	+0.20	1	+0.34	+0.08	1	−0.13	+0.12	1	−0.25	+0.14
1.0	.41	.27	1	.26	.15	5	+0.15	+0.12	2	+0.03	+0.14
1.9	.59	.41	2	.37	.35	2	+0.22	+0.06	2	+0.10	+0.08
2.4	.65	.52	2	.40	.37	4	+0.25	+0.15	3	+0.13	+0.17
2.9	.62	.45	3	.46	.49	2	+0.16	−0.04	2	+0.04	−0.02
3.4	.63	.44	2	.47	.49	4	+0.16	−0.05	3	+0.04	−0.03
3.9	.77	.58	8	.97	.79	2	−0.20	−0.21	3	−0.32	−0.19
4.9	.84	.70	8	.63	.62	4	+0.21	+0.08	5	+0.09	+0.10
5.4	.75	.52	4	.66	.60	3	+0.09	−0.08	3	−0.03	−0.06
6.4	.79	.53	7	.62	.47	2	+0.17	+0.06	3	+0.05	+0.08
6.9	.65	.45	4	.56	.47	3	+0.09	−0.02	3	−0.03	0.00
7.4	.48	.45	2	.69	.49	2	−0.21	−0.04	2	−0.33	−0.02
7.9	.41	.19	6	.23	.29	6	+0.18	−0.10	6	+0.06	−0.08
8.4	.44	20	4	.40	.36	1	+0.04	−0.16	2	−0.08	−0.14
8.9	.49	.27	2	.21	.16	4	+0.28	+0.11	3	+0.16	+0.13
9.4	.32	.05	5	.19	.14	3	+0.13	−0.09	4	+0.01	−0.07

two observers must therefore vary in a complex manner impossible to determine at present. In earlier numbers of the Bulletins, Seares[1] and Haynes found a constant difference between their measures apparently unaffected by the magnitude difference measured. The comparison of the present results brings out a difficulty not experienced by them.

We must now proceed to adjust the measures of the two observers with the unwarranted but necessary assumption that the systematic difference between them is constant so long as the same two stars are compared, regardless of any change in the bright-

[1]Laws Observatory Bulletin No. 9,140, 1907.

ness of either one. The weighted means of the values $\triangle(v\text{-}a)$ and $\triangle(v\text{-}b)$ in columns (8) and (9) of Table II are computed; the residuals in columns (11) and (12) are taken from these means. Returning now to Table I and subtracting columns (6)-(7) we have for each observation the quantity $b\text{-}a$, or the difference in brightness between the comparision stars derived from measures of the variable star referred to the comparison stars in close succession. The mean of these quantities for each observer is taken and their difference, which represents the systematic difference in comparing the stars a and b. The results obtained here are assembled in Table III. The second and third columns of the table

TABLE III

MEAN DIFFERENCES (BAKER-WYLIE)

Differences	Actual	Prob. Error	Adopted
	mag.	mag.	mag.
$\triangle(v\text{-}a)$	+0.123	±0.023	+0.12
$\triangle(v\text{-}b)$	−0.015	±0.018	−0.02
$\triangle v$	+0.054	±0.015	+0.05
$\triangle(b\text{-}a)$	+0.149	±0.018	+0.14
$b\text{-}a$ (Baker)	+0.195	±0.012	+0.19
$b\text{-}a$ (Wylie)	+0.046	±0.014	+0.05

contain the mean differences obtained by the procedure outlined above, and their probable errors. To bring the measures of the two observers into agreement I have assumed as a standard for Baker that the comparison star b is of magnitude 7.07, the value assigned to this star in the Harvard Durchmusterung. Combining this with the mean differences of the table, the following values are derived for the comparison stars:

for Baker	Comp. star $a = 6^m.88$,	Comp. star $b = 7^m.07$
Wylie	$7^m.00$	$7^m.05$

From column four of the table we see that the differences adopted are in close agreement with those computed from the measures.

Having established the brightness of the comparison stars necessary to bring the results for the variable into concordance, columns (8) and (9) of Table I may be filled in from the observed differences $v\text{-}a$ and $v\text{-}b$ in the two columns preceding, and the means taken to form column (10). Here we have as definitive ordinates the magnitude of the variable star for each observation. It remains to adjust the abscissae.

A discussion of the observations made at times when the light of the star is changing most rapidly yields as a correction to the preliminary period $\triangle P = +0^d.05 \pm 0^d.023$. When the observations are collected on the corrected period the time of maximum light requires the correction $\triangle \text{Maximum} = +0.2$ days. The new period is sufficiently close

to permit an evaluation of the number of fluctuations of the star between the epoch of the present series and the four observations by Mme. Ceraski; an error of one fluctuation in the interval from the last of these four, changes the value of the period by 0.04 days. An inspection of the four observations made at Moscow indicates that they are fortunately placed for comparison with the present series. On three of the plates the star has the same intensity while on the fourth it is 0.8 magnitudes fainter, a difference slightly greater than the total range derived from the visual observations. We may suppose therefore that the first three plates were secured near the times of maximum light and that the fourth exposure found the star near minimum. Table IV shows the period obtained by comparing each of the early observations with the times of maximum and minimum light established from the Laws Observatory measures. In column two the tenth

TABLE IV

COMPARISON WITH MOSCOW OBSERVATIONS

Phase	Julian Date	Interval	E	Period	Wt.	O-C
	days	days		days		days
Maximum	2414925.2	4720.7	−494	9.556	2	+0.003
Maximum	5224.4	4421.5	463	9.550	2	−0.003
Maximum	5252.3	4393.6	460	9.551	2	−0.002
Minimum	7472.2	2168.9	227	9.555	1	+0.002
			Period =	9d553	±0d001	

of the day is assumed; column three contains the interval in days between the early observation and the present epoch. The number of fluctuations of the star during the interval follows in the fourth column, whence the period from this comparison in column five. The weighted mean of the four periods, with its probable error, is appended. Thus we find for the definitive elements of YZ Sagittarii:

$$\text{Max.} = \text{J.D. } 2419645.9 + 9^{d}.553 \cdot E, \text{ G.M.T. (Final Elements)}$$

Actually the value 9.552 days for the period is used for the final phases, because the reductions were far advanced before the change in the epoch could be noted. The addition of 0.2 days to the time of maximum above mentioned changes the period by 0.001 days, a change too small to warrant a recomputation of the phases.

The observations are now grouped with regard to phase into 23 normal places. They appear in Table V and are represented graphically in Figure 1. A curve drawn to best satisfy the normal places, the full curve of the figure, requires two halts in the fluctuation of the variable, one during decreasing and the other during increasing light. The

TABLE V

NORMAL PLACES OF YZ SAGITTARII

No.	Phase	Mag.	No. Obs.	O-C	O-C′	No.	Phase	Mag.	No. Obs.	O-C	O-C′
	days			mag.	mag.		days			mag.	mag.
1	0.71	7.22	6	0.00	0.00	13	5.58	7.64	6	−0.02	−0.02
2	1.23	7.30	5	0.00	+0.01	14	5.92	7.68	4	+0.04	+0.04
3	1.71	7.38	2	−0.01	+0.02	15	6.25	7.56	2	−0.05	−0.05
4	2.03	7.47	6	0.00	+0.07	16	6.46	7.54	5	−0.04	−0.04
5	2.47	7.47	4	0.00	0.00	17	6.87	7.57	9	+0.04	+0.04
6	2.91	7.47	4	0.00	−0.08	18	7.57	7.29	8	−0.01	−0.06
7	3.23	7.54	5	0.00	−0.06	19	8.05	7.30	6	+0.02	+0.01
8	3.62	7.68	6	0.00	+0.04	20	8.50	7.25	5	−0.02	0.00
9	4.01	7.73	4	+0.04	+0.05	21	8.89	7.27	6	+0.02	+0.06
10	4.58	7.66	7	−0.04	−0.04	22	9.22	7.19	3	0.00	+0.01
11	4.90	7.71	4	+0.01	+0.01	23	9.51	7.14	4	−0.01	−0.02
12	5.08	7.74	6	+0.05	+0.05						

TABLE VI

MEAN LIGHT CURVE OF YZ SAGITTARII

Phase	C	C′	Phase	C	C′	Phase	C	C′
days	mag.	mag.	days	mag.	mag.	days	mag.	mag.
0.0	7.15	7.16	3.2	7.52	7.59	6.4	7.59	7.59
0.2	7.16	7.17	3.4	7.59	7.62	6.6	7.57	7.57
0.4	7.17	7.18	3.6	7.67	7.64	6.8	7.54	7.54
0.6	7.20	7.20	3.8	7.68	7.66	7.0	7.50	7.50
0.8	7.23	7.23	4.0	7.69	7.67	7.2	7.44	7.44
1.0	7.26	7.26	4.2	7.69	7.69	7.4	7.36	7.38
1.2	7.29	7.29	4.4	7.70	7.70	7.6	7.30	7.35
1.4	7.32	7.31	4.6	7.70	7.70	7.8	7.29	7.32
1.6	7.36	7.34	4.8	7.70	7.70	8.0	7.28	7.30
1.8	7.41	7.37	5.0	7.69	7.69	8.2	7.28	7.28
2.0	7.46	7.40	5.2	7.68	7.68	8.4	7.27	7.26
2.2	7.47	7.43	5.4	7.67	7.67	8.6	7.26	7.24
2.4	7.47	7.46	5.6	7.66	7.66	8.8	7.26	7.22
2.6	7.47	7.49	5.8	7.65	7.65	9.0	7.24	7.20
2.8	7.47	7,52	6.0	7.63	7.63	9.2	7.19	7.18
3.0	7.47	7.56	6.2	7.61	7.61	9.4	7.16	7.17

TABLE VII

PROBABLE ERROR OF A SINGLE OBSERVATION

	Irregular Curve			Regular Curve		
	Baker	Wylie	All	Baker	Wylie	All
	mag.	mag.	mag.	mag.	mag.	mag.
v-a	±0.076	±0.090	±0.082	±0.080	±0.095	±0.087
v-b	±0.077	±0.085	±0.081	±0.081	±0.085	±0.082
v	±0.058	±0.071	±0.064	±0.064	±0.074	±0.068

dotted curve is the one that represents the variation of the light, if we suppose this to proceed regularly and neglect as fortuitous the orderly manner in which the normal places depart from this curve. The argument against the reality of the irregular curve is that the normal places agree with the curve much better where the irregularities occur than at other points. This is perhaps offset by the fact that the departures O-C′ from the regular curve are greatest where we have supposed the halts to occur, and that these discrepancies have the appearance of being systematic. The ordinates of the two curves

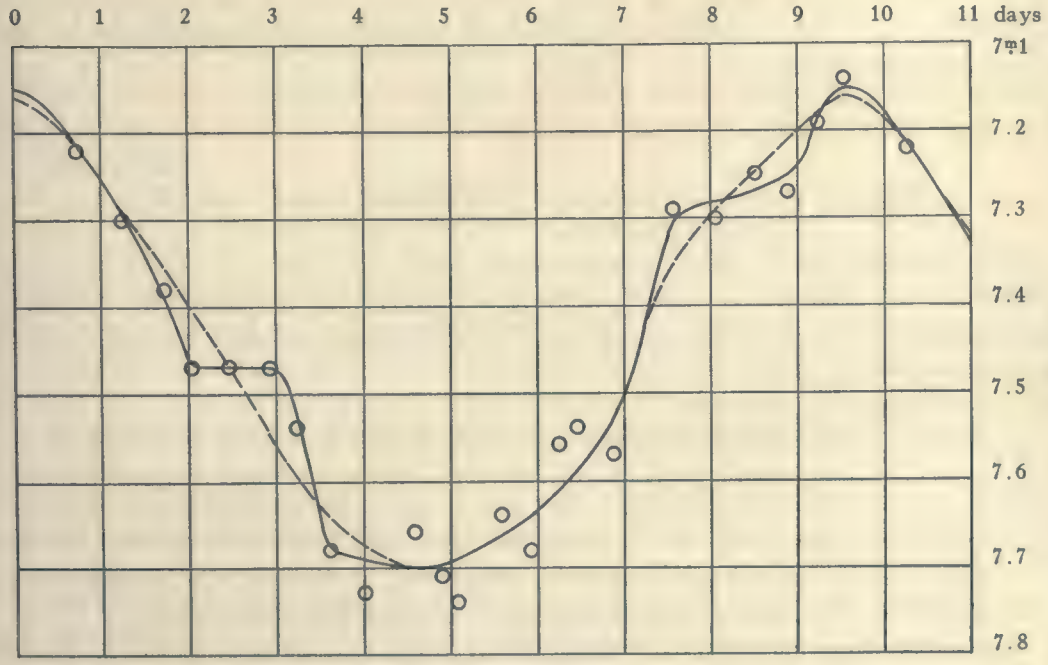


Figure 1. Light Curve of YZ Sagittarii

are given in Table VI for equal phase intervals; column C contains the ordinates for the irregular curve, column C′ for the regular curve. As we see from the figure the two curves are identical between phases $0^d.6$ and $1^d.2$ and between $4^d.4$ and $7^d.2$. The departures of the separate observations from these curves form the last two columns of Table I; residuals for the normal places occupy a similar position in Table V. C and C^1 as above refer respectively to the ordinates of the irregular and regular mean light curves. A discussion of the residuals for single observations may well include those for the separate measures. These magnitudes appear in Table I, columns (8) and (9). The corresponding residuals are not here set down because of lack of space in the table. The probable errors are now

derived from the residuals last named and from those that appear in Table I, and are assembled to form Table VII.

The star YZ Sagittarii is a short period variable of a type resembling Zeta Geminorum, since the minimum phase occurs midway between the times of maximum light. The elements that represent the light variation are:

$$\text{Max.} = \text{J.D. } 2419645.9 + 9^{d}.553 \cdot E, \text{ G.M.T. (Final Elements)}$$

The range of fluctuation, based on the adopted magnitude 7.07 for comparison star *b*, is between $7^{m}.15$ and $7^{m}.70$. Evidence is presented to show that irregularities in the light curve are not improbable.

THE LYRID VARIABLE Z VULPECULAE (26.1900)

The variability of Z Vulpeculae was suspected in 1900 by A. S. Flint[1] at the Washburn Observatory. It was independently discovered in 1908 by Mr. T. H. Astbury[2] who concluded that it must be of the Algol type. The elements as later derived by Astbury[3] are as follows:

$$\text{Min.} = \text{J.D. } 2418285.250 + 2^{d}.445 \cdot E, \text{ G.M.T. (Elements by Astbury)}$$

This result was obtained by combining his observations with minima observed by Flint and by M. F. de Roy at Antwerp. The latter is here said to have found that the increase and decrease of light are not symmetrical. According to Astbury the magnitude ranges from about $7^{m}.3$ to $8^{m}.5$ and the change of light occupies about six hours.

This star was placed on the program of the Laws Observatory for observation with the polarizing photometer, in order to study the character of the entire light curve. The present discussion is based on 68 observations made by the writer that appear in Table VIII. The arrangement of the table is similar to that of Table I described above. The phase is here reduced to the sun, and referred to minimum light according to the elements adopted below. The observing data for the variable and for the two comparison stars are as follows:

[1] Astronomical Journal, 21, 74, 1901.
[2] Astronomische Nachrichten, 179, 225, 1908.
[3] Astronomische Nachrichten, 182, 389, 1909.

Variable	=BD.+25°3803, 7m5, R.A. (1900)	19h 17m6, Decl. (1900)	+25° 23′
Comp.star a=	+25°3802, 7m0,	19h 17m4	+25° 25′
Comp.star b=	+25°3805, 8m8,	19h 17m7	+25° 17′

The Harvard Durchmusterung gives for BD.+25°3802 magnitude 7.26. In the Potsdam Durchmusterung[1] its magnitude is 7.71.

Referring again to Table VIII and subtracting columns (6)-(7), we have for each observation the difference in magnitude between the comparison stars, and taking the mean we find:

$$b\text{-}a = +1^{m}.54.$$

In the reductions I have adopted for comparison star *a* the value found at Harvard; the brightness of star *b* follows from the difference just derived. The adopted values for the comparison stars are:

$$\text{Comp. star } a = 7^{m}.26, \quad \text{Comp. star } b = 8^{m}.80.$$

The observations are now formed into 30 normal places with regard to phase. These appear in Table IX and are represented graphically in Figure 2, with the light curve drawn to best satisfy them. It is found that the epoch for minimum phase does not agree exactly with the minimum computed from Astbury's elements. A negative correction of 0.015 days is demanded. This correction has been made throughout the present reductions, retaining Astbury's period, and the following elements have been adopted:

$$\text{Heliocent. Min.} = \text{J.D. } 2419679.675 + 2^{d}.445 \cdot E, \text{ G.M.T. (Adopted Elements)}$$

From the figure it appears that the light of Z Vulpeculae varies continuously and that the star belongs to the Beta Lyrae type of eclipsing variables. At maximum light its magnitude is $7^{m}.06$, while at the principal minimum it has decreased to $8^{m}.79$. The brightness at secondary minimum is $7^{m}.44$, a drop of more than a third of a magnitude. The secondary minimum appears to occur at phase $1^{d}.20$, slightly earlier than a time midway between two principal minima, indicating some eccentricity of the orbit and an inclination of the line of apsides to the line of sight. The departures of the separate measures from the mean light curve give the probable errors of a single measure as follows:

for *v-a*,	Probable Error =	± 0m066,
v-b,	=	± 0m080,
v ,	=	± 0m054.

[1]Potsdam Publications, vol. 17, 1907.

TABLE VIII

OBSERVATIONS OF Z VULPECULAE

No.	Obs.	Date, G.M.T.	Julian Day	Helioc. Phase	*v-a*	*v-b*	*v(a)*	*v(b)*	*v*	O-C
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		1912 d. h. m.		days	mag.	mag.	mag.	mag.	mag.	mag.
1	B	July 23 15 28	2419607.644	1.623	−0.21	−1.71	7.05	7.09	7.07	−05
2	B	23 15 36	607.650	1.629	−0.10	−1.64	7.16	7.16	7.16	+ 4
3	B	25 15 59	609.666	1.190	+0.03	−1.34	7.29	7.46	7.38	− 6
4	B	25 16 08	609.672	1.196	0.00	−1.29	7.26	7.51	7.38	− 6
5	B	25 17 47	609.741	1.265	+0.11	−1.67	7.37	7.13	7.25	−15
6	B	25 17 56	609.747	1.271	+0.40	−1.31	7.66	7.49	7.58	+18
7	B	26 15 50	610.660	2.184	+0.05	−1.41	7.31	7.39	7.35	+ 7
8	B	26 15 59	610.666	2.190	−0.06	−1.43	7.20	7.37	7.28	− 1
9	B	26 18 23	610.766	2.290	+0.12	−1.49	7.38	7.31	7.34	− 6
10	B	26 18 58	610.790	2.314	+0.17	−1.42	7.43	7.38	7.40	− 5
11	B	26 19 08	610.797	2.321	+0.20	−1.30	7.46	7.50	7.48	0
12	B	26 19 33	610.815	2.338	+0.36	−1.24	7.62	7.56	7.59	− 1
13	B	26 19 41	610.820	2.344	+0.30	−1.16	7.56	7.64	7.60	− 3
14	B	26 20 10	610.840	2.364	+0.51	−0.99	7.77	7.81	7.79	− 1
15	B	26 20 16	610.844	2.368	+0.58	−0.95	7.84	7.85	7.84	0
16	B	26 20 40	610.861	2.385	+0.72	−0.92	7.98	7.88	7.93	− 7
17	B	26 20 46	610.865	2.389	+0.71	−0.73	7.97	8.07	8.02	− 2
18	B	26 21 09	610.881	2.405	+0.82	−0.46	8.08	8.34	8.21	+ 1
19	B	26 21 24	610.892	2.416	+1.03	−0.54	8.29	8.26	8.28	− 5
20	B	26 21 47	610.908	2.432	+1.43	−0.16	8.69	8.64	8.66	+10
21	B	27 15 46	611.657	0.726	−0.14	−1.67	7.12	7.13	7.12	+ 4
22	B	28 18 12	612.758	1.827	−0.17	−1.67	7.09	7.13	7.11	+ 5
23	B	30 15 22	614.640	1.254	+0.01	−1.28	7.27	7.52	7.40	− 2
24	B	30 15 32	614.647	1.261	+0.15	−1.57	7.41	7.23	7.32	− 8
25	B	31 14 30	615.604	2.218	+0.05	−1.47	7.31	7.33	7.32	0
26	B	31 16 14	615.676	2.290	+0.04	−1.37	7.30	7.43	7.36	− 4
27	B	31 16 22	615.682	2.296	+0.04	−1.50	7.30	7.30	7.30	−11
28	B	31 18 18	615.762	2.376	+0.67	−0.62	7.93	8.18	8.06	+15
29	B	31 18 26	615.768	2.382	+0.80	−0.82	8.06	7.98	8.02	+ 6
30	B	Aug. 1 16 56	616.706	0.864	−0.03	−1.70	7.23	7.10	7.16	− 2
31	B	3 14 41	618.612	0.316	0.00	−1.61	7.26	7.19	7.22	− 2
32	B	3 16 15	618.677	0.381	−0.04	−1.49	7.22	7.31	7.26	− 2
33	B	4 16 46	619.699	1.402	−0.18	−1.46	7.08	7.34	7.21	− 7
34	B	4 17 02	619.710	1.414	−0.08	−1.23	7.18	7.57	7.38	+11
35	B	5 14 12	620.592	2.295	+0.28	−1.16	7.54	7.64	7.59	+18
36	B	5 15 02	620.626	2.330	+0.21	−1.30	7.47	7.50	7.48	− 6
37	B	5 16 10	620.674	2.377	+0.53	−0.83	7.79	7.97	7.88	− 3
38	B	5 16 41	620.695	2.399	+0.88	−0.86	8.14	7.94	8.04	− 9
39	B	10 16 10	625.674	0.012	+1.59	+0.10	8.85	8.90	8.88	+16
40	B	16 14 40	631.611	1.040	+0.12	−1.39	7.38	7.41	7.40	+ 6
41	B	Sept. 17 15 08	663.631	1.143	+0.26	−1.47	7.52	7.33	7.42	0
42	B	22 14 48	668.617	1.218	+0.10	−1.21	7.36	7.59	7.48	+ 4
43	B	22 14 54	668.621	1.222	+0.26	−1.33	7.52	7.47	7.50	+ 6
44	B	30 14 46	676.615	1.852	−0.18	−1.93	7.08	6.87	6.98	− 8
45	B	30 14 51	676.619	1.855	−0.11	−1.91	7.15	6.89	7.02	− 4

TABLE VIII—Continued

OBSERVATIONS OF Z VULPECULAE

No.	Obs.	Date, G.M.T.	Julian Day	Helioc. Phase	v-a	v-b	$v(a)$	$v(b)$	v	O-C
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		1912 d. h. m.		days	mag.	mag.	mag.	mag.	mag.	mag.
46	B	Oct. 3 15 54	2419679.662	2.444	+1.42	−0.05	8.68	8.75	8.72	− 1
47	B	3 16 10	679.674	0.000	+1.51	−0.07	8.77	8.73	8.75	− 4
48	B	3 16 27	679.685	0.012	+1.37	−0.18	8.63	8.62	8.62	−10
49	B	3 16 42	679.696	0.022	+1.31	−0.27	8.57	8.53	8.55	− 2
50	B	3 16 59	679.708	0.034	+1.12	−0.30	8.38	8.50	8.44	+ 4
51	B	3 17 14	679.718	0.044	+1.23	−0.54	8.49	8.26	8.38	+11
52	B	4 13 43	680.572	0.898	−0.09	−1.88	7.17	6.92	7.04	−17
53	B	14 13 02	690.543	1.048	+0.13	−1.61	7.39	7.19	7.29	− 5
54	B	29 14 20	705.597	1.372	−0.08	−1.52	7.18	7.28	7.23	− 8
55	B	29 14 26	705.601	1.376	+0.05	−1.49	7.31	7.31	7.31	+ 1
56	B	Nov. 2 13 14	709.551	0.415	−0.06	−1.54	7.20	7.26	7.23	+ 6
57	B	2 13 20	709.556	0.420	−0.11	−1.51	7.15	7.29	7.22	+ 6
58	B	2 15 46	709.657	0.521	−0.18	−1.60	7.08	7.20	7.14	+ 4
59	B	9 12 36	716.525	0.024	+1.20	−0.42	8.46	8.38	8.42	−11
60	B	9 12 53	716.537	0.035	+1.22	−0.40	8.48	8.40	8.44	+ 4
61	B	9 13 23	716.558	0.056	+0.93	−0.66	8.19	8.14	8.16	+ 3
62	B	9 14 04	716.586	0.085	+0.44	−0.87	7.70	7.93	7.82	− 2
63	B	Dec. 14 12 15	751.510	0.637	−0.34	−1.87	6.92	6.93	6.92	−14
64	B	14 12 20	751.514	0.641	−0.08	−1.82	7.18	6.98	7.08	+ 2
65	B	19 11 58	756.499	0.715	−0.05	−1.47	7.21	7.33	7.27	+19
66	B	19 12 03	756.502	0.719	+0.12	−1.73	7.38	7.07	7.22	+14
67	B	21 11 54	758.496	0.257	+0.02	−1.58	7.28	7.22	7.25	− 5
68	B	21 12 00	758.500	0.262	0.00	−1.54	7.26	7.26	7.26	− 4

TABLE IX

NORMAL PLACES FOR Z VULPECULAE

No.	Helioc. Phase	Mag.	No. Obs.	O-C	No.	Helioc. Phase	Mag.	No. Obs.	O-C
	days			mag.		days			mag.
1	0.012	8.75	2	+0.02	16	1.263	7.39	4	−0.01
2	0.023	8.48	2	−0.06	17	1.391	7.28	4	0.00
3	0.034	8.44	2	+0.04	18	1.626	7.12	2	0.00
4	0.044	8.38	1	+0.10	19	1.827	7.11	1	+0.05
5	0.056	8.16	1	+0.03	20	1.853	7.00	2	−0.06
6	0.085	7.82	1	−0.02	21	2.197	7.32	3	+0.02
7	0.259	7.26	2	−0.04	22	2.293	7.40	4	0.00
8	0.383	7.23	4	+0.04	23	2.322	7.45	3	−0.04
9	0.521	7.14	1	+0.04	24	2.341	7.60	2	0.00
10	0.639	7.00	2	−0.06	25	2.366	7.82	2	0.00
11	0.720	7.20	3	+0.12	26	2.380	7.97	4	+0.02
12	0.881	7.10	2	−0.10	27	2.394	8.03	2	−0.05
13	1.044	7.34	2	0.00	28	2.410	8.24	2	−0.02
14	1.143	7.42	1	0.00	29	2.432	8.66	1	+0.10
15	1.207	7.44	4	0.00	30	2.449	8.74	2	−0.04

The greater error in the measured difference *v-b* is due probably to the faintness of star *b* in the 4½-inch telescope.

The difference between the time of minimum adopted for the Laws Observatory results and that computed by Astbury's formula suggests that the period of this star may be improved. The times of minimum light observed by Astbury and de Roy ap-

TABLE X

MEAN LIGHT CURVE OF Z VULPECULAE

Helioc. Phase	Mag.	Helioc. Phase	Mag.	Helioc. Phase	Mag.	Helioc. Phase	Mag.
days		days		days		days	
0.00	8.79	0.45	7.14	1.25	7.42	2.05	7.16
0.01	8.75	0.50	7.11	1.30	7.37	2.10	7.21
0.02	8.60	0.55	7.08	1.35	7.33	2.15	7.26
0.03	8.46	0.60	7.06	1.40	7.28	2.20	7.30
0.04	8.32	0.65	7.06	1.45	7.24	2.25	7.36
0.05	8.20	0.70	7.08	1.50	7.20	2.30	7.42
0.06	8.09	0.75	7.10	1.55	7.16	2.35	7.68
0.07	8.00	0.80	7.13	1.60	7.13	2.36	7.76
0.08	7.90	0.85	7.17	1.65	7.10	2.37	7.85
0.10	7.72	0.90	7.21	1.70	7.08	2.38	7.94
0.15	7.45	0.95	7.26	1.75	7.06	2.39	8.04
0.20	7.37	1.00	7.30	1.80	7.06	2.40	8.14
0.25	7.31	1.05	7.34	1.85	7.06	2.41	8.26
0.30	7.26	1.10	7.39	1.90	7.07	2.42	8.40
0.35	7.22	1.15	7.42	1.95	7.10	2.43	8.54
0.40	7.18	1.20	7.44	2.00	7.12	2.44	8.69

TABLE XI

COMPARISON WITH PREVIOUS OBSERVATIONS OF MINIMA

Observer	Julian Date	Interval	E	Period	Wt.	O-C
	days	days		days		days
Astbury	2418199.39	1480.285	−603	2.45487	6	−0.00005
Astbury	204.33	1475.345	601	482	6	− 10
Astbury	285.25	1394.425	568	497	5½	+ 5
de Roy	432.60	1247.075	508	487	5	− 5
de Roy	437.50	1242.175	506	489	5	− 3
de Roy	459.54	1220.135	497	500	5	+ 8
de Roy	464.52	1215.155	495	486	5	− 6
Astbury	496.38	1183.295	482	497	5	+ 5
Astbury	523.35	1156.325	471	504	4½	+ 12
Astbury	528.36	1151.315	469	483	4½	− 5
A. and de R.	550.38	1129.295	460	499	4½	+ 7
de Roy	555.34	1124.335	458	488	4½	− 4
Astbury	604.39	1075.285	438	499	4½	+ 7
				2d.45492		±0d.00001

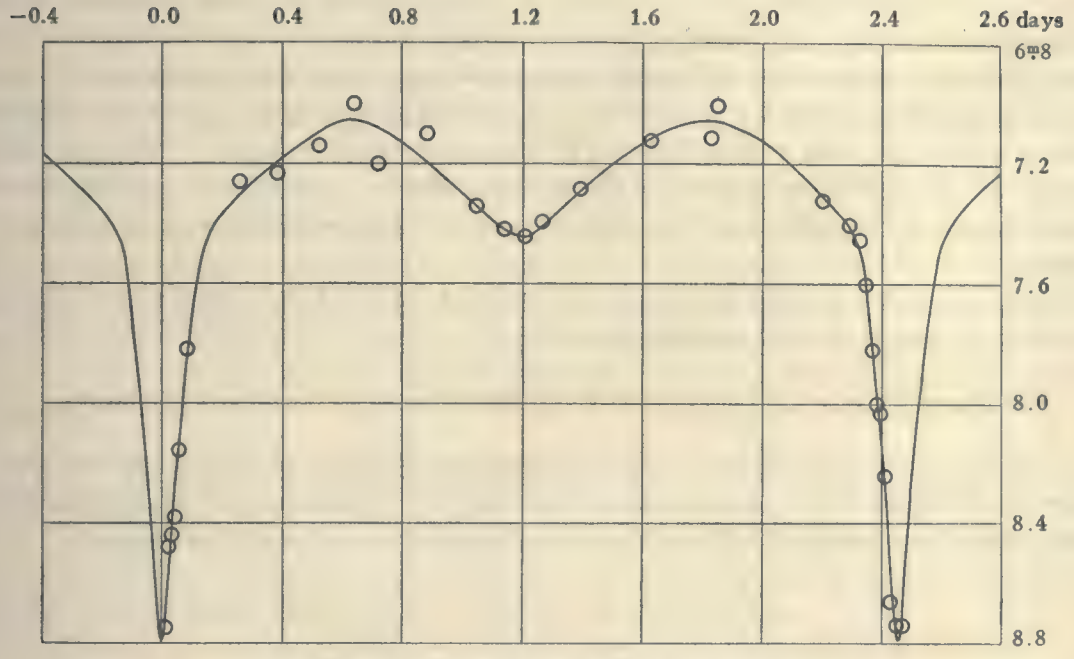


Figure 2. Light Curve of Z Vulpeculae

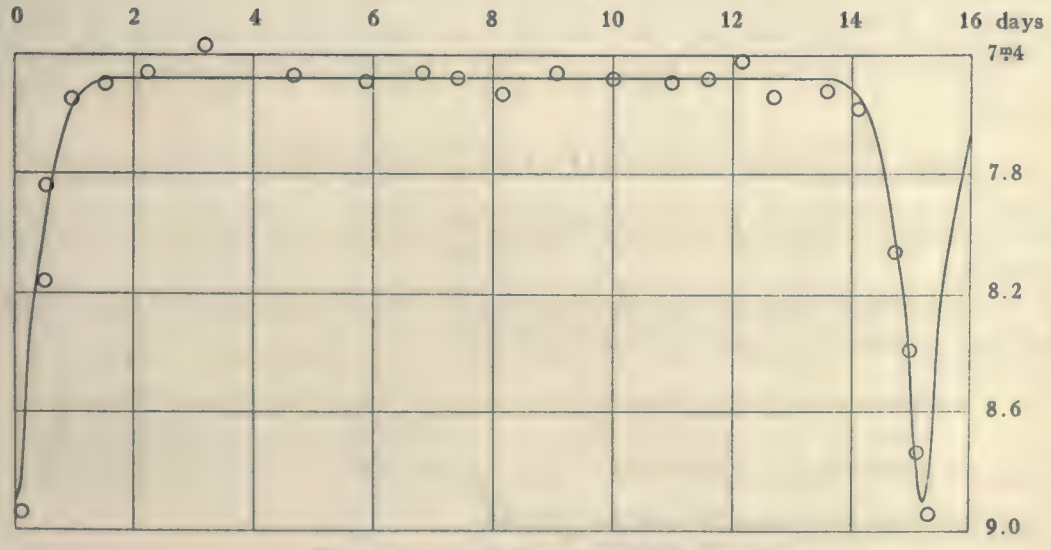


Figure 3. Light Curve of RZ Scuti

pear in Table XI. Four observations of earlier date by Flint are not included here, because the times are given only to tenths of a day and the decrease in weight due to this is not balanced by a sufficiently greater interval of time. The interval between these times and the time adopted for the present series appears in the third column of the table, followed in the next column by the number of fluctuations during the interval. A division gives the period for each comparison. The mean period with its probable error is appended. As the difference 0.00008 days between the two periods is not sufficient to seriously affect the results from the present series of observations, a recomputation is not made.

The star Z Vulpeculae is an eclipsing variable of the Beta Lyrae type. The elements that represent the light variation are:

$$\text{Heliocent. Min.} = \text{J.D. } 2419679.675 + 2^{d}.45492 \cdot E, \text{ G.M.T. (Final Elements)}$$

The range of fluctuation, based on the adopted magnitude 7.26 for comparison star *a*, is between $7^{m}.06$ and $8^{m}.79$. At secondary minimum the brightness is $7^{m}.44$, and the phase does not exactly correspond with a time midway between two principal minima.

THE ALGOL VARIABLE RZ SCUTI (35.1908)

The variable character of RZ Scuti was discovered by Miss Cannon[1]. The star appears faint on two of the 35 plates of this region secured between 1889 and 1904 at the Harvard College Observatory. The variation is announced as probably of the Algol type. This was confirmed by Zinner[2] who from 70 observations made in 1911 derived the elements:

$$\text{Min.} = \text{J.D. } 2419276.2507 + 15^{d}.13194 \cdot E, \text{ G.M.T. (Zinner's Elements)}$$

He finds a range of magnitude between $7^{m}.4$ and $8^{m}.3$, and about 72 hours for the duration of the eclipse.

The observing data for the variable and for the two comparison stars employed for the Laws Observatory measures are as follows:

Variable	= BD. −9°4736, $7^{m}.5$,	R.A. (1900) $18^{h}\ 21^{m}.1$,	Decl. (1900) −9° 15′,
Comp.star *a* =	−8°4606, $7^{m}.5$,	$18^{h}\ 21^{m}.1$,	−8° 15′,
Comp.star *b* =	−8°4611, $6^{m}.7$,	$18^{h}\ 21^{m}.8$,	−8° 06′.

[1]Harvard College Observatory Circular, No. 137, 1908.

[2]Astronomische Nachrichten, 190, 377, 1912.

The Harvard Durchmusterung gives for BD. −8°4606 magnitude 7.50, and for BD. −8°-4611 magnitude 6.55.

The following discussion is based on 92 observations of the variable made in 1912 with the polarizing photometer, the intention being to cover all parts of the light curve. These observations appear in Table XII. A preliminary discussion indicates that the probable error of a single measure *v-a* is much smaller than for *v-b;* also that it is smaller than the probable error of a single observation of the variable referred to both comparison stars. It follows either that star *b* is too bright to be well observed, or that its light is not constant. In either event it is clear that the measures referred to this star should be rejected as they detract from the accuracy of the results. The observed magnitudes of the variable in Table XII are thus referred to comparison star *a* alone. A comparison of the present results with the time of minimum given by Zinner yields the following elements:

$$\text{Min.} = \text{J.D. } 2419640.90 + 15^{d}.194 \cdot E, \text{ G.M.T. (Improved Elements)}$$

The adopted magnitude of star *a* is $7^{m}.50$, the value assigned at Harvard. Collecting separately the measures *v-a* made by each observer during constant light of the star, we have from the means these values for the variable star:

from 25 observations by Baker, $v = 7^{m}.52 \pm 0^{m}.01$,
from 43 observations by Wylie, $v = 7^{m}.44 \pm 0^{m}.01$.

The probable error of a single measure by Baker is $\pm 0^{m}.086$, by Wylie $\pm 0^{m}.078$. The systematic difference between the two observers is established; and if we suppose that the difference is constant during the changing light of the variable star, we have these corrections to the observed magnitude of the variable:

$$\Delta\text{Mag. (Baker)} = -0^{m}.04, \quad \Delta\text{Mag. (Wylie)} = +0^{m}.04.$$

With the adopted magnitude of star *a* and these corrections the definitive magnitude of the variable for each observation is given in column (8) of the table.

The observations are now formed into 23 normal places with regard to phase. These appear in table XIII and are represented graphically in Figure 3, with the mean light curve drawn to best satisfy them. While the curve at minimum is not well defined because of the limited opportunity to observe this phase, the character of the light variation is shown throughout the period.

The star RZ Scuti is an eclipsing variable of the Algol type. The elements that

TABLE XII

OBSERVATIONS OF RZ SCUTI

No.	Obs.	Date, G.M.T.	Julian Day	Phase	Readings		v-a	v	O-C
(1)	(2)	(3)	(4)	(5)	(6)		(7)	(8)	(9)
		1912 d. h. m.		days	a	v	mag.	mag.	
1	B	July 7 16 48	2419591.70	11.58	28.7	28.5	+0.01	7.47	− 1
2	B	7 17 17	591.72	11.60	30.0	29.1	+0.06	7.52	+ 4
3	B	8 15 17	592.64	12.51	24.8	26.3	−0.11	7.35	−13
4	B	8 15 31	592.65	12.52	29.7	25.9	+0.28	7.74	+26
5	B	9 15 24	593.64	13.52	25.2	25.3	0.00	7.46	− 2
6	B	9 16 13	593.68	13.55	29.4	29.2	+0.01	7.47	− 1
7	B	9 16 24	593.68	13.56	27.0	23.8	+0.20	7.66	+18
8	B	12 14 57	596.62	1.30	31.3	32.9	−0.09	7.37	−13
9	B	12 15 13	596.63	1.32	29.9	29.4	+0.03	7.49	− 1
10	B	14 15 20	598.64	3.32	33.1	33.2	0.00	7.46	− 2
11	B	14 15 38	598.65	3.33	23.6	26.5	−0.23	7.23	−25
12	B	18 14 44	602.61	7.30	23.6	23.4	+0.02	7.48	0
13	B	18 14 59	602.62	7.31	22.8	23.0	−0.02	7.44	− 4
14	B	19 14 42	603.61	8.29	20.0	21.0	−0.09	7.37	−11
15	B	19 15 11	603.63	8.32	24.2	22.2	+0.18	7.64	+16
16	B	20 14 35	604.61	9.29	22.0	23.7	−0.15	7.31	−17
17	B	20 14 47	604.62	9.30	21.9	21.7	+0.02	7.48	0
18	B	22 14 40	606.61	11.29	25.7	24.0	+0.13	7.59	+11
19	B	22 14 52	606.62	11.30	25.6	25.6	0.00	7.46	− 2
20	B	23 14 19	607.60	12.28	17.8	19.9	−0.24	7.22	−26
21	B	23 14 34	607.61	12.29	23.2	21.7	+0.14	7.60	+12
22	B	25 14 50	609.62	14.30	22.4	20.8	+0.15	7.61	− 1
23	B	25 15 05	609.63	14.31	22.0	20.8	+0.13	7.59	− 3
24	B	26 14 42	610.61	0.10	23.4	12.1	+1.38	8.84	+ 4
25	B	26 15 00	610.62	0.11	23.4	11.1	+1.58	9.04	+26
26	B	27 14 14	611.59	1.08	19.2	18.1	+0.12	7.58	+ 4
27	B	27 14 29	611.60	1.09	18.9	21.0	−0.22	7.24	−30
28	B	30 14 49	614.62	4.10	23.6	24.9	−0.11	7.35	−13
29	B	31 14 59	615.62	5.11	23.8	22.1	+0.16	7.62	+14
30	B	Aug. 1 16 31	616.69	6.18	23.6	23.8	−0.01	7.45	− 3
31	B	3 14 23	618.60	8.09	25.2	24.6	+0.05	7.51	+ 3
32	W	4 15 15	619.64	9.12	27.4	29.0	−0.12	7.42	− 6
33	B	5 14 32	620.61	10.09	29.9	29.1	+0.05	7.51	+ 3
34	B	10 14 44	625.61	15.10	31.6	17.0	+1.27	8.73	− 7
35	W	11 15 03	626.63	0.92	28.8	29.3	−0.03	7.51	− 9
36	W	12 15 58	627.67	1.96	30.8	30.4	+0.01	7.55	+ 7
37	W	12 16 13	627.68	1.97	31.5	31.0	+0.03	7.57	+ 9
38	B	16 14 05	631.59	5.88	29.8	30.7	−0.05	7.41	− 7
39	W	17 14 14	632.59	6.89	27.4	27.4	0.00	7.54	+ 6
40	W	17 14 32	632.61	6.90	25.2	26.8	−0.13	7.41	− 7
41	W	18 14 46	633.62	7.91	34.0	33.0	+0.06	7.60	+12
42	W	21 14 48	636.62	10.91	24.2	25.4	−0.09	7.45	− 3
43	W	21 15 02	636.63	10.92	27.1	31.4	−0.30	7.24	−24
44	W	23 16 08	638.67	12.97	24.3	27.1	−0.21	7.33	−15
45	W	24 14 30	639.60	13.90	22.8	20.2	+0.26	7.80	+30

TABLE XII—Continued

OBSERVATIONS OF RZ SCUTI

No.	Obs.	Date, G.M.T.	Julian Day	Phase	Readings		$v-a$		O-C
(1)	(2)	(3)	(4)	(5)	(6)		(7)	(8)	(9)
		1912 d. h. m.		days	a	v	mag.	mag.	
46	W	Aug. 24 14 43	2419639.61	13.91	20.4	22.3	−0.20	7.34	−16
47	W	25 16 10	640.67	14.97	23.6	15.7	+0.85	8.39	− 5
48	W	27 14 52	642.62	1.72	16.1	15.8	+0.05	7.59	+11
49	W	27 15 02	642.63	1.73	16.1	15.8	+0.04	7.58	+10
50	W	30 14 56	645.62	4.72	24.8	27.0	−0.17	7.37	−11
51	W	30 15 04	645.63	4.73	27.1	27.2	−0.01	7.53	+ 5
52	W	31 14 24	646.60	5.70	29.4	28.3	+0.08	7.62	+14
53	W	31 14 34	646.61	5.71	26.9	27.5	−0.05	7.49	+ 1
54	W	Sept. 1 14 48	647.62	6.72	28.2	31.7	−0.22	7.32	−16
55	W	1 14 59	647.62	6.72	30.0	29.2	+0.05	7.59	+11
56	W	3 14 23	649.60	8.70	22.0	22.8	−0.07	7.47	− 1
57	W	3 14 35	649.61	8.71	22.7	23.9	−0.10	7.44	− 4
58	W	4 14 30	650.60	9.70	24.4	25.3	−0.06	7.48	0
59	W	4 14 43	650.61	9.71	25.7	27.4	−0.12	7.42	− 6
60	W	5 14 41	651.61	10.71	28.2	27.6	+0.04	7.58	+10
61	W	5 14 51	651.62	10.72	28.8	27.8	+0.07	7.61	+13
62	W	6 14 34	652.61	11.71	22.6	24.2	−0.13	7.41	− 7
63	W	7 14 25	653.60	12.70	22.0	19.6	+0.24	7.78	+30
64	W	7 14 35	653.61	12.71	20.2	20.8	−0.06	7.48	0
65	W	8 14 40	654.61	13.71	22.8	20.7	+0.21	7.75	+27
66	W	8 14 58	654.62	13.72	20.6	21.2	−0.06	7.48	0
67	W	9 14 27	655.60	14.70	24.0	17.3	+0.68	8.22	+22
68	W	9 14 41	655.61	14.71	19.7	16.6	+0.36	7.90	−12
69	W	10 14 06	656.59	0.49	22.5	17.8	+0.48	8.02	+ 2
70	W	10 14 20	656.60	0.50	21.3	14.9	+0.75	8.29	+31
71	W	10 14 25	656.60	0.51	18.5	16.4	+0.26	7.80	−15
72	W	10 14 37	656.61	0.52	18.1	16.0	+0.26	7.80	−13
73	W	10 14 42	656.61	0.52	18.6	15.0	+0.45	7.99	+ 6
74	W	10 14 57	656.62	0.53	17.2	15.5	+0.23	7.77	−15
75	W	11 14 01	657.58	1.49	29.0	28.3	+0.05	7.59	+11
76	W	11 14 12	657.59	1.50	26.0	27.7	−0.13	7.41	− 7
77	W	12 14 10	658.59	2.50	23.9	25.4	−0.13	7.41	− 7
78	W	12 14 22	658.60	2.50	24.9	28.1	−0.25	7.29	−19
79	W	17 14 12	663.59	7.50	19.8	19.8	−0.01	7.53	+ 5
80	W	17 14 25	663.60	7.51	20.9	21.8	−0.08	7.46	− 2
81	W	21 14 13	667.59	11.50	24.1	24.4	−0.03	7.51	+ 3
82	W	23 13 44	669.57	13.48	19.4	22.7	−0.32	7.22	−26
83	W	23 13 53	669.58	13.48	21.5	21.4	+0.01	7.55	+ 7
84	B	Oct. 4 13 09	680.55	9.26	24.7	22.9	+0.16	7.62	+14
85	W	13 12 44	689.53	3.05	26.8	29.0	−0.16	7.38	−10
86	W	13 12 54	689.54	3.06	26.1	28.0	−0.14	7.40	− 8
87	W	20 12 23	696.52	10.03	38.3	37.6	+0.03	7.57	+ 9
88	W	20 12 33	696.52	10.04	28 7	30.6	−0.13	7.41	− 7
89	W	22 12 18	698.51	12.03	39.3	39.8	−0.03	7.51	+ 3
90	W	22 12 28	698.52	12.04	31.5	34.6	−0.18	7.36	−12
91	B	26 12 12	702.51	0.83	21.7	18.5	+0.33	7.79	+14
92	B	26 12 22	702.52	0.84	23.9	21.9	+0.18	7.64	0

TABLE XIII

NORMAL PLACES OF RZ SCUTI

No.	Phase	Mag.	No. Obs.	O-C	No.	Phase	Mag.	No. Obs.	O-C
	days			mag.		days			mag.
1	0.11	8.94	2	+0.14	13	9.06	7.46	6	−0.02
2	0.50	8.16	2	+0.16	14	9.92	7.48	5	0.00
3	0.52	7.84	4	−0.12	15	10.98	7.49	6	+0.01
4	0.95	7.55	5	−0.03	16	11.59	7.48	4	0.00
5	1.51	7.50	6	+0.02	17	12.16	7.42	4	−0.06
6	2.23	7.46	4	−0.02	18	12.68	7.54	5	+0.06
7	3.19	7.37	4	−0.11	19	13.58	7.52	7	+0.04
8	4.67	7.47	4	−0.01	20	14.10	7.58	4	+0.02
9	5.87	7.49	4	+0.01	21	14.71	8.06	2	+0.02
10	6.81	7.46	4	−0.02	22	14.97	8.39	1	0.00
11	7.40	7.48	4	0.00	23	15.10	8.73	1	0.00
12	8.15	7.53	4	+0.05					

represent the light variations are:

$$\text{Min.} = \text{J.D. } 2419640.90 + 15^{d}.194 \cdot E, \text{ G.M.T. (Improved Elements)}$$

The light remains constant at magnitude 7.48 between phases $1^{d}.6$ and $13^{d}.6$ and decreases during eclipse to magnitude 8.90. The duration of eclipse is 3.2 days, practically in agreement with the duration given by Zinner. There is no evidence of a secondary minimum.

It will now be of interest to collect the probable errors of the measures on the three stars thus far discussed. The values derived above are for single measures, four settings on the variable and four on the comparison star. They are as follows:

Star	Comparison	Baker	Wylie
YZ Sagittarii	*v-a*	±0.076	±0.090
(irregular curve)	*v-b*	77	85
Z Vulpeculae	*v-a*	66	
	v-b	80	
RZ Scuti	*v-a*	86	78
[(constant light)			
Mean Probable Error =		±0.077	±0.084

The probable error of a single measure of a Pleiades pair with the glass wedge photometer was found by Seares[1] to be ±0.066, and the mean of corresponding values since derived by him and by other observers is in close agreement with this. The only record I can

[1]Laws Observatory Bulletin No. 7,104,1905.

find of the probable error of a single measure with the polarizing photometer is that derived from 35 observations of X Lacertae by Haynes[1]. This was found to be $\pm 0^m.103$, a value exceeding any in the list for the present series. The photometer was attached to the 7½-inch telescope. The conclusion is that the accuracy of the polarizing photometer has probably been increased by its use in connection with the smaller telescope. The same result would be anticipated for the 7½-inch telescope if an eyepiece of longer focus is substituted. Additional improvement in the results is to be desired, and it is hoped that the polarizing photometer will prove as efficient as the wedge photometer with further experience in its use.

PRELIMINARY NOTE ON THE VARIATION OF TU CASSIOPEIAE (17.1911)

The variability of TU Cassiopeiae was discovered in 1911 by Miss Cannon[2] at the Harvard College Observatory. From an examination of 46 plates of this region a fluctuation of 1.4 magnitudes was found and the period appears to be short. The spectrum is of Class G. From five observations made at Bamberg between June and October 1911, Zinner confirmed the variability of the star and derived the elements:

$$\text{Max.} = \text{J.D. } 2419244 + 59^d.E, \text{ G.M.T. (Elements by Zinner)}$$

The range of magnitude according to Zinner is between $7^m.7$ and $8^m.5$.

The star has been on the program of the Laws Observatory since July 1912 and the observations, now partly reduced, indicate a short period as Miss Cannon has suggested. The preliminary elements are:

$$\text{Max.} = \text{J.D. } 2419631.8 + 2^d.137 \cdot E, \text{ G.M.T. (Preliminary Elements)}$$

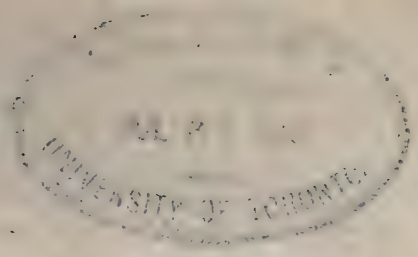
The fluctuation appears to be of the Cepheid type.

[1]Laws Observatory Bulletin No. 13, 197, 1907.
[2]Harvard College Observatory Circular No. 164, 1911.

Columbia, Missouri, 1913 February 8.

70

July 31



THE UNIVERSITY OF MISSOURI BULLETIN

ASTRONOMICAL SERIES

LAWS OBSERVATORY BULLETIN NO. 21

THE ROTATING ELLIPSOID RU CAMELOPARDALIS

By Harlow Shapley

1. *Introduction.* The light received at any time from a binary system of eclipsing stars is given by the relation,

$$L = [L_1 + (1 - \alpha)L_2]\ (1 - \varepsilon^2 \sin^2 i \cos^2\theta)^{-\frac{1}{2}}, \qquad (1)$$

where L_1 is the light of the eclipsing star, L_2 is the light of the star that undergoes eclipse, α is the percentage of the eclipse at the given time, and the radical involves the prolateness of the component stars.[1] For spherical stars the radical reduces to unity and outside of eclipse, when $\alpha = 0$, the light is simply the combined brightness of the two stars, that is, the constant L_0. For a system of ellipsoidal stars the amount of the light variation due to rotation can be computed readily from the shape of the non-eclipse portion of the light curve, and its effect removed, leaving the characteristic eclipse curve with constant maximum brightness. In the equation above θ is the true anomaly, counted from principal minimum, i is the inclination of the orbit to the plane tangent to the celestial sphere, and ε is the eccentricity of the equatorial section of the assumed truly ellipsoidal stars. Professor Russell has pointed out[2] that by plotting the square of the observed light intensity against the corresponding $\cos^2\theta$, the points representing the light curve outside of eclipse will fall on a straight line whose slope is $\varepsilon^2 \sin^2 i$. When the eclipse begins the points drop below this line. The inclination can be determined, with the other elements, from the "rectified" eclipse curve, and the actual ellipticity of the stars is immediately derived. From the light curve it is possible, however, to obtain only two of the three axes of the ellipsoids. The axis of rotation, which is of course the shortest

[1] H. N. Russell, On the Determination of the Orbital Elements of Eclipsing Variable Stars, *Astrophysical Journal*, 36, 64.

[2] *loc. cit.*

of the three, is indeterminate, but it may be estimated, at least roughly, on dynamical principles.[1]

The relations stated above are based on the assumption that the stellar surfaces appear as uniformly illuminated disks. Assuming on the other hand that the disks are darkened to zero at the limb, it is found that the ellipticity of the components of an eclipsing system can be determined as readily as for the "uniform" hypothesis.[2] The non-eclipse portion of the curve in this case is represented by the equation,

$$L = L_0 (1 - Z \cos^2\theta), \qquad (2)$$

where L is the observed light intensity, L_0 is a constant, and Z is the darkened "rectification" factor, given by

$$Z = (\tfrac{1}{3}\varepsilon^2 + \tfrac{1}{175}\varepsilon^4 \ldots.) \sin^2 i. \qquad (3)$$

It is now necessary to plot $\cos^2\theta$ against the first power of the observed light intensity, and the determination of the ellipticity is evident. In practice it is found that the hypothesis of darkening gives a smaller value of the ellipticity.

This method of treating the light variations of rotating elliptical stars is by no means restricted to eclipsing binary systems, but can be applied as well to the rotation of an isolated ellipsoidal stellar mass. Considering that we find in eclipsing systems all the various stages of development, from stars in contact or overlapping to the widely separated pair, we are led to expect the occasional discovery of an elongated star not yet divided into components. Such a rotating mass, if it closely follows the physical laws applicable to masses of homogeneous incompressible liquids, would be successively spheroidal, ellipsoidal, and pear-shaped, with continually increasing density and with a decreasing period of rotation up to the time of division. In the study of the orbits of eclipsing binaries, which has been carried on at Princeton during the last two years, the light curves of three stars have been found to be most satisfactorily explained on the hypothesis that the objects are rotating ellipsoids.

The southern variable S Antliae was originally classed as an eclipsing star, later as a short period variable. Professor E. C. Pickering has determined the curve very accurately with the Meridian Photometer,[3] and Sperra has observed the star extensively.[4] A study of the individual observations shows that the theory of a rotating ellipsoid satisfies the data completely. The star varies from $6^m.37$ to $6^m.73$; its period

[1] *Astrophysical Journal*, **36**, pp. 61, 62.

[2] H. N. Russell and Harlow Shapley, On Darkening at the Limb in Eclipsing Variables, *Astrophysica Journal*, **36**, 397ff.

[3] *Harvard Annals*, **46**, 155; see also *Harvard Annals*, **69**, 44, and *Harvard Circular* No. 41.

[4] *Astronomical Journal*, **18**, 38.

of light change is 0.324 days, and its spectrum is of type F.

SZ Tauri, a variable discovered by Schwarzchild[1] in 1910, has been observed photographically in Göttingen and Potsdam.[2] At maximum the brightness is $7^m.16$, and at minimum six-tenths of a magnitude fainter. The period of the light change is 3.1484 days; the spectrum is of type F8. A good representation of the observations is obtained on the hypothesis that the star is a rotating ellipsoid.

The third ellipsoidal variable is RU Camelopardalis, a star whose light variation was discovered in 1907 by Madame Ceraski at Moscow.[3] As the light curve has been derived from observations made at the Laws Observatory, the detailed discussion of the work and of the results obtained from it will be taken up in the following pages.

2. *Photometric observations of RU Camelopardalis.*[4] Observations of this star have been made for the purpose of determining the period of light variation by Blajko,[5] Luizet,[6] and Ichinohe,[7] and the last two have published light curves. All three observers use the Argelander method of visual estimates. Observations with a wedge photometer were begun at the Laws Observatory in March, 1907, and were continued with more or less interruption for exactly two years. Eighty percent of the 292 measures were made by Mr. E. S. Haynes, and the remainder by Professor F. H. Seares and by the writer. I am greatly indebted to Mr. Haynes for transferring to me all of the data relating to the star. A considerable part of the reductions had been completed by him before the records were sent.

In Table I is given in detail the observational material. Each original measure consisted of four settings on the variable and four on a comparison star. Two comparison stars were always used on each night. To save space in printing I have referred all the measures to one comparison star and have combined them into 146 means, each representing sixteen settings. In column (2) the first letter is the initial of the observer, and the second indicates the instrument—W for the Zeiss-Wedge, and Z for the Zöllner-Müller photometer. The 7½-inch equatorial was used for all the observations except in the few cases where the third letter in the fourth column indicates that the Gans 4½-inch equatorial was employed. The fifth column contains the phases computed from the accepted light elements; the sixth contains the observed magnitude difference, and the last two columns contain, respectively, the number of the normal in which the observation is included and the observed magnitude difference between the two comparison stars.

[1] *Astronomische Nachrichten* 4425.

[2] K. Schwarzchild, Über den Lichtwechsel des Veränderlichen, 41. 1910 Tauri, *Astronomische Nachrichten* 4532.

[3] *Astronomische Nachrichten* 4157.

[4] The position (taken from *Harvard Annals*, **56**, 192) for 1900 is: $\alpha = 7^h\ 10^m.9$, $\delta = 69°\ 52'$.

[5] *Astronomische Nachrichten* 4196.

[6] *Astronomische Nachrichten* 4196; *ibid.* 4614.

[7] *Astronomische Nachrichten* 4319.

TABLE I

OBSERVATIONS OF RU CAMELOPARDALIS

No.	Obs. Instr.	Date, G.M.T.				Julian Day	Phase	△ Mag. (*b*—*v*)	Normal No.	*b*—*a*
(1)	(2)	(3)				(4)	(5)	(6)	(7)	(8)
		1907	d.	h.	m.		days	mag.		mag.
1	HW	Mar.	23	15	11	2417658.63	+ 3.33	+0.14	11	0.38
2	HW		25	19	06	7660.80	+ 5.50	+ .41	13	.68
3	HW		31	20	01	7666.83	−10.65	+ .60	1	.38
4	HW	Apr.	1	19	58	7667.83	− 9.65	+ .54	2	.55
5	HW		8	19	49	7674.83	− 2.65	− .02	7	.33
6	HW		9	18	24	7675.77	− 1.71	− .26	8	.28
7	HW		13	17	32	7679.73	+ 2.25	+ .30	10	.65
8	HW		20	17	43	7686.74	+ 9.26	+ .82	14	.42
9	HW		21	18	52	7687.79	+10.31	+ .62	15	.36
10	HW		23	17	28	7689.73	− 9.92	+ .84	2	.41
11	HW	May	1	18	10	7697.76	− 1.89	− .08	7	.18
12	HW		11	16	18	7707.68	+ 8.03	+ .61	14	.58
13	HW		16	15	44	7712.66	− 9.16	+ .61	2	.46
14	HW		19	15	46	7715.66	− 6.16	+ .43	4	.41
15	HW		20	15	33	7716.65	− 5.17	+ .50	5	.52
16	HW		22	14	34	7718.61	− 3.21	+ .05	6	.18
17	HW	June	2	15	26	7729.64	+ 7.82	+ .62	14	.25
18	HW		10	15	12	7737.63	− 6.36	+ .55	4	.43
19	HW		11	15	21	7738.64	− 5.35	+ .44	5	.51
20	HW		12	14	48	7739.62	− 4.37	+ .40	5	.48
21	HW		14	14	56	7741.62	− 2.37	.00	6	.41
22	HW		15	15	11	7742.63	− 1.36	− .02	7	.58
23	HW		16	16	18	7743.68	− 0.31	− .12	8	.42
24	HZ		17	16	09	7744.67	+ 0.68	− .12	9	.68
25	HZ		25	15	00	7752.62	+ 8.63	+ .56	14	.08
26	HZ		26	15	01	7753.63	+ 9.64	+ .88	15	.06
27	HZ		27	15	32	7754.65	+10.66	+ .72	15	.22
28	HZ		28	14	38	7755.61	−10.55	+ .97	1	.64
29	HZ		28	14	47	7755.62	−10.54	+ .82	1	.21
30	HZ	July	2	14	38	7759.61	− 6.55	+ .50	4	.46
31	HZ		2	14	46	7759.62	− 6.54	+ .74	4	.19
32	HZ		3	14	44	7760.61	− 5.55	+ .58	5	.08
33	HZ		4	14	41	7761.61	− 4.55	+ .40	5	.32
34	HZ		5	14	48	7762.62	− 3.54	+ .28	6	.74
35	HZ		6	15	06	7763.63	− 2.53	+ .07	7	.30
36	HZ		7	15	40	7764.65	− 1.51	− .14	8	.23
37	HZ		8	15	22	7765.64	− 0.52	− .13	8	.37
38	HZ		11	15	15	7768.64	+ 2.48	+ .15	10	.09
39	HZ		19	15	14	7776.63	+10.47	+ .92	15	.31
40	HZ		26	14	32	7783.61	− 4.73	+ .10	5	.46
41	HZ		29	14	42	7786.61	− 1.73	− .18	7	.22
42	HZ		30	14	23	7787.60	− 0.74	− .24	8	.36
43	HZ		31	14	42	7788.61	+ 0.27	− .34	9	.27
44	HZ	Aug.	3	14	28	7791.60	+ 3.26	+ .35	11	.36
45	HZ		8	14	47	7796.62	+ 8.28	+ .82	14	.12

TABLE I—Continued

OBSERVATIONS OF RU CAMELOPARDALIS

No.	Obs. Instr.	Date, G.M.T.			Julian Day	Phase	△ Mag. (b—v)	Normal No.	b—a
(1)	(2)	(3)			(4)	(5)	(6)	(7)	(8)
		1907 d.	h.	m.		days	mag.		mag.
46	HZ	Aug. 9	14	44	2417797.61	+ 9.27	+0.79	15	0.34
47	HZ	10	14	27	7798.60	+10.26	+ .94	15	.18
48	HZ	18	22	16	7806.93	− 3.58	+ .37	6	.17
49	HZ	Sept. 12	22	58	7831.96	− 0.72	− .06	8	.31
50	HZ	18	22	06	7837.92	+ 5.24	+ .44	12	.67
51	HZ	23	20	54	7842.87	+10.19	+ .88	15	.24
52	HZ	Oct. 4	19	55	7853.83	− 1.02	− .18	8	.27
53	HZ	5	22	02	7854.92	+ 0.07	− .18	9	.58
54	HZ	8	20	06	7857.84	+ 2.99	− .34		.44
55	HZ	10	21	46	7859.91	+ 5.06	+ .41	12	.40
56	HZ	11	20	22	7860.85	+ 6.00	+ .49	13	.40
57	SZ	12	19	32	7861.81	+ 6.96	+ .48	14	.26
58	HZ	23	19	12	7872.80	− 4.22	+ .54	5	.66
59	HW	23	19	44	7872.82	− 4.20	+ .36	5	.54
60	HZG	Nov. 3	21	15	7883.89	+ 6.87	+ .58	13	.10
61	HZG	5	20	06	7885.84	+ 8.82	+ .71	14	.16
62	HZG	12	17	54	7892.75	− 6.45	+ .36		.24
63	HZG	Dec. 1	18	23	7911.77	− 9.60	+ .86	2	.47
64	HZG	4	18	17	7914.76	− 6.61	+ .66	4	.60
		1908							
65	HZG	Jan. 1	15	46	7942.66	− 0.88	+ .16		.42
66	HW	1	21	06	7942.88	− 0.66	+ .02	8	.45
67	HW	4	12	33	7945.52	+ 1.98	+ .16	9	.57
68	HW	8	15	00	7949.62	+ 6.08	+ .58	13	.63
69	HW	13	14	56	7954.62	+11.08	+ .83	15	.42
70	HW	14	13	34	7955.56	−10.15	+ .98	2	.53
71	HW	16	14	16	7957.59	− 8.12	+ .70	3	.44
72	HW	17	15	21	7958.64	− 7.07	+ .80	3	.52
73	HW	18	13	43	7959.57	− 6.14	+ .67	4	.45
74	HW	21	13	55	7962.58	− 3.13	+ .30	6	.45
75	HW	24	14	20	7965.60	− 0.11	− .02	8	.53
76	HW	27	14	49	7968.62	+ 2.91	+ .28	10	.49
77	HW	29	14	22	7970.60	+ 4.89	+0.58	11	.31
78	HW	Feb. 6	17	48	7978.74	− 9.14	+1.01	2	.55
79	HW	16	19	04	7988.79	+ 0.91	+0.46		.34
80	HW	21	17	30	7993.73	+ 5.85	+ .66	12	.32
81	HW	22	20	03	7994.84	+ 6.96	+ .54	14	.32
82	HW	27	14	00	7999.58	−10.48	+ .58	1	.30
83	HW	Mar. 6	16	10	8007.67	− 2.39	+ .09	6	.49
84	HW	9	16	56	8010.71	+ 0.66	− .02	9	.23
85	HW	11	15	34	8012.65	+ 2.59	+ .22	10	.17
86	HW	13	15	56	8014.66	+ 4.60	+ .22	11	.26
87	HW	15	15	56	8016.66	+ 6.60	+ .73	13	.24
88	HW	24	19	16	8025.80	− 6.43	+ .46	3	.14
89	HW	Apr. 2	19	02	8034.79	+ 2.56	+ .22	10	.04
90	HW	3	17	08	8035.71	+ 3.48	+ .12	. 10	.30

TABLE I—Continued

OBSERVATIONS OF RU CAMELOPARDALIS

No.	Obs. Instr.	Date, G.M.T.				Julian Day	Phase	△ Mag. (*b*—*v*)	Normal No.	*b*—*a*
(1)	(2)	(3)				(4)	(5)	(6)	(7)	(8)
		1908	d.	h.	m.		days	mag.		mag.
91	HW	Apr.	5	18	58	2418037.79	+ 5.56	+0.38	12	0.29
92	HW		11	18	05	8043.75	−10.65	+ .70	1	.05
93	HW		18	17	04	8050.71	− 3.69	+ .21	6	.38
94	HW		22	16	36	8054.69	+ 0.29	− .46		
95	HW		27	17	57	8059.75	+ 5.35	+ .28	12	.14
96	HW	May	9	16	58	8071.71	− 4.86	+ .28	5	.25
97	HW		9	21	54	8071.91	− 4.66	+ .85		.15
98	SW		11	16	18	8073.68	− 2.89	+ .05	6	.28
99	SW		14	17	04	8076.71	+ 0.14	− .20	9	.33
100	HW		16	16	26	8078.68	+ 2.11	− .06	9	.31
101	HW		18	16	52	8080.70	+ 4.13	− .26	11	.17
102	SW		20	16	06	8082.67	+ 6.10	+ .53	13	.37
103	SW		29	17	15	8091.72	− 7.02	+ .67	3	.43
104	SW		30	17	34	8092.73	− 6.01	+ .66	4	.69
105	SW		31	15	52	8093.66	− 5.08	+ .06		.07
106	SW	June	1	17	15	8094.72	− 4.02	+ .32	5	.57
107	SW		5	17	08	8098.71	− 0.03	− .32	8	.46
108	SW		8	18	06	8101.75	+ 3.01	+ .24	11	.34
109	HW		9	16	46	8102.70	+ 3.96	+ .34	11	.25
110	SW		10	18	02	8103.75	+ 5.01	+ .36	12	.33
111	SW		11	14	50	8104.62	+ 5.88	+ .41	13	.36
112	SW		14	20	54	8107.87	+ 9.13	+ .88	14	.35
113	HW		15	16	26	8108.68	+ 9.94	+ .78	15	.26
114	SW		17	14	42	8110.61	−10.31	+ .78	1	.33
115	HW		18	15	12	8111.63	− 9.29	+ .64	2	.22
116	SW		19	14	36	8112.61	− 8.31	+ .78	3	.35
117	HW		20	14	58	8113.62	− 7.30	+ .66	3	.27
118	SW		21	16	10	8114.67	− 6.25	+ .60	4	.42
119	SW		22	16	38	8115.69	− 5.23	+ .49	5	.54
120	SW		24	17	31	8117.73	− 3.19	+ .31	6	.26
121	HW		25	17	52	8118.74	− 2.18	+ .02	7	.32
122	SW		29	16	15	8122.68	+ 1.76	+ .18	9	.42
123	HW	July	4	15	04	8127.63	+ 6.71	+ .64	13	.28
124	HW		5	15	28	8128.64	+ 7.72	+ .56	14	.41
125	HW		7	16	02	8130.67	+ 9.75	+ .68	15	.25
126	SW		8	17	52	8131.74	+10.62	+ .92	15	.27
127	HW		9	15	49	8132.66	−10.43	+ .96	1	.24
128	HW		10	15	38	8133.65	− 9.44	+ .66	2	.41
129	HW		11	16	13	8134.68	− 8.41	+ .72	3	.36
130	HW		16	15	43	8139.65	− 3.44	+ .14	6	.41
131	HW		18	15	59	8141.67	− 1.42	− .04	7	.49
132	SW		20	15	18	8143.64	+ 0.55	− .02	9	.31
133	SW		21	15	54	8144.66	+ 1.57	− .03	10	.38
134	HW		23	21	52	8146.91	+ 3.82	+ .17	11	.40
		1909								
135	HW	Feb.	25	15	01	8363.63	− 1.18	− .08	7	.41

TABLE I—Continued

OBSERVATIONS OF RU CAMELOPERDALIS

No.	Obs. Instr.	Date, G.M.T.	Julian Day	Phase	△ Mag. (b—v)	Normal No.	b—a
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		1909 d. h. m.		days	mag.		mag.
136	ShW	Feb. 26 16 50	2418364.70	− 0.11	−0.28	8	0.26
137	HW	27 14 44	8365.61	+ 0.80	− .14	9	.47
138	ShW	Mar. 1 14 38	8367.61	+ 2.80	+ .26	10	.52
139	HW	1 14 44	8367.61	+ 2.80	+ .25	10	.50
140	ShW	3 16 43	8369.70	+ 4.89	+ .48	12	.24
141	HW	6 15 50	8372.66	+ 7.85	+ .73	14	.29
142	ShW	10 14 56	8376.62	−10.36	+ .82	1	.36
143	ShW	13 15 31	8379.65	− 7.33	+ .62	3	.36
144	HW	17 14 24	8383.60	− 3.38	+ .41	6	.30
145	HW	18 15 30	8384.65	− 2.33	+ .26	7	.33
146	ShW	20 16 56	8386.71	− 0.27	+ .02	8	.31

NOTES TO THE TABLE OF OBSERVATIONS

1907, July 3. b—a is negative; on all other dates it is positive.
1907, Oct. 8. Order of stars has been changed; reject the observation.
1907, Oct. 12. The first observation was probably made with the Zöllner photometer attached to the Gans telescope. The record as it stands would indicate that both observations were made with the 7½-inch, but with different photometers.
1908, April 22. Comparison stars not directly measured.
1908, July 5. Aperture stopped down to four inches.
1908, July 16. Aperture stopped down to four inches.
Observations 54, 62, 65, 79, 94, 97, and 105 are rejected in the formation of normal places because of the large errors, which are known to be due to clouds, irregularities in the electric current for the artificial star, or uncertainty of the readings.

The stars used for comparison with RU Camelopardalis are a=BD.+70°450 and b=BD.+70°447 Mr. Haynes has made a thorough investigation of the measured magnitude differences between the two stars and concludes that a systematic variation exists. In Table II the measures have been grouped in order of time. Plotting the numbers in column four against those in column five, and drawing a smooth curve, a continuously varying comparison star difference is obtained, which has been applied to the original readings to derive the magnitude differences in the sixth column of Table I. The adopted comparison star curve can be obtained from the quantities in the sixth column of Table II. Its amplitude is $0^m.3$, with an indication of periodicity in a cycle of about 250 days. The satisfactory representation of the comparison star normals is shown by the residuals in the last column. As 145 measures were used, this variation can be considered as definite. It probably is due to the slight variability of one of the comparison stars, but there is no indication as to which star's light fluctuates. All the measures on the variable have been referred to each of the comparison stars in turn and plotted. Since no dissimilarity was detected in the resulting curves, all the observations have been arbitrarily referred to b.

TABLE II

GROUPS OF MEASURES OF THE COMPARISON STAR DIFFERENCES

No.	Limiting Julian Day	No. Obs'ns.	Mean Julian Day	Observed $b-a$	Curve $b-a$	O—C
				mag.	mag.	mag.
1	7658–79	7	7669	+0.46	+0.38	+0.08
2	7686–97	4	7690	.34	.42	− .08
3	7707–18	5	7714	.43	.43	.00
4	7729–44	8	7740	.47	.38	+ .09
5	7752–68	14	7760	.27	.32	− .05
6	7776–91	6	7786	.33	.26	+ .07
7	7796–06	4	7800	.20	.24	− .04
8	7831–42	3	7838	.41	.33	+ .08
9	7853–61	6	7858	.39	.39	.00
10	7872–14	7	7892	.40	.46	− .06
11	7942–59	9	7952	.49	.49	.00
12	7962–78	5	7969	.47	.47	.00
13	7988–07	5	7997	.36	.36	.00
14	8010–25	5	8016	.21	.24	− .03
15	8034–59	6	8044	.20	.19	+ .01
16	8071–82	7	8077	.27	.25	+ .02
17	8091–08	11	8100	.37	.32	+ .05
18	8110–22	9	8115	.35	.35	.00
19	8127–46	12	8136	.35	.39	− .04
20	8363–86	12	8374	.36	.36	.00

TABLE III

PROBABLE ERRORS OF MEASURES ($b-a$) REFERRED TO CURVE

	Haynes	Seares	Shapley	Polarizing Photometer	Wedge Photometer	All
Number of observations	117	21	6	40	104	144
Probable error of one measure	±0m09	±0m10	±0m07	±0m13	±0m08	±0m09
Probable error of mean	± .01	±0.02	±0.03	±0.02	±0.01	±0.01

TABLE IV

OBSERVED TIMES OF MINIMA AND THEIR REPRESENTATION BY LUIZET'S LIGHT ELEMENTS.

No.	Julian Day	Weight	O—Luizet	No.	Julian Day	Weight	O—Luizet
			days				days
1	7676.5	2	−1.0	10	8010.8	2	+0.7
2	7699.0	1	−0.6	11	8032.1	2	−0.1
3	7722.0	1	+0.2	12	8054.3	1	−0.1
4	7744.0	3	0.0	13	8077.0	4	+0.4
5	7766.3	4	+0.1	14	8099.0	3	+0.3
6	7788.2	2	−0.1	15	8121.1	4	+0.2
7	7855.1	2	+0.2	16	8143.0	2	−0.1
8	7943.6	1	+0.1	17	8364.5	2	−0.3
9	7966.0	3	+0.3	18	8387.4	1	+0.4

Table III contains a synopsis of the probable errors and indicates the general accuracy of the work. A single measure with the Zöllner photometer by Seares is not included in the table. The observations with the Zöllner photometer seem to be of an inferior grade, relatively, but there appears to be no reason why they should be excluded from this discussion.

3. *Elements of Light Variation.* From a discussion of the times of all the maxima contained in the table of observations, Mr. Haynes derived a period of 22.20 days. The minima are much more sharply defined than the maxima, but fewer of them had been observed. A more complete discussion of the elements is the recent study by Luizet, whose results, obtained by a least squares solution, are based on observations covering

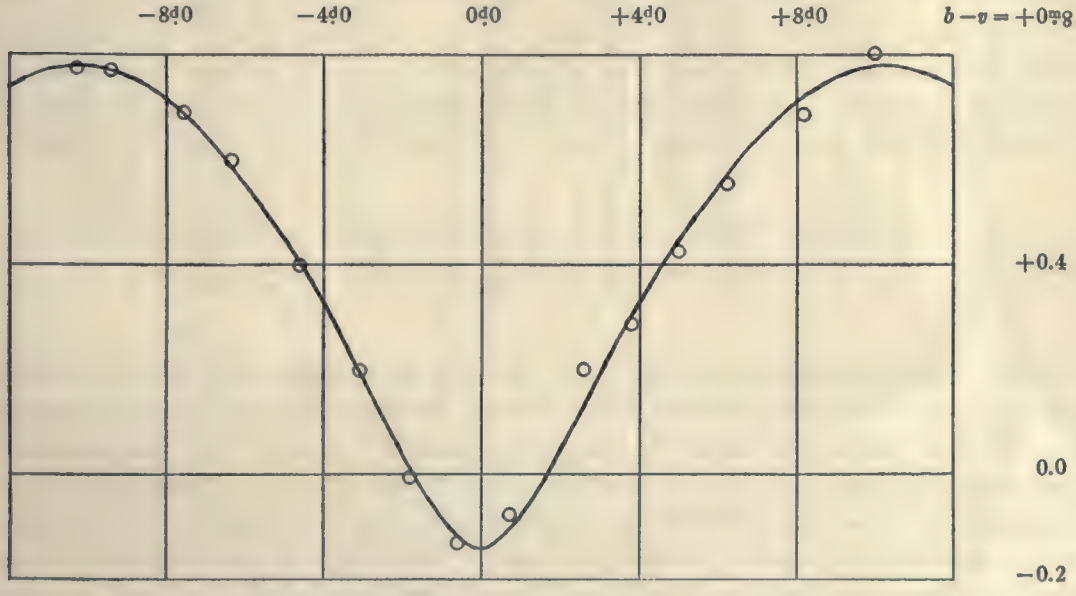


Figure 1. MAGNITUDE CURVE OF RU CAMELOPARDALIS

five years.[1] I have estimated from the detailed plots of our observations the times of the eighteen minima contained in Table IV, and have compared them with the times predicted by Luizet's elements. The representation of my estimates is shown by the residuals in columns four and eight, and is of such a satisfactory character that I have adopted as definitive,

$$\text{Min.} = \text{J.D. } 2417610.96 + 22^{d}.172 E, \text{ G.M.T. (Elements by Luizet)}$$

[1] *Astronomische Nachrichten* 4614.

4. *The Light Curve.* Examining the question of a different range of variation at alternate minima, two sets of normal magnitudes were computed; but finding successive minima to be of sensibly the same shape and depth, I have combined the two into the single set of fifteen normal points of columns two and five of Table V.

TABLE V

NORMAL MAGNITUDES OF RU CAMELOPARDALIS

No.	Phase from Minimum		No. Obs'ns.	Observed Brightness		O—C	
	Days	$\cos^2\theta$		Magn. $b-v$	(Intensity)^2	Int.	Magn.
1	−10.50	0.007	8	+0.78	0.982	$-0^{I^2}012$	−0.007
2	− 9.54	.047	8	+ .77	.964	+ .003	+ .001
3	− 7.50	.236	8	+ .68	.816	+ .009	+ .006
4	− 6.33	.389	8	+ .60	.705	+ .024	+ .019
5	− 4.75	.611	11	+ .40	.487	− .012	− .013
6	− 3.16	.812	11	+ .20	.337	+ .003	+ .005
7	− 1.92	.927	9	− .01	.229	− .011	− .025
8	− 0.65	.992	12	− .14	.180	− .006	− .017
9	+ 0.90	.984	10	− .07	.205	+ .011	+ .030
10	+ 2.60	.870	9	+ .20	.337	+ .050	+ .088
11	+ 3.88	.726	8	+ .28	.391	− .013	− .018
12	+ 5.28	.536	7	+ .44	.525	− .035	− .035
13	+ 6.22	.403	8	+ .55	.643	− .026	− .021
14	+ 8.13	.165	11	+ .67	.802	− .063	− .041
15	+10.20	.016	11	+ .81	1.037	+ .049	+ .027

The points are plotted in Figure 1. They show a variation of about nine-tenths of a magnitude. According to Luizet's estimates the star at maximum is of magnitude 7.93. The light curve published by him is in reasonable accord with the one deduced from our observations. In fact, it was the shape of his light curve that suggested this investigation. The small oscillation in the maximum light is of very low weight, according to his observations, and is probably not real. The range of variation is of the same order of magnitude as found at the Laws Observatory, but Luizet's value is necessarily somewhat doubtful because of the uncertainty of his scale value and of the adopted magnitudes of his comparison stars. For the same reason his curve during the interval of most rapid light changes is scarcely adapted for analytical study. The curve determined by Ichinohe's eye-estimates shows an irregular maximum, but is based on too few observations to throw any doubt on the photometric work.

5. *Interpretation of the Light Changes.* Whether we consider RU Camelopardalis a single rotating ellipsoid or a system of two elliptical stars, the period of rotation is necessarily twice the period of the light change. The angle of rotation can be used to express the interval from minimum just as well as the time in days. The mean times,

t_n, in the table of normals have accordingly been converted into $\cos^2\theta$ by means of the relation $\theta_n = \frac{2\pi t_n}{P}$, where t and the period of rotation, P, are expressed in the same units. Similarly we can convert the observed brightness of the star from stellar magnitude into the square of the light intensity. For this latter purpose I have taken as unit light, $b-v = +0^m.79$. Plotting the square of the light against $\cos^2\theta$, we have at once a representation of the light curve that will enable us to discuss the probable cause of the variation.

In Figure 2 the points taken from the descending branch are indicated by circles, and points from the ascending branch by dots. The curve is practically symmetrical. If the star were a binary and the variation partly due to eclipses we would find the plotted points curving downward as $\cos^2\theta$ approaches unity. That is, while the eclipse is in progress L would decrease more rapidly than could be accounted for by the rotation of an elliptical star. If the light curve were that of a typical Cepheid variable the plotted points would, in general, rise above the straight line that represents the points near unit brightness. It is evident from the diagram that neither of these assumptions is consistent with the observations, which are, however, very accurately represented throughout the

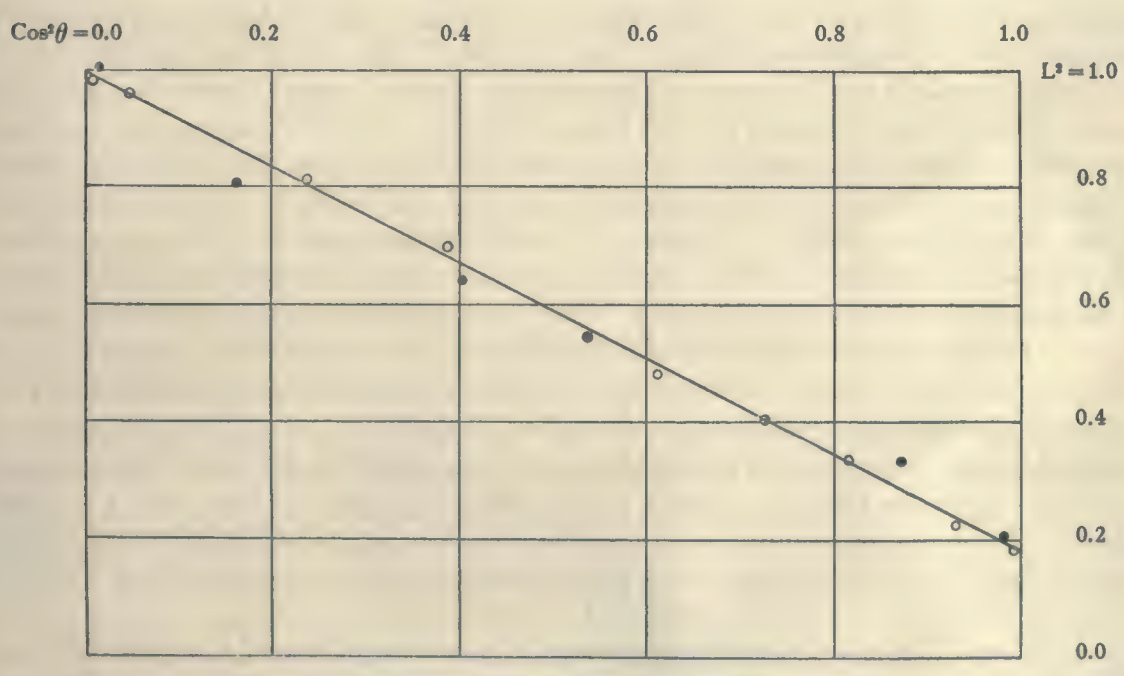


Figure 2. Ellipticity Diagram for Uniform Brightness

whole range by a straight line. The slope of this line is $\epsilon^2\sin^2 i = 0.82 \pm 0.005$. In column seven of the table of normals are given the residuals from the line. The average deviation of a normal point is ± 0.022 in units of L^2.

With the value of $\epsilon^2\sin^2 i$ just derived I have computed the ellipticity-magnitude curve, which is drawn in Figure 1 in the usual manner with the time in days as abscissae. The residuals from this curve, which are given in the last column of Table V, show that the representation of the observations is all that could be desired. The average deviation of a normal point is $\pm 0^m.016$.

The conclusion to be drawn from the above investigation is that the light variations of RU Camelopardalis can be satisfactorily accounted for on the hypothesis of a single, uniformly luminous, ellipsoidal body rotating in a period of 44.344 days. The possibility might be suggested that this is not a single body, but two elliptical stars revolving around their center of gravity in an orbit so highly inclined to the line of sight that the stars eclipse very small portions of each other's light, or do not eclipse at all. RR Centauri is a system of this kind,[1] and several other eclipsing stars approach that condition. From a recent solution for the orbit of RR Centauri I find that more than 70 per cent of the range of variation is due to the change of the apparent area of the highly elliptical stars as they rotate. The orbit is inclined more than 40 degrees to the line of sight and the eclipses are accordingly very shallow. That this is not also the case for RU Camelopardalis can be seen readily from the greater range of variation, and from the general conditions existing in close binary systems. For, if a binary, the component stars must be nearly if not quite in contact, and to obtain eclipses as small as is necessary to even roughly conform with the observations, the inclination of the orbit to the line of sight must be very high. The most favorable circumstance we can impose on these observations are minima of $0^m.15$, which for stars in contact would give an inclination to the line of sight of nearly 40 degrees. Now the quantity determined from the straight line is to be divided by the square of the inclination to the tangent plane in order to obtain the ellipticity of the stars. For the supposed system, the necessary magnitude of the inclination would give a value of the ellipticity which would mean that the major axis of the elliptical equator is more than three times the length of the minor axis—a condition that we are not prepared to accept in the light of our present knowledge concerning close double stars. In fact, we are inclined to believe that the plane of rotation of the single ellipsoid passes very near the earth, for otherwise the great elongation of the rotating mass might involve serious questions of stability.

It should be noted that the hypothesis of a rotating ellipsoid is not the only possible interpretation of the light curves of this star and the two others mentioned in the

[1] *Monthly Notices of the Royal Astronomical Society*, **63**, 537.

introduction. The rotation of a spherical or a spheroidal star, one side of which is intrinsically brighter than the other, would give rise to a similar light variation, and by the proper distribution of light over the star's surface it would be possible to reproduce exactly these observed curves. The period of such a rotating body would be one-half the period of the ellipsoid discussed above. While such an interpretation is possible it seems, however, to be much less likely than the other, for we know that a rotating ellipsoidal mass of gas is a very probable condition in the early evolution of a binary system; but no equally good reason appears for the existence of stars with one side bright. Spectroscopic observations, if possible for these stars, could distinguish between the alternatives. The spotted star would show a shift of the spectral lines, while the ellipsoid would show nothing more than a broadening of the lines at the time of maximum light.

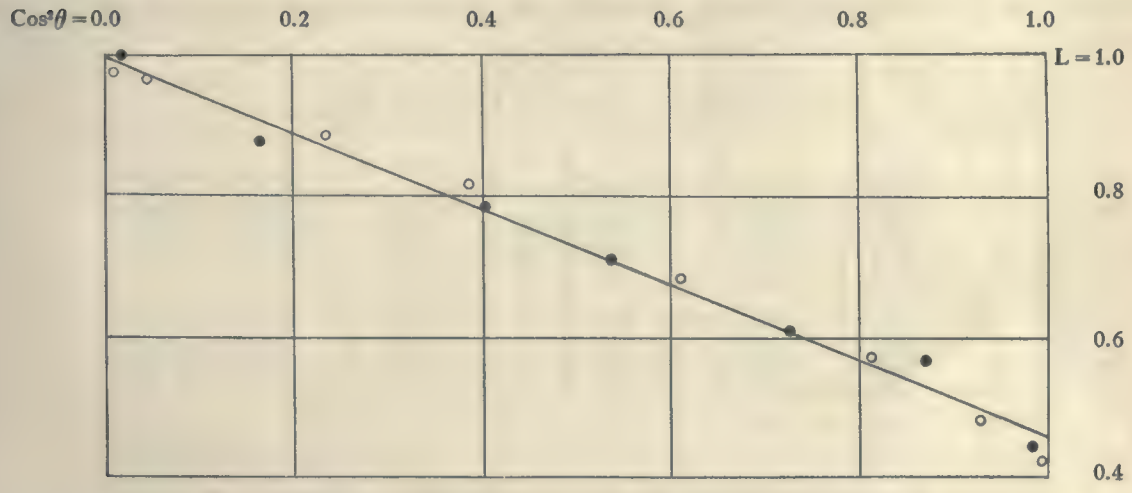


Figure 3. Ellipticity Diagram for Complete Darkening

In the discussion up to this point the rotating ellipsoid has been treated as a uniformly illuminated disk. Considering it now from the standpoint of a disk darkened at the limb, we have merely to plot the values of $\cos^2\theta$ against 1—L. Maximum light (which we adopt as unit intensity) is now most satisfactorily taken at $b—v = +0^m.81$. This plot of the normal points is shown in Figure 3, and the residuals from the best straight line are given in Table VI, where the values of $\cos^2\theta$ are repeated. The representation of the observations is not quite so satisfactory in this case as for an assumed uniform disk, but the errors are still within the probable uncertainty of the observations. The value of the slope is now $Z = 0.54 \pm 0.006$, and we obtain from this quantity the min-

imum value of the eccentricity of the equatorial section of the star,

$$\epsilon^2 \sin^2 i = \tfrac{5}{4} Z - \tfrac{5}{28} Z^2 = 0.62.$$

Again the actual elongation of the star depends on the angle between the plane of rotation and the line of sight. The possibility of a binary system with shallow or no eclipses is practically excluded on the same grounds as before.

The method and formulae used just above are applicable only for complete darkening at the limb. It appears from the nature of the deviations from the two straight lines that an intermediate degree of darkening would represent the observations with still greater accuracy; but it would serve no purpose to extend the study of these observations, which are already satisfactorily represented by the simpler hypotheses. Judging from the evidence deduced from the orbits of eclipsing binaries it is probable that the solution for a darkened disk more nearly represents the actual condition of the star.

TABLE VI

REPRESENTATION OF NORMALS BY DARKENED ELLIPSOID

No.	$\cos^2\theta$	1—L	O—C	No.	$\cos^2\theta$	1—L	O—C
1	0.007	0.027	+0L023	9	0.984	0.555	+0L023
2	.047	.036	+ .011	10	.870	.430	− .040
3	.236	.113	− .015	11	.726	.386	− .006
4	.389	.176	− .034	12	.536	.289	.000
5	.611	.314	− .016	13	.403	.213	− .005
6	.812	.430	− .009	14	.165	.121	+ .032
7	.927	.530	+ .029	15	.015	.000	− .008
8	.992	.583	+ .046				

Concerning the spectral classification Professor E. C. Pickering writes in a recent letter: "R U Camelopardalis proves to have a very peculiar spectrum, which appears to change. Miss Cannon thinks that it may be a form of the fourth type, class N."

If we designate the major and minor axes of the equatorial section of the ellipsoid by a and b, respectively, (taking $i = 90°$), we find on the "uniform" hypothesis, $b = 0.42a$, and for complete darkening, $b = 0.62a$. The curves and results for S Antliae and SZ Tauri will be published elsewhere in the near future, together with some suggestions concerning the polar flattening and the possible density of ellipsoidal variables.

Princeton, N. J. 1913, Feb. 24.

THE UNIVERSITY OF MISSOURI BULLETIN

ASTRONOMICAL SERIES

LAWS OBSERVATORY BULLETIN NO. 22

THE VISUAL AND PHOTOGRAPHIC LIGHT VARIATIONS OF RR LYRAE

By C. C. Kiess

Since the discovery of its variability in 1899,[1] the light-changes of the cluster variable RR Lyrae ($\alpha = 19^h\ 22^m.3$; $\delta = +42°\ 36'$) have been observed by Wendell[2], v. Zeipel[3], Hertzsprung[4], Fontana[5], Townley and Kiess[6]. From the data published by these observers the character of its variation has been determined, and a mean period of 0.566826 day deduced.

During the summers of 1912-13, while at the Lick Observatory, I secured a number of spectrograms of the star for the determination of radial velocity. The velocity proved to be variable in a period identical with the light period. It was during the course of this work that there appeared to be a discrepancy of 1.6 hours between the times of maximum light as predicted from the epoch of visual maximum given by Wendell, and the epoch of photographic maximum given by Hertzsprung. Recently Shapley has announced that from a series of simultaneous visual and photographic observations he found no indication of a lag between photographic and visual maxima[7]; and that, not only for RR Lyrae but also for other stars of this class, successive maxima do not recur with clock-like regularity, but may precede or follow the predicted maximum by a short interval of time[8].

It was to give definite answer, if possible, to some of the questions which have arisen concerning the light-variations of RR Lyrae, that the writer began a series of visual and photographic observations of this star shortly after coming to the Laws Observatory in the autumn of 1913. It is the purpose of this paper to bring together the results that have been obtained thus far, and to point out the extent to which they give answer to the questions stated above.

1. H. C. O. Circular, No. 54, 1901.
2. Annals, H. C. O., 69. Chap. 5, Part I, 1909; Chap. 8, Part II, 1914.
3. Astronomische Nachrichten, 177, 372, 1908.
4. Vierteljahrschrift, 229, 1912.
5. Memorie della Società degli Spettroscopisti italiani, Vol. II, Ser. 2*a*, 1913.
6. Lick Observatory Bulletin, 7, 141-2, 1913.
7. Popular Astronomy, 22, 144, 1914.
8. Astrophysical Journal, 40, 452, 1914.

The visual observations were made with the polarizing photometer attached to either the 7½-inch or the 4½-inch equatorial. All the observations secured prior to June 19, 1914, were made with the larger telescope of which the aperture was reduced to approximately three inches. The smaller telescope was used in making the observations of June 19 and September 20, 1914. Previous bulletins of the Laws Observatory contain adequate descriptions of the photometer and its auxiliary apparatus. It is only necessary to add that after November 10, 1913, the colorimeter was removed and blue glass inserted in its stead.

All the observations dated 1913 were made in the order $tv_4a_4b_4v_4t$, which means that four settings each were made on the variable and comparison stars, the time being recorded at the beginning and close of a complete set of comparisons. The observations dated 1914 were made either in the order $a_4tv_8tb_4$ or $b_4tv_8ta_4$. Care was always taken to have the image of the real star midway between the images of the artificial stars, and to have the line passing through them parallel to the line joining the eyes of the observer.

TABLE I

COMPARISON STARS

Star	B D	Spectrum	Visual Magnitude		Phot. Magnitude	
			Harvard	Adopted	Harvard	Adopted
a	+42°3331		8.37	8.40		8.32
b	+42°3325	A	6.90	6.90	6.52	6.86
c	+42°3340		7.83			7.89
d	+41°3352	A	7.48		7.36	7.36
e	+43°3229	G	5.95		6.82	6.66

The variable was compared with two near-by stars designated as *a* and *b* in Table I. The adopted difference in magnitude between *a* and *b* is the mean of 138 determinations and is equal to 1.50 mag. with a probable error ±0.015 mag. It is worthy of note that the mean difference *a*-*b* determined with the 4½-inch telescope is 1.68 mag., while that determined with the 7½-inch telescope is 1.43 mag. It was decided, however, to adopt the mean of all the determinations as the true magnitude difference rather than use different values for the magnitudes of the comparison stars according to the telescope used.

The visual observations are brought together in Table II, of which the column-headings are self-explanatory. In the third column the phases refer to maximum determined with the epoch given in the fourth column and the final elements given below. After the phases were computed the observations were then combined with those made by the writer at the Lick Observatory to form a mean curve. The observations were separated into twenty groups from which the normal places given in Table IV were computed. The curve passing through these points is shown in Figure 1. The last column of Table II gives the departures of the individual observations from this curve.

TABLE II

VISUAL OBSERVATIONS OF RR LYRAE

No.	Julian Day G. M. T.	Helioc. Phase	Epoch	v	O–C	No.	Julian Day G. M. T.	Helioc. Phase	Epoch	v	O–C
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
	2420	day		mag.	mag.		2420	day		mag.	mag.
1	052.591	0.564	9166	7.18	+0.14	46	065.651	0.020	.....	7.33	+0.26
2	.601	0.007	9167	7.18	+0.14	47	.657	0.026	.....	7.18	+0.08
3	.609	0.015	.....	7.17	+0.10	48	.667	0.036	.....	7.26	+0.10
4	.617	0.023	.....	7.11	+0.02	49	.673	0.042	.....	7.17	−0.05
5	.627	0.033	.....	7.34	+0.20	50	.677	0.046	.....	7.30	+0.07
6	.633	0.039	.....	7.18	−0.01	51	.685	0.054	.....	7.67	+0.42
7	.640	0.046	.....	7.38	+0.15	52	.692	0.061	.....	7.27	+0.01
8	.649	0.055	.....	7.19	−0.06	53	066.545	0.347	9191	7.70	−0.16
9	.657	0.063	.....	7.35	+0.08	54	.551	0.353	.....	7.65	−0.20
10	.666	0.072	.....	7.38	+0.08	55	.564	0.366	.....	7.70	−0.14
11	.680	0.086	.....	7.19	−0.15	56	.576	0.378	.....	7.84	0.00
12	.689	0.095	.....	7.26	−0.11	57	.620	0.422	.....	7.64	−0.14
13	.693	0.099	.....	7.30	−0.09	58	.640	0.442	.....	7.93	+0.13
14	.698	0.104	.....	7.49	+0.08	59	.653	0.455	.....	7.84	+0.03
15	.715	0.121	.....	7.59	+0.12	60	.666	0.468	.....	8.09	+0.27
16	.720	0.126	.....	7.41	−0.07	61	.673	0.475	.....	7.87	+0.03
17	053.562	0.402	9168	7.63	−0.17	62	.680	0.482	.....	7.87	+0.05
18	.569	0.409	.....	7.74	−0.06	63	082.609	0.540	9219	7.59	+0.41
19	.578	0.418	.....	7.64	−0.15	64	.617	0.548	.....	7.11	0.00
20	.584	0.424	.....	7.70	−0.08	65	.626	0.557	.....	7.10	+0.04
21	.593	0.433	.....	7.74	−0.04	66	.630	0.561	.....	7.04	0.00
22	.601	0.441	.....	7.72	−0.07	67	.634	0.565	.....	6.97	−0.06
23	.610	0.450	.....	7.66	−0.14	68	.639	0.003	9220	7.14	−0.10
24	.619	0.459	.....	7.75	−0.06	69	.642	0.006	.....	7.14	−0.10
25	.628	0.468	.....	7.76	−0.05	70	.647	0.011	.....	6.90	−0.15
26	.635	0.475	.....	7.70	−0.14	71	.651	0.015	.....	6.80	−0.26
27	.643	0.483	.....	7.86	+0.05	72	.657	0.021	.....	7.01	−0.07
28	.649	0.489	.....	7.68	−0.10	73	.666	0.030	.....	7.08	−0.04
29	065.539	0.475	9189	8.08	+0.23	74	.675	0.039	.....	7.45	+0.27
30	.546	0.482	.....	7.69	−0.12	75	.679	0.043	.....	7.24	+0.03
31	.554	0.490	.....	7.62	−0.15	76	.687	0.051	.....	7.30	+0.06
32	.562	0.498	.....	7.61	−0.11	77	.693	0.057	.....	7.22	−0.02
33	.569	0.505	.....	7.66	0.00	78	.699	0.063	.....	7.14	−0.12
34	.576	0.512	.....	7.83	+0.28	79	.707	0.071	.....	7.36	−0.07
35	.588	0.524	.....	7.22	−0.13	80	.712	0.076	.....	7.29	−0.01
36	592	0.528	.....	7.25	−0.03	81	.717	0.081	.....	7.54	+0.21
37	.597	0.533	.....	7.26	+0.02	82	114.548	0.168	9276	7.46	−0.08
38	.601	0.537	.....	7.25	+0.04	83	.556	0.176	.....	7.70	+0.13
39	.608	0.544	.....	7.11	−0.02	84	.566	0.186	.....	7.46	−0.13
40	.614	0.550	.....	7.10	0.00	85	.573	0.193	.....	7.66	+0.04
41	.622	0.558	.....	7.08	+0.02	86	290.618	0.523	9586	7.60	+0.15
42	.627	0.563	.....	7.31	+0.27	87	.630	0.535	.....	7.34	+0.12
43	.634	0.003	9190	7.37	+0.33	88	.635	0.540	.....	7.03	−0.13
44	.640	0.009	.....	7.44	+0.39	89	.645	0.550	.....	7.19	+0.09
45	.645	0.014	.....	7.01	−0.06	90	.651	0.556	.....	7.10	+0.04

TABLE II—Continued

VISUAL OBSERVATIONS OF RR LYRAE

No.	Julian Day G. M. T.	Helioc. Phase	Epoch	v	O–C	No.	Julian Day G. M. T.	Helioc. Phase	Epoch	v	O–C
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
	2420	day		mag.	mag.		2420	day		mag.	mag.
91	290.656	0.561	.....	7.10	+0.06	117	303.727	0.029	.....	7.11	0.00
92	.660	0.565	.....	7.01	−0.02	118	.738	0.040	.....	7.25	+0.04
93	.665	0.003	9587	6.89	−0.14	119	396.583	0.490	9773	7.68	−0.09
94	.670	0.008	.....	7.04	0.00	120	.591	0.498	.....	7.58	−0.14
95	.675	0.013	.....	6.94	−0.10	121	.599	0.506	.....	7.90	+0.27
96	.681	0.019	.....	7.08	0.00	122	.606	0.513	.....	7.79	+0.23
97	.691	0.029	.....	6.98	−0.13	123	.612	0.519	.....	7.38	−0.06
98	.707	0.045	.....	7.05	−0.17	124	.618	0.525	.....	7.45	+0.10
99	303.623	0.491	9609	7.91	+0.14	125	.625	0.532	.....	7.28	+0.03
100	.629	0.497	.....	8.05	+0.31	126	.630	0.537	.....	7.04	−0.16
101	.634	0.502	.....	7.89	+0.20	127	.636	0.543	.....	7.13	−0.01
102	.640	0.508	.....	7.62	0.00	128	.641	0.548	.....	6.89	−0.23
103	.647	0.515	.....	7.54	+0.01	129	.647	0.554	.....	7.10	+0.02
104	.652	0.520	.....	7.23	−0.16	130	.653	0.560	.....	7.04	0.00
105	.657	0.525	.....	7.17	−0.16	131	.657	0.564	.....	6.92	−0.11
106	.663	0.531	.....	7.27	+0.02	132	.661	0.001	9774	7.17	+0.13
107	.667	0.535	.....	7.19	−0.02	133	.665	0.005	.....	6.94	−0.10
108	.671	0.539	.....	7.07	−0.09	134	.669	0.009	.....	7.03	−0.01
109	.677	0.545	.....	7.20	+0.06	135	.673	0.013	.....	6.69	−0.36
110	.684	0.552	.....	7.15	+0.06	136	.678	0.018	.....	6.87	−0.20
111	.689	0.557	.....	6.87	−0.19	137	.683	0.023	.....	7.18	+0.09
112	.694	0.562	.....	6.98	−0.06	138	.689	0.029	.....	7.33	+0.19
113	.699	0.001	9610	6.97	−0.06	139	.695	0.035	.....	7.08	−0.07
114	.703	0.005	.....	7.07	+0.03	140	.700	0.040	.....	7.58	+0.38
115	.710	0.012	.....	7.35	+0.29	141	.707	0.047	.....	7.30	+0.07
116	.719	0.021	.....	7.25	+0.17	142	.713	0.053	.....	7.24	0.00

TABLE III

MAGNITUDE DIFFERENCES

Stars	Δ Mag.	Prob. Error	Prob. Error Single Obs'n
d–a	−0.96	±0.005	±0.066
d–b	+0.50	±0.003	±0.048
d–c	−0.53	±0.003	±0.042
d–e	+0.70	±0.004	±0.056

The photographic observations were made with the 5-inch camera which is attached to the 7½-inch telescope. The clear aperture of the camera is 12.50 cm. and its focal length 66.08 cm. The plates were exposed 1.5 mm. behind the focal plane, the resulting star images being discs approximately one-third of a millimeter in diameter. Usually, from seven to ten sets of images were photographed on a plate by moving it in declination. The total exposure time averaged eighty minutes. On a clear day, subsequent to the exposure in the camera, the plates were exposed to the northern sky in the sensitometer to secure images of known intensity, to furnish the scale used in the reductions. The plates were then developed for seven minutes in hydrochinon kept at a constant temperature of 20° C. Seed 27 plates were used throughout the entire series of observations.

The opacities of the star images were measured with the microphotometer. The sensitometer images on the plates served to calibrate the wedge of this instrument. For each plate, therefore, there is an opacity curve whose ordinates are the wedge readings of the microphotometer and whose abscissae are magnitude differences. The difference in magnitude between any pair of stars as read from the curves would be the true difference if the stars were at the same zenith distance and if the paths which their light traversed in the lens system of the camera were equivalent. Since neither of these conditions can be realized, corrections must be applied to the magnitude differences read from the curves to obtain the true differences. The corrections for differential absorption in the earth's atmosphere were computed from tables given by Wirtz[1]; and the corrections for differential absorption in the lens system of the camera from tables supplied by Dr. Baker.*

TABLE IV

VISUAL LIGHT CURVE OF RR LYRAE. NORMAL PLACES

Θ	$t-t_o$	mag.	Θ	$t-t_o$	mag.	Θ	$t-t_o$	mag.
0°	0ᵈ000	7.03	102°	0ᵈ161	7.52	291°	0ᵈ458	7.81
12	0.019	7.07	121	0.190	7.61	303	0.477	7.84
25	0.040	7.22	149	0.234	7.73	320	0.504	7.69
37	0.058	7.25	178	0.280	7.85	325	0.512	7.54
50	0.079	7.32	213	0.335	7.87	335	0.528	7.28
64	0.100	7.39	246	0.388	7.83	347	0.546	7.13
83	0.130	7.50	272	0.429	7.78			

1. Astronomische Nachrichten, 154, 362, 1900.

* An account of the extra-focal photometric work in progress at the Laws Observatory will appear in another bulletin.—R. H. Baker.

TABLE V

JOURNAL OF PLATES

Plate No.	No. of Exposures	Dates of Exposures G. M. T.						Epoch	Notes
(1)	(2)	(3)						(4)	(5)
154	9	1914 April	21^d	17^h	00^m0 to	18^h	25^m0	9505	Test plate. Images underexposed.
163	7	May	16	15	57.0 to	17	09.0	9549	Images of *a* and *c* underexposed.
164	7		16	17	23.0 to	18	35.0	9550	Seventh image of *c* marred.
165	7		16	18	50.0 to	20	02.0	9550	
166	6		17	15	19.5 to	16	34.5	9551	
167	6		17	16	51.5 to	18	06.5	9551	Images of *b* and *e* overexposed.
171	6		19	15	54.5 to	17	09.5	9555	
186	7		29	17	03.0 to	18	15.0	9572	
187	7		29	18	28.0 to	19	40.0	9573	Images are rings, not discs. Only *c* and *d* used in the measurements.
188	7	June	10	15	16.0 to	16	28.0	9594	Ditto.
212	8		20	18	06.0 to	19	16.0	9611	Images of *b* and *e* overexposed.
213	8		20	19	41.0 to	20	59.0	9611	Clouds during first four exposures. Images of *b* and *e* overexposed.
312	9	September	18	13	51.0 to	14	59.0	9770	
313	10		18	15	09.0 to	16	21.0	9770	Images of *a* underexposed
314	8		18	16	43.0 to	17	53.0	9770	
315	8		18	18	05.0 to	19	15.0	9770	
329	10		21	13	49.0 to	15	01.0	9775	
330	10		21	15	18.0 to	16	30.0	9775	Clouds during exposures 6, 7, 8, 9?
335	8		25	15	45.0 to	16	55.0	9782	
336	10		25	17	13.0 to	18	25.0	9782	
337	7		25	18	36.0 to	19	36.0	9783	
360	10	October	19	12	56.0 to	14	08.0	9824	
361	10		19	14	17.0 to	15	29.0	9825	First image of *a* marred.
362	7		19	15	58.0 to	16	58.0	9825	
377	7		23	14	37.0 to	15	37.0	9832	
378	6		23	15	50.0 to	16	40.0	9832	
381	10	November	9	12	38.0 to	13	50.0	9861	
382	10		9	13	59.0 to	15	13.0	9862	
394	8		16	12	47.0 to	13	57.0	9874	
395	6		16	14	19.0 to	15	09.0	9874	Air foggy. Only first four exposures used

The variable was compared with the five stars given in Table I. The adopted photographic magnitudes are based on that of star *d* as standard because it was measured on all the plates except the first. On some of the plates the images of *b* and *e* were overexposed, and were not used; and on some of the plates the images of *a* and *c* were underexposed, and, accordingly, were not used. These instances are recorded in the notes to Table V. The magnitude differences between star *d* and the other comparison stars are given in Table III. Tables V and VII contain the results of the photographic observations.

Using the elements given below, the photographic observations were brought together into a mean curve. An attempt was then made to represent this curve by a Fourier's series of the form:

$$m = f(\Theta) = \tfrac{1}{2}c_0 \sin\phi_0 + c_1 \sin(\Theta + \phi_1) + c_2 \sin(2\Theta + \phi_2) + c_3 \sin(3\Theta + \phi_3) + \ldots\ldots$$

where m is the magnitude at any phase Θ after maximum, and

$$P\Theta = 360^\circ(t - t_0).$$

To evaluate the constants, the ordinates of the mean curve were measured at intervals of 15°. The resulting equation for the magnitude is

$$\begin{aligned} m = 7.78 &+ 0.292 \sin(\Theta + 232^\circ.5) + 0.122 \sin(2\Theta + 229^\circ.0) + 0.085 \sin(3\Theta + 263^\circ.9) \\ &+ 0.042 \sin(4\Theta + 274^\circ.1) + 0.048 \sin(5\Theta + 284^\circ.6) + 0.017 \sin(6\Theta + 343^\circ.6) \\ &+ 0.010 \sin(7\Theta + 323^\circ.1) + 0.007 \sin(8\Theta + 344^\circ.1) + 0.007 \sin(9\Theta + 74^\circ.1) \\ &+ 0.011 \sin(10\Theta + 296^\circ.6) + 0.013 \sin(11\Theta + 257^\circ.0). \end{aligned}$$

This equation is represented in Figure 2, the coordinates being given in Table VI. The plotted observations for each night were then superposed on this curve and the departures of the individual observations from it were read off. These residuals are given in the last column of Table VII. A more faithful representation would be secured if the interval between the ordinates of the mean curve were made smaller.

TABLE VI

PHOTOGRAPHIC LIGHT CURVE OF RR LYRAE

Θ	$t-t_0$	mag.	Θ	$t-t_0$	mag.	Θ	$t-t_0$	mag.
0°	0^{d}0000	7.25	120°	0^{d}1889	7.84	240°	0^{d}3779	8.00
5	0.0079	7.26	125	0.1968	7.84	245	0.3857	8.01
10	0.0157	7.30	130	0.2046	7.84	250	0.3936	8.01
15	0.0236	7.36	135	0.2125	7.83	255	0.4015	8.00
20	0.0315	7.39	140	0.2204	7.84	260	0.4093	7.99
25	0.0394	7.40	145	0.2283	7.87	265	0.4172	7.99
30	0.0472	7.42	150	0.2361	7.90	270	0.4251	8.00
35	0.0551	7.44	155	0.2440	7.93	275	0.4329	8.01
40	0.0630	7.48	160	0.2518	7.95	280	0.4408	8.01
45	0.0708	7.52	165	0.2597	7.96	285	0.4487	8.00
50	0.0787	7.55	170	0.2676	7.97	290	0.4566	7.98
55	0.0866	7.56	175	0.2754	7.98	295	0.4644	7.99
60	0.0944	7.55	180	0.2833	8.00	300	0.4724	8.00
65	0.1023	7.55	185	0.2912	8.01	305	0.4803	8.02
70	0.1102	7.57	190	0.2991	8.01	310	0.4881	8.02
75	0.1180	7.61	195	0.3069	7.99	315	0.4960	7.99
80	0.1259	7.65	200	0.3149	7.98	320	0.5039	7.94
85	0.1338	7.67	205	0.3228	7.96	325	0.5118	7.86
90	0.1417	7.68	210	0.3306	7.94	330	0.5196	7.76
95	0.1495	7.69	215	0.3385	7.93	335	0.5275	7.66
100	0.1574	7.70	220	0.3464	7.93	340	0.5354	7.56
105	0.1653	7.73	225	0.3543	7.94	345	0.5432	7.45
110	0.1731	7.77	230	0.3621	7.95	350	0.5511	7.35
115	0.1810	7 81	235	0.3700	7.98	355	0.5590	7.27

TABLE VII

PHOTOGRAPHIC OBSERVATIONS OF RR LYRAE

Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	m	O–C	Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	v	O–C
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	day	mag.	mag.			2420	day	mag.	mag.
154	1	244.7083	0.5252	8.10	+0.35	186	3	282.7271	0.0011	7.33	+0.07
	2	.7153	0.5322	7.87	+0.21		4	.7354	0.0094	7.22	0.00
	3	.7222	0.5391	7.85	+0.30		5	.7438	0.0178	7.23	0.00
	4	.7292	0.5461	7.36	−0.09		6	.7521	0.0261	7.24	−0.03
	5	.7361	0.5530	7.31	−0.03		7	.7604	0.0344	7.33	−0.01
	6	.7447	0.5616	7.27	−0.01						
	7	.7535	0.0036	7.27	0.00	187	1	282.7694	0.0434	7.36	0.00
	8	.7604	0.0105	7.23	−0.05		2	.7778	0.0518	7.40	+0.02
	9	.7674	0.0175	7.25	−0.07		3	.7861	0.0601	7.51	+0.11
							4	.7944	0.0684	7.48	+0.06
163	1	269.6646	0.5420	7.59	−0.02		5	.8028	0.0768	7.51	+0.04
	2	.6729	0.5503	7.48	−0.02		6	.8112	0.0852	7.53	+0.01
	3	.6813	0.5587	7.42	−0.01		7	.8195	0.0935	7.55	+0.02
	4	.6896	0.0001	7.32	0.00						
	5	.6979	0.0084	7.28	0.00	188	1	294.6361	0.0070	7.27	+0.01
	6	.7063	0.0168	7.34	+0.05		2	.6445	0.0154	7.41	+0.11
	7	.7146	0.0251	7.31	−0.02		3	.6528	0.0237	7.40	+0.04
							4	.6611	0.0320	7.51	+0.12
164	1	269.7243	0.0348	7.42	+0.02		5	.6694	0.0403	7.49	+0.08
	2	.7326	0.0431	7.38	−0.04		6	.6778	0.0487	7.54	+0.11
	3	.7410	0.0515	7.46	+0.02		7	.6861	0.0570	7.57	+0.12
	4	.7493	0.0598	7.47	−0.01						
	5	.7576	0.0681	7.53	+0.03	212	1	304.7542	0.4891	7.98	−0.04
	6	.7660	0.0765	7.62	+0.08		2	.7611	0.4960	7.96	−0.03
	7	.7743	0.0848	7.49	+0.02		3	.7681	0.5030	7.94	0.00
							4	.7750	0.5099	7.91	+0.02
165	1	269.7847	0.0952	7.54	−0.04		5	.7819	0.5168	7.82	+0.01
	2	.7931	0.1036	7.50	−0.07		6	.7889	0.5238	7.64	−0.05
	3	.8014	0.1119	7.54	−0.03		7	.7958	0.5307	7.48	−0.11
	4	.8097	0.1202	7.57	−0.03		8	.8028	0.5377	7.43	−0.08
	5	.8181	0.1286	7.65	0.00						
	6	.8264	0.1369	7.60	−0.09	213	1	304.8201	0.5550	(7.49)	.....
	7	.8347	0.1452	7.65	−0.05		2	.8285	0.5634	(7.46)	.....
							3	.8382	0.0063	(7.35)	.....
166	1	270.6385	0.3822	8.02	+0.01		4	.8465	0.0146	7.32	+0.02
	2	.6490	0.3927	7.94	−0.07		5	.8535	0.0216	7.31	−0.03
	3	.6594	0.4031	8.00	0.00		6	.8604	0.0285	7.32	−0.05
	4	.6698	0.4135	8.00	+0.01		7	.8674	0.0355	7.30	−0.09
	5	.6802	0.4239	7.97	−0.03		8	.8743	0.0424	7.32	−0.08
	6	.6906	0.4343	8.04	+0.03						
						312	1	394.5771	0.1855	7.81	−0.07
167	1	270.7024	0.4461	8.01	0.00		2	.5854	0.1938	7.77	−0.07
	2	.7129	0.4566	7.99	0.00		3	.5910	0.1994	7.76	−0.08
	3	.7233	0.4670	8.01	+0.02		4	.5965	0.2049	7.74	−0.10
	4	.7337	0.4774	8.03	+0.01		5	.6021	0.2105	7.81	−0.02
	5	.7441	0.4878	8.05	+0.03		6	.6076	0.2160	7.84	+0.01
	6	.7545	0.4982	7.98	0.00		7	.6132	0.2216	7.82	−0.02
							8	.6187	0.2271	7.84	−0.02
171	1	272.6628	0.1393	7.72	+0.04		9	.6243	0.2327	7.88	−0.01
	2	.6733	0.1498	7.71	+0.02						
	3	.6837	0.1602	7.70	−0.01	313	1	394.6313	0.2397	7.96	+0.04
	4	.6941	0.1706	7.80	+0.04		2	.6368	0.2452	8.08	+0.14
	5	.7045	0.1810	7.78	−0.03		3	.6424	0.2508	8.00	+0.05
	6	.7149	0.1914	7.75	−0.09		4	.6479	0.2563	7.91	−0.04
							5	.6535	0.2619	8.03	+0.06
186	1	282.7104	0.5512	7.39	0.00		6	.6590	0.2674	8.01	+0.04
	2	.7188	0.5596	7.38	+0.02		7	.6646	0.2730	7.98	0.00

TABLE VII—Continued

PHOTOGRAPHIC OBSERVATIONS OF RR LYRAE

Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	v	O–C	Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	v	O–C
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	day	mag.	mag.			2420	day	mag.	mag.
313	8	394.6701	0.2785	8.10	+0.11	336	5	401.7396	0.5456	7.33	+0.01
	9	.6757	0.2841	8.01	0.00		6	.7451	0.5511	7.24	0.00
	10	.6813	0.2897	8.01	0.00		7	.7507	0.5567	7.20	+0.01
							8	.7563	0.5623	7.19	0.00
314	1	394.6965	0.3049	7.97	−0.03		9	.7618	0.0010	7.21	+0.01
	2	.7035	0.3119	8.00	+0.01		10	.7674	0.0066	7.29	+0.07
	3	.7104	0.3188	7.97	0.00						
	4	.7174	0.3258	7.99	+0.03	337	1	401.7750	0.0142	7.24	−0.02
	5	.7243	0.3327	7.89	−0.05		2	.7819	0.0211	7.37	+0.06
	6	.7313	0.3397	7.93	0.00		3	.7889	0.0281	7.32	−0.01
	7	.7382	0.3466	7.89	−0.04		4	.7958	0.0350	7.32	−0.02
	8	.7451	0.3535	7.88	−0.06		5	.8028	0.0420	7.42	+0.06
							6	.8097	0.0489	7.45	+0.07
315	1	394.7535	0.3619	8.11	+0.15		7	.8167	0.0559	7.47	+0.06
	2	.7604	0.3688	8.06	+0.08						
	3	.7674	0.3758	8.01	+0.01	360	1	425.5389	0.5370	7.49	+0.06
	4	.7743	0.3827	8.06	+0.05		2	.5445	0.5426	7.45	0.00
	5	.7813	0.3897	8.06	+0.05		3	.5500	0.5481	7.41	0.00
	6	.7882	0.3966	7.98	−0.02		4	.5556	0.5537	7.33	0.00
	7	.7951	0.4035	7.91	−0.08		5	.5611	0.5592	7.30	+0.03
	8	.8021	0.4105	7.96	−0.03		6	.5667	0.5648	7.30	+0.06
							7	.5722	0.0035	7.23	0.00
329	1	397.5757	0.3497	7.89	−0.04		8	.5778	0.0091	7.26	+0.02
	2	.5813	0.3553	7.98	+0.04		9	.5833	0.0146	7.26	0.00
	3	.5868	0.3608	8.05	+0.09		10	.5889	0.0202	7.22	−0.07
	4	.5924	0.3664	7.99	−0.02						
	5	.5979	0.3719	7.97	−0.01	361	1	425.5951	0.0264	7.34	0.00
	6	.6035	0.3775	7.88	−0.12		2	.6007	0.0320	7.33	−0.03
	7	.6090	0.3830	7.92	−0.09		3	.6063	0.0376	7.31	−0.06
	8	.6146	0.3886	7.93	−0.08		4	.6118	0.0431	7.31	−0.07
	9	.6201	0.3941	7.93	−0.08		5	.6174	0.0487	7.32	−0.08
	10	.6257	0.3997	7.96	−0.04		6	.6229	0.0542	7.36	−0.05
							7	.6285	0.0598	7.39	−0.03
330	1	397.6375	0.4115	8.07	+0.08		8	.6340	0.0653	7.37	−0.09
	2	.6431	0.4171	8.00	0.00		9	.6396	0.0709	7.45	−0.04
	3	.6486	0.4226	8.01	+0.01		10	.6451	0.0764	7.42	−0.09
	4	.6542	0.4282	7.99	−0.01						
	5	.6597	0.4337	7.97	−0.04	362	1	425.6653	0.0966	7.57	+0.04
	6	.6653	0.4393	....	.....		2	.6722	0.1035	7.54	+0.01
	7	.6708	0.4448	....	.....		3	.6792	0.1105	7.57	+0.02
	8	.6764	0.4504	....	.....		4	.6861	0.1174	7.59	+0.01
	9	.6819	0.4559	7.91	−0.07		5	.6931	0.1244	7.69	+0.07
	10	.6875	0.4615	7.93	−0.05		6	.7000	0.1313	7.67	+0.03
							7	.7069	0.1382	7.63	+0.01
335	1	401.6563	0.4623	7.97	+0.04						
	2	.6632	0.4692	7.96	+0.01	377	1	429.6090	0.0722	7.43	−0.09
	3	.6701	0.4761	7.96	0.00		2	.6160	0.0792	7.51	−0.04
	4	.6771	0.4831	7.97	+0.01		3	.6229	0.0861	7.50	−0.06
	5	.6840	0.4900	7.92	−0.02		4	.6299	0.0931	7.61	+0.06
	6	.6910	0.4970	7.93	+0.03		5	.6368	0.1000	7.59	+0.04
	7	.6979	0.5039	7.87	+0.02		6	.6438	0.1070	7.61	+0.04
	8	.7049	0.5109	7.78	+0.01		7	.6507	0.1139	7.63	+0.04
336	1	401.7174	0.5234	7.63	+0.03	378	1	429.6597	0.1229	7.65	+0.01
	2	.7229	0.5289	7.55	+0.01		2	.6667	0.1299	7.68	+0.01
	3	.7285	0.5345	7.42	−0.01		3	.6736	0.1368	7.63	−0.04
	4	.7340	0.5400	7.36	0.00		4	.6805	0.1437	7.66	−0.02

TABLE VII—Continued

PHOTOGRAPHIC OBSERVATIONS OF RR LYRAE

Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	v	O—C	Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	v	O–C
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	day	mag.	mag.			2420	day	mag.	mag.
378	5	429.6875	0.1507	7.66	−0.03	382	6	446.6118	0.0693	7.57	−0.01
	6	.6944	0.1576	7.63	−0.07		7	.6174	0.0749	7.58	−0.01
							8	.6229	0.0804	7.63	+0.02
381	1	446.5264	0.5507	7.35	+0.02		9	.6285	0.0860	7.59	−0.01
	2	.5319	0.5562	7.30	0.00		10	.6340	0.0915	7.59	0.00
	3	.5375	0.5618	7.33	+0.03						
	4	.5431	0.0006	7.32	+0.01	394	1	453.5326	0.1879	7.85	+0.01
	5	.5486	0.0061	7.39	+0.05		2	.5396	0.1949	7.85	+0.01
	6	.5542	0.0117	7.41	+0.03		3	.5465	0.2018	7.85	+0.01
	7	.5597	0.0172	7.38	−0.02		4	.5535	0.2088	7.84	+0.01
	8	.5653	0.0228	7.41	−0.02		5	.5604	0.2157	7.82	−0.01
	9	.5708	0.0283	7.38	−0.06		6	.5674	0.2227	7.85	0.00
	10	.5764	0.0339	7.43	−0.01		7	.5743	0.2296	7.82	−0.06
							8	.5813	0.2366	7.83	−0.07
382	1	446.5826	0.0401	7.49	+0.02						
	2	.5889	0.0464	7.48	0.00	395	1	453.5965	0.2518	7.95	0.00
	3	.5951	0.0526	7.48	−0.03		2	.6035	0.2588	7.91	−0.05
	4	.6007	0.0582	7.51	−0.03		3	.6104	0.2657	7.96	−0.01
	5	.6063	0.0638	7.59	+0.01		4	.6174	0.2727	7.93	−0.05

TABLE VIII

MAXIMA OF RR LYRAE

Date	Julian Date Heliocentric Maximum	Epoch	Character	Observer	(O–C)I	(O–C)II
1899 July 20	2414856.5000+Δt_0	0	visual	Wendell	.....	.....
1899 Sept. 23	4921.6770	115	visual	Wendell	+0^{d}0042	+0^{d}0029
1899 Sept. 27	4925.6393	122	visual	Wendell	− 12	− 26
1912 Aug. 20	9635.9754	8432	visual	Townley	− 8	+ 27
1912 Sept. 13	9659.7739	8474	visual	Townley	− 92	− 57
1912 Oct. 16	9692.6556	8532	visual	Kiess	− 35	− 3
1912 Oct. 21	9697.7604	8541	visual	Kiess	00	+ 31
1912 Oct. 25	9701.7212	8548	visual	Kiess	− 70	− 39
1913 Oct. 24	2420065.6192	9190	visual	Kiess	− 123	− 117
1913 Nov. 10	0082.6425	9220	visual	Kiess	+ 63	+ 67
1914 April 21	0244.7513	9506	phot	Kiess	+ 24	+ 16
1914 May 16	0269.6989	9550	phot	Kiess	+ 96	+ 86
1914 May 29	0282.7379	9573	phot	Kiess	+ 115	+ 105
1914 June 6	0290.6671	9587	visual	Kiess	+ 51	+ 39
1914 June 19	0303.6920	9610	visual	Kiess	− 70	− 83
1914 Sept. 20	0396.6611	9774	visual	Kiess	+ 25	+ 3
1914 Sept. 25	0401.7574	9783	phot	Kiess	− 27	− 47
1914 Oct. 19	0425.5714	9825	phot	Kiess	+ 46	+ 22
1914 Nov. 9	0446.5351	9862	phot	Kiess	− 44	− 69

A new solution for the period of the variable, based on all the available observed maxima, has been carried out. In this solution the photographic maxima published in this paper have been included because they have been satisfactorily represented by elements referring to visual maximum. Consequently, based on the data of Table VIII, eighteen equations of condition were written of the form

$$t_E = t_o + \Delta t_o + EP + E^2 dP$$

where P denotes the period, t the date of a maximum of order E, and $t_o = 2414856.50000$, the approximate date of the first maximum observed. A least squares solution yielded the following results:

$\Delta t_o = -0^d.0124 \pm 0^d.0034$.
$P = 0^d.5668274 \pm 0^d.0000025$ (Elements I)
$dP = 0$ (assumed)

$\Delta t_o = -0^d.0109 \pm 0^d.0031$
$P = 0^d.5668265 \pm 0^d.0000022$ (Elements II)
$dP = +0^d.492 \times 10^{-9} \pm 0^d.230 \times 10^{-9}$

The results of the representation of the observation equations with these two sets of elements are shown in the last two columns of Table VIII. The representation with the second elements is the more satisfactory of the two; therefore, I have adopted as final elements:

$$(\text{Helioc. Max.})_E = \text{J. D. } 2414856.4891 + 0^d.5668265 \cdot E, + 0^d.492 \times 10^{-9} \cdot E^2, \text{ G.M.T.}$$

With but four exceptions the individual residuals in the last column of Table VIII are less than the intervals of time separating the observations used to determine the maxima to which they refer. These larger residuals are not excessive and may be accounted for by errors of observation as well as by irregularities in the period. Although the data here presented do not strongly support Shapley's statement that the period is slightly irregular, yet they do not offer contradiction. For, to detect such irregularities the observed maxima should be separated by as short intervals of time as possible. This condition is not realized in the data of Table VIII. Frequent attempts were made to observe maxima on successive nights, but cloudy weather prevented the success of the plan.

A comparison of the visual and photographic light curves of RR Lyrae shows that there is no marked difference between the two. On both, the ascent from minimum to maximum occurs in 0.090 day, and the descent from maximum to minimum in 0.290 day. The minimum phase lasts approximately one-third of the entire period. The descending branches of the curves are marked by slight fluctuations, those on the visual curve not being so pronounced as those on the photographic. This difference, however, can not

be considered real because of the tendency to smoothen any apparent irregularities when drawing a mean curve through the plotted observations. Both the visual and photographic observations show that the maximum brightness of the variable is not constant but may fluctuate within a range of approximately one-tenth magnitude.

Particularly noteworthy is the fact that the photographic range of RR Lyrae is less than (or at most, equal to) the visual. Generally, for stars of the δ-Cephei type the photographic range is greater than the visual. And this has been found to be true for three cluster type variables which have been studied photographically and visually, namely: SU Draconis, SW Draconis and XZ Cygni. For these stars the photographic light curves are due to Martin and Plummer,[1] and the visual curves to Sperra[2] and Enebo.[3] For XX Cygni, however, which has been investigated by various observers and notably by Kron,[4] the photographic range is less than the visual. The accompanying table presents the available data on this subject:

Star	Period	Phot. Range	Vis. Range	Phot./ Vis.
	d	mag.	mag.	
XX Cygni	0.1348652	0.63	0.77	0.82
XZ Cygni	0.466586	1.40	0.62	2.26
RR Lyrae	0.5668265	0.77	0.84	0.92
SW Draconis	0.569666	1.00	0.81	1.23
SU Draconis	0.6604347	1.10	0.65	1.69

In a recent study of the best observed variables of the cluster type, Plummer[5] has reached the conclusion that the minimum phase can in no case with certainty be said to be stationary. A similar conclusion may be drawn concerning RR Lyrae. The harmonic analysis of the photographic light curve produces at minimum a series of waves of diminishing amplitude, which follow with a fair degree of fidelity the course taken by the observations. Not only the observations at minimum, but also those covering the entire descending branch of the curve show oscillations of short period and small amplitude, which the harmonic curve cannot follow. The remarkable agreement between the observations made on different nights but at the same phase leads to the belief that the observed fluctuations are real and are not to be accounted for as errors of observation. The course taken by the observations is shown by the dotted line in Figure 2. It would seem, therefore, that the true basis for separating the short period variables into sub-classes is length of period rather than any essential differences in the light curves.

1. Monthly Notices R. A. S., **73**, 166 and 440, 1913; **74**, 225, 1914.
2. Astronomische Nachrichten, **184**, 241-52, 1910.
3. Beob. Veränderlicher Sterne, angestellt auf Dombaas (Norwegen), II.
4. Publ. des Astroph. Obs. zu Potsdam, **22**, Nr. 65, 1912.
5. Monthly Notices R. A. S., **73**, 657, 1913.

*Note added November 22, 1915.—From a recent study of XX Cygni made at the Mount Wilson Solar Observatory (Astrophysical Journal, **42**, 161, 1915), Dr. and Mrs. Shapley conclude that the visual range does not exceed the photographic.

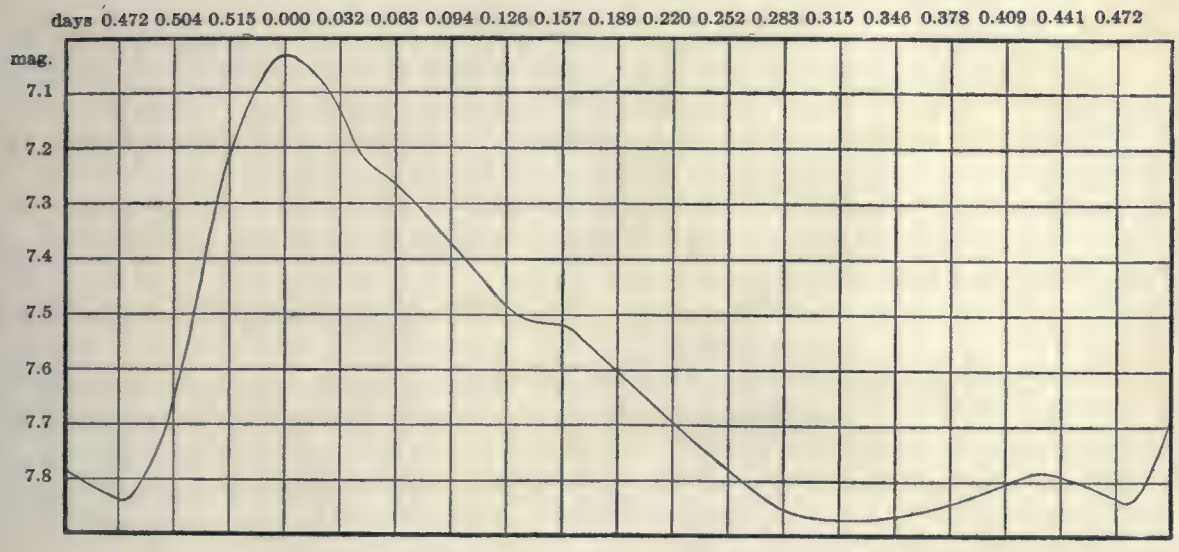


Figure 1. Visual Light Curve of RR Lyrae

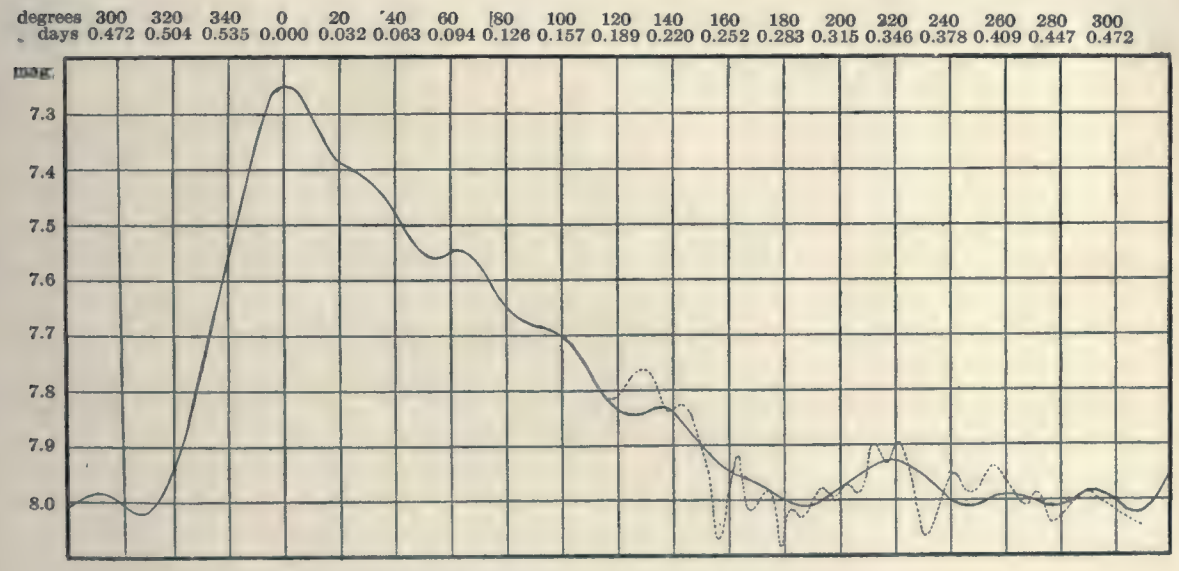


Figure 2. Photographic Light Curve of RR Lyrae

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The following papers contain observations of, or information concerning, RR Lyrae:

Sixty-Four New Variable Stars; E. C. Pickering; Harvard College Observatory, Circular No. 54, 1901.

Variable Stars of Short Period; O. C. Wendell; Annals, Harvard College Observatory, **69**, Chap. 5, 1909.

Observations of Variables; O. C. Wendell; *Ibid.*, Chap. 8, 1914.

Ergebnisse von Photometerbeobachtungen veränderlicher Sterne kurzer Periode; H. v. Zeipel; Astronomische Nachrichten, **177**, 372, 1908.

The Cluster Variable RR Lyrae; C. C. Kiess; Publications of the Astronomical Society of the Pacific, **25**, 121, 1913.

The Cluster Variable RR Lyrae; C. C. Kiess; Lick Observatory Bulletin, **7**, 140, 1913.

Bestätigung der Eigenbewegung von RR Lyrae; E. Hertzsprung; Astronomische Nachrichten, **196**, 208, 1913.

Osservazioni Fotometriche di RR Lyrae; V. Fontana; Memorie della Società degli Spettroscopisti Italiani, Vol. II, Serie 2*a*, 1913.

Columbia, Missouri, 1915 June 4.

THE UNIVERSITY OF MISSOURI BULLETIN

ASTRONOMICAL SERIES

LAWS OBSERVATORY BULLETIN NO. 23

THE VISUAL AND PHOTOGRAPHIC LIGHT VARIATIONS OF RT AURIGAE

By C. C. Kiess

The variability of RT Aurigae ($\alpha = 6^h\ 23^m.0; \delta = +30°\ 33'$) was discovered by Astbury[1] in 1905, although the star was suspected as variable by Müller and Kempf[2] during the course of their work on the Potsdam Photometric Durchmusterung. From a series of observations made by himself and other observers during the years 1905-07, Astbury[3] has shown that the variation is of the δ-Cephei type, the period being 3.7282 days. Photometric observations of RT Aurigae have been made also by v. Zeipel[4] and by Wendell.[5] The work of these observers confirms the character of the variable and the period, but their observations are too few in number to permit of a detailed description of the light curve. Duncan[6] has submitted the star to spectroscopic analysis and has found that its radial velocity is variable in a period identical with the light period.

To secure, if possible, material for a more exact description of the light fluctuations of the variable, the writer began a series of photographic and visual photometric observations of the star in January, 1914. The period of observation extends from that date to April, 1915. During this time sixty-six visual observations and nearly four hundred photographic observations have been secured.

The visual observations were made with the polarizing photometer which has been adequately described in earlier bulletins of the Laws Observatory. All the observations prior to September, 1914, were made with the 7½-inch equatorial of which the aperture was reduced to approximately 1½ inches. All the observations following that date were made with the 4½-inch telescope of which the aperture was stopped down slightly more than 50 per cent. The method of observing has been described in Bulletin No. 22 of this series.

The variable was compared with the star designated as *a* in Table I, the order of the observation being $a_4tv_8ta_8tv_8ta_4$. For a few of the observations only half the usual number of settings were made. These are marked with an asterisk (*) in Table III in which the results of the visual observations are given. The phases in column 3 refer to the maximum computed with the elements given below and the epoch given in column 4.

1. Journal British Astronomical Association; **15**, 244, 1905.
2. Publ. des Astroph. Obs. zu Potsdam; **13**, 442, 1899.
3. Journal British Astronomical Association; **18**, 85, 1907.
4. Astronomische Nachrichten; **177**, 371, 1908.
5. Annals H. C. O.; **69**, 123, 1913.
6. Lick Observatory Bulletin; **5**, 81, 1908.

The photographic observations were made with the 12.50 cm. camera. The plates were exposed 5 mm. behind the focal plane of the camera where the resulting star images are discs approximately one millimeter in diameter. From four to twelve sets of images were photographed on a plate by moving it in declination, the exact number of sets depending on the total exposure time which averaged eighty minutes. Seed 27 plates were used thruout the entire series of observations. They were developed for ten minutes in hydrochinon kept at a constant temperature of 20° C.

TABLE I

COMPARISON STARS

Star	B D	Spect.	Visual Mag. Harvard	Photographic Mag.	
				Harvard	Adopted
a	+30° 1232		6.89		
b	+29° 1190	A		5.97	5.96
c	+30° 1211	A		6.47	6.48
d	+29° 1213	A	6.52	6.22	6.17
f	+33° 1356	A	6.38	6.16	6.16
g	+29° 1293	A	5.54	5.52	5.45

Table IV contains a journal of the plates secured for the present investigation. All the plates numbered from 141 to 422 were exposed to the northern sky in the sensitometer to secure images of known intensity to furnish the curve used in the reductions. The stars lettered *b*, *c*, *d*, *f* and *g* in Table I were selected for measurement with the microphotometer. Stars *b*, *c* and *g* were measured on a majority of the plates, while *d* and *f* were measured on all the plates. The final magnitudes of the comparison stars deduced from the plates containing sensitometer images served to furnish the reduction curves for plates numbered 111 to 138 which lack sensitometer images. These final magnitudes corrected for loss of light in the earth's atmosphere and in the lens system of the camera were plotted as abscissae against the microphotometer wedge readings as ordinates. From the resulting curve the magnitude of the variable was determined. The differences in magnitude between the comparison stars as derived from plates 141 to 422 are given in Table II.

The photographic observations are brought together in Table V. As in Table III, the phases refer to the epoch of maximum computed with the elements given below. The magnitudes of the variable are given in column 5, those derived from plates 141 to 422 being the mean of the values given by direct comparison with stars *d* and *f*.

Astbury has derived the following elements for the variable:

$$\text{Max.} = \text{J.D. } 2417173.36 + 3^{d}7282 \cdot E, \text{ G.M.T.}$$ (Elements by Astbury)

TABLE II

MAGNITUDE DIFFERENCES

Stars	Δ Mag.	Prob. Error	Prob. Error Single Obs'n
d-b	+0.21	±0.003	±0.029
d-c	−0.31	±0.003	±0.027
d-f	+0.01	±0.003	±0.038
d-g	+0.72	±0.003	±0.033

TABLE III

VISUAL OBSERVATIONS OF RT AURIGAE

No.	Julian Day G. M. T.	Helioc. Phase	Epoch	*v*	O–C	No.	Julian Day G. M. T.	Helioc. Phase	Epoch	*v*	O–C
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
	2420	days		mag.	mag.		2420	days		mag.	mag.
1	146.552	1.832	797	5.94	+0.24	34	254.580	1.738	826	5.76	+0.07
2	153.552	1.375	799	5.68	+0.06	35	260.650	0.351	828	5.26	−0.20
3	167.557	0.467	803	5.39	+0.09	36	261.589	1.290	828	5.58	+0.02
4	168.537	1.447	803	5.75	+0.07	37	261.640	1.341	828	5.49	−0.10
5	168.658	1.568	803	5.54	−0.17	38	267.586	3.558	829	5.28	+0.04
6	179.618	1.343	806	5.55	−0.04	39*	270.601	2.845	830	5.79	−0.14
7	180.548	2.273	806	5.82	0.00	40	425.829	1.501	872	5.66	−0.03
8	193.583	0.395	810	5.36	0.00	41	429.784	1.728	873	5.64	−0.04
9*	197.583	0.667	811	5.46	+0.03	42	432.678	0.895	874	5.34	−0.11
10	200.618	3.702	811	5.52	+0.46	43	434.667	2.884	874	5.41	−0.52
11	200.707	0.063	812	5.32	+0.19	44	435.698	0.187	875	5.01	−0.23
12	201.557	0.913	812	5.50	+0.05	45	438.662	3.151	875	5.77	−0.08
13	201.679	1.035	812	5.27	−0.09	46	441.671	2.432	876	5.55	−0.30
14*	205.554	1.181	813	5.36	−0.10	47	442.687	3.448	876	5.17	−0.25
15	205.693	1.320	813	5.70	+0.12	48	445.649	2.682	877	5.75	−0.15
16	205.753	1.380	813	5.56	−0.06	49	446.670	3.703	877	5.19	+0.13
17	206.554	2.181	813	5.84	+0.04	50	447.684	0.989	878	5.23	−0.13
18	206.632	2.259	813	5.72	−0.09	51*	505.570	2.956	893	5.99	+0.04
19	206.716	2.343	813	5.75	−0.08	52	505.670	3.056	893	5.89	−0 02
20	209.557	1.455	814	5.68	0.00	53	506.596	0.254	894	5.43	−0.01
21	214.698	2.867	815	5.92	−0.01	54	509.586	3.244	894	5.67	−0.09
22	228.587	1.843	819	6.01	+0.30	55	509.708	3.366	894	5.53	−0.05
23	228.682	1.938	819	5.82	+0.09	56	512.565	2.495	895	5.88	+0.01
24	232.582	2.110	820	5.75	−0.03	57	512.697	2.627	895	5.97	+0.07
25	232.668	2.196	820	5.77	−0.03	58	535.544	3.104	901	5.48	−0.38
26	234.582	0.382	821	5.31	−0.07	59	539.548	3.380	902	5.66	+0.12
27	234.621	0.421	821	5.39	+0.04	60	539.733	3.565	902	5.20	−0.03
28	234.663	0.463	821	5.38	+0.07	61	545.557	1.932	904	5.74	+0.01
29	234.721	0.521	821	5.31	+0.02	62	573.653	0.201	912	5.45	+0.04
30	235.565	1.364	821	5.64	+0.03	63	591.574	3.208	916	5.86	+0.06
31	238.573	0.644	822	5.74	+0.34	64	591.660	3.294	916	5.84	+0.14
32	244.585	2.928	823	6.02	+0.07	65	599.577	0.026	919	5.09	−0.02
33	248.584	3.198	824	5.88	+0.07	66	600.585	1.034	919	5.45	+0.08

TABLE IV

JOURNAL OF PLATES

Plate No.	No. of Exposures	Dates of Exposures G. M. T.						Epoch	Notes
(1)	(2)	(3)						(4)	(5)
		1914	d	h	m	h	m		
111	7	January	14	12	56.0	to 13	56.0	797	
112	12		20	13	57.0	to 15	47.0	799	
113	7		20	16	02.0	to 17	02.0	799	
114	6		20	17	22.0	to 18	12.0	799	Exposure stopped by clouds.
116	7		31	13	30.0	to 14	30.0	802	Plate fogged by moon light.
117	8		31	14	42.0	to 15	52.0	802	Plate fogged by moon light.
118	12		31	16	17.0	to 18	07.0	802	
119	7	February	14	16	21.0	to 17	21.0	806	Air very foggy.
120	7		16	13	36.0	to 14	36.0	806	
121	10		16	15	00.0	to 16	30.0	806	
122	10		16	16	42.0	to 18	12.0	806	
132	10		21	16	20.0	to 17	50.0	807	Air foggy during last three exposures.
133	12		24	16	05.0	to 17	55.0	808	
134	8		27	14	06.0	to 15	16.0	809	
135	8		27	15	28.0	to 16	38.0	809	
136	8		27	16	50.0	to 18	00.0	809	
137	8	March	1	15	03.0	to 16	13.0	810	Plate fogged by moon light.
138	8		1	16	25.0	to 17	35.0	810	
141	9		17	15	55.0	to 17	15.0	814	
142	4		20	15	39.5	to 16	10.0	815	Air very foggy.
146	8		22	15	09.0	to 16	19.0	815	
148	9		28	14	03.0	to 15	23.0	817	
149	9	April	12	14	15.0	to 15	35.0	821	
316	8	September	18	19	51.0	to 21	01.0	864	
317	8		18	21	13.0	to 22	23.0	864	
338	7		25	20	12.0	to 21	12.0	865	
339	7		25	21	24.0	to 22	24.0	865	
363	6	October	19	17	26.0	to 18	26.0	872	
364	6		19	18	52.0	to 19	52.0	872	
365	6		19	20	06.0	to 21	06.0	872	
379	6		23	17	21.0	to 18	21.0	873	
380	6		23	19	06.0	to 20	06.0	873	Clouds during last exposure.
389	3	November	13	15	02.5	to 16	17.5	879	Clouds during entire exposure.
396	6		20	16	28.0	to 17	28.0	880	
397	6		20	17	42.0	to 18	42.0	880	
398	6		20	18	56.0	to 19	56.0	880	
		1915							
399	5	January	8	13	02.5	to 14	02.5	894	Air foggy.
400	6		8	14	37.5	to 15	52.5	894	
401	5		11	12	44.5	to 13	44.5	894	
402	5		11	14	22.5	to 15	22.5	894	
403	6		11	15	38.0	to 16	38.0	894	
404	7		11	17	16.0	to 18	28.0	894	
406	5		14	13	50.5	to 14	50.5	895	
407	7		14	15	06.0	to 16	18.0	895	
409	6	February	10	13	52.0	to 14	52.0	902	

TABLE IV—Continued

JOURNAL OF PLATES

Plate No.	No. of Exposures	Dates of Exposures G. M. T.						Epoch	Notes
(1)	(2)	(3)						(4)	(5)
		1915	d	h	m	h	m		
410	6	February	10	15	12.0	to 16	12.0	902	
411	5		10	16	26.0	to 17	14.0	902	
412	5		15	13	38.5	to 14	38.5	904	
413	5		15	15	02.5	to 16	02.5	904	
414	5		15	16	33.5	to 17	33.5	904	Air very foggy during last two exposures.
415	6		16	14	29.5	to 15	44.5	904	
416	5		16	16	02.5	to 17	02.5	904	
417	5	April	3	14	20.5	to 15	20.5	916	
418	4		10	14	07.5	to 14	52.5	918	
419	4		10	15	14.5	to 15	59.5	918	
420	4		11	14	13.5	to 14	58.5	919	
421	4		11	15	24.5	to 16	09.5	919	
422	6		14	14	22.5	to 15	37.5	919	

From the observations of Wendell a maximum occurred on J. D. 2418280.699; and from my photographic observations a maximum occurred on J. D. 2420401.967. With these data I have derived the following elements:

$$\text{Helioc. Max.} = \text{J.D. } 2417173.459 + 3^{d}72806 \cdot E, \text{ G.M.T. (Final Elements)}$$

It was with these elements that the phases given in Tables III and V were computed. The observations were then plotted according to phase and the curves shown in Figures 1 and 2 drawn through them. These curves attempt to show the actual course of the observations. The accuracy with which this is done may be estimated from the residuals given in the last columns of Tables III and V. The dotted portions of Figure 2 represent those phases of the light cycle for which observations are lacking.

A comparison of the two curves, which are drawn to the same scale, shows the characteristic that is generally common to stars of the δ-Cephei type, namely, that the photographic amplitude is greater than the visual. In the case of RT Aurigae the ratio is:

Phot. range / Vis. range = 1.20/0.85 = 1.41. The descending branches are marked by numerous fluctuations, those of greatest amplitude occurring 0.5 day and 1.0 day after maximum. In discussing his observations of this star Astbury[1] says: "It may be worth while to mention that the apparent fall and recovery at about 0.5 days after maximum may not be altogether due to faulty observation." This fall and recovery are amply confirmed by the photographic observations.

1. Journal British Astronomical Association; 18, 87, 1907.

TABLE V

PHOTOGRAPHIC OBSERVATIONS OF RT AURIGAE

Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	v	O–C	Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	v	O–C
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days	mag.	mag.			2420	days	mag.	mag.
111	1	147.5389	2.8188	6.14	0.00	118	5	164.7063	1.3450	5.87	−0.02
	2	.5458	.8257	6.12	−0.01		6	.7132	.3519	5.85	−0.05
	3	.5528	.8327	6.14	0.00		7	.7201	.3588	5.90	−0.02
	4	.5597	.8396	6.16	0.00		8	.7271	.3658	5.92	−0.01
	5	.5667	.8466	6.21	+0.02		9	.7340	.3727	5.93	0.00
	6	.5736	.8535	6.21	+0.01		10	.7410	.3797	5.93	−0.01
	7	.5806	.8605	6.19	−0.01		11	.7479	.3866	5.94	0.00
							12	.7549	.3936	5.92	0.00
112	1	153.5813	1.4048	5.99	+0.09						
	2	.5882	.4117	5.98	+0.08	119	1	178.6813	0.4069	5.44	−0.03
	3	.5951	.4186	5.96	+0.05		2	.6882	.4138	5.46	0.00
	4	.6021	.4256	6.02	+0.09		3	.6951	.4207	5.45	−0.01
	5	.6090	.4325	5.99	+0.04		4	.7021	.4277	5.45	0.00
	6	.6160	.4395	6.00	+0.03		5	.7090	.4346	5.44	0.00
	7	.6229	.4464	6.02	+0.04		6	.7160	.4416	5.44	+0.01
	8	.6299	.4534	6.00	+0.01		7	.7229	.4485	5.40	−0.03
	9	.6368	.4603	6.00	0.00						
	10	.6438	.4673	5.99	0.00	120	1	180.5667	2.2922	6.19	+0.01
	11	.6507	.4742	5.98	−0.01		2	.5736	.2991	6.15	−0.03
	12	.6576	.4811	6.00	+0.02		3	.5806	.3061	6.14	−0.03
							4	.5875	.3130	6.12	−0.04
113	1	153.6681	1.4916	5.97	0.00		5	.5944	.3199	6.15	0.00
	2	.6750	.4985	5.95	0.00		6	.6014	.3269	6.14	0.00
	3	.6819	.5054	5.97	+0.02		7	.6083	.3338	6.14	0.00
	4	.6889	.5124	5.94	0.00						
	5	.6958	.5193	5.96	+0.02	121	1	180.6250	2.3505	6.14	−0.02
	6	.7028	.5263	5.98	+0.04		2	.6319	.3574	6.17	0.00
	7	.7097	.5332	5.99	+0.04		3	.6389	.3644	6.12	−0.05
							4	.6458	.3713	6.14	−0.03
114	1	153.7236	1.5471	5.95	−0.01		5	.6528	.3783	6.18	+0.01
	2	.7306	.5541	5.96	−0.01		6	.6597	.3852	6.16	0.00
	3	.7375	.5610	5.97	−0.01		7	.6667	.3922	6.14	−0.01
	4	.7444	.5679	5.93	−0.05		8	.6736	.3991	6.15	0.00
	5	.7514	.5749	6.00	+0.01		9	.6805	.4060	6.16	0.00
	6	.7583	.5818	6.03	+0.04		10	.6875	.4130	6.18	+0.01
116	1	164.5625	1.2012	5.86	0.00	122	1	180.6958	2.4213	6.18	0.00
	2	.5695	.2082	5.85	−0.01		2	.7028	.4283	6.21	0.00
	3	.5764	.2151	5.85	−0.02		3	.7097	.4352	6.21	−0.01
	4	.5833	.2220	5.87	0.00		4	.7167	.4422	6.25	+0.01
	5	.5903	.2290	5.89	0.00		5	.7236	.4491	6.21	−0.03
	6	.5972	.2359	5.92	+0.01		6	.7306	.4561	6.25	0.00
	7	.6042	.2429	5.88	−0.03		7	.7375	.4630	6.23	0.00
							8	.7444	.4699	6.21	0.00
117	1	164.6125	1.2512	5.95	+0.02		9	.7514	.4769	6.21	0.00
	2	.6194	.2581	5.93	0.00		10	.7583	.4838	6.25	+0.04
	3	.6264	.2651	5.95	0.00						
	4	.6333	.2720	5.96	0.00	132	1	185.6806	3.6776	5.08	+0.01
	5	.6403	.2790	5.94	−0.01		2	.6875	.6845	5.16	+0.10
	6	.6472	.2859	5.98	+0.03		3	.6944	.6914	5.14	+0.08
	7	.6542	.2929	5.94	0.00		4	.7014	.6984	5.16	+0.10
	8	.6611	.2998	5.94	+0.01		5	.7083	.7053	5.16	+0.10
							6	.7153	.7123	5.16	+0.11
118	1	164.6785	1.3172	5.88	−0.01		7	.7222	.7192	5.21	+0.16
	2	.6854	.3241	5.85	−0.03		8	.7292	.7262	5.21	+0.16
	3	.6924	.3311	5.90	+0.01		9	.7361	0.0050	5.25	+0.20
	4	.6993	.3380	5.91	+0.02		10	.7431	.0120	5.28	+0.22

TABLE V—Continued

PHOTOGRAPHIC OBSERVATIONS OF RT AURIGAE

Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	v	O–C	Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	v	O–C
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days	mag.	mag.			2420	days	mag.	mag.
133	1	188.6701	2.9388	6.11	0.00	141	1	209.6632	1.5615	5.98	0.00
	2	.6771	.9458	6.10	0.00		2	.6701	.5684	5.96	−0.02
	3	.6840	.9527	6.11	+0.01		3	.6771	.5754	6.00	+0.01
	4	.6910	.9597	6.09	0.00		4	.6840	.5823	6.00	+0.01
	5	.6979	.9666	6.13	+0.01		5	.6910	.5893	6.00	+0.02
	6	.7049	.9736	6.13	0.00		6	.6979	.5962	5.94	−0.03
	7	.7118	.9805	6.09	−0.03		7	.7049	.6032	5.94	−0.02
	8	.7188	.9875	6.09	−0.03		8	.7118	.6101	5.92	−0.04
	9	.7257	.9944	6.12	0.00		9	.7187	.6170	5.86	−0.09
	10	.7326	3.0013	6.13	+0.01						
	11	.7396	.0083	6.13	+0.02	142	1	212.6524	0.8224	5.68	−0.05
	12	.7465	.0152	6.16	+0.05		2	.6597	.8297	5.75	0.00
							3	.6667	.8367	5.83	+0.08
134	1	191.5875	2.1279	6.11	0.00		4	.6736	.8436	5.78	+0.02
	2	.5944	.1348	6.11	0.00						
	3	.6014	.1418	6.13	+0.01	146	1	214.6313	2.8011	6.17	−0.03
	4	.6083	.1487	6.14	0.00		2	.6382	.8080	6.18	0.00
	5	.6153	.1557	6.15	0.00		3	.6451	.8149	6.17	0.00
	6	.6222	.1626	6.17	+0.01		4	.6521	.8219	6.12	−0.01
	7	.6292	.1696	6.12	−0.03		5	.6590	.8288	6.13	0.00
	8	.6361	.1765	6.16	−0.01		6	.6660	.8358	6.15	0.00
							7	.6729	.8427	6.14	−0.02
135	1	191.6444	2.1848	6.13	−0.05		8	.6799	.8497	6.10	−0.10
	2	.6514	.1918	6.18	0.00						
	3	.6583	.1987	6.16	−0.02	148	1	220.5854	1.2985	5.94	+0.01
	4	.6653	.2057	6.18	0.00		2	.5924	.3055	5.91	0.00
	5	.6722	.2126	6.12	−0.05		3	.5993	.3124	5.93	+0.03
	6	.6792	.2196	6.18	+0.01		4	.6063	.3194	5.93	+0.04
	7	.6861	.2265	6.16	0.00		5	.6132	.3263	5.98	+0.09
	8	.6931	.2335	6.21	+0.07		6	.6201	.3332	5.98	+0.09
							7	.6271	.3402	5.92	+0.02
136	1	191.7014	2.2418	6.11	−0.02		8	.6340	.3471	5.90	0.00
	2	.7083	.2487	6.12	−0.01		9	.6410	.3541	5.92	−0.02
	3	.7153	.2557	6.14	0.00						
	4	.7222	.2626	6.15	0.00	149	1	235.5938	1.3932	5.90	0.00
	5	.7292	.2696	6.16	0.00		2	.6007	.4001	5.92	+0.03
	6	.7361	.2765	6.19	+0.01		3	.6076	.4070	5.85	−0.04
	7	.7431	.2835	6.18	0.00		4	.6146	.4140	5.95	+0.04
	8	.7500	.2904	6.19	0.00		5	.6215	.4209	5.93	0.00
							6	.6285	.4279	5.90	−0.03
137	1	193.6271	0.4392	5.33	−0.09		7	.6354	.4348	5.92	−0.03
	2	.6340	.4461	5.35	−0.06		8	.6424	.4418	5.92	−0.04
	3	.6410	.4531	5.42	+0.01		9	.6493	.4487	5.92	−0.05
	4	.6479	.4600	5.38	−0.01						
	5	.6549	.4670	5.39	0.00	316	1	394.8271	0.3204	5.47	+0.04
	6	.6618	.4739	5.37	0.00		2	.8340	.3273	5.44	0.00
	7	.6688	.4809	5.37	0.00		3	.8410	.3343	5.45	0.00
	8	.6757	.4878	5.34	−0.02		4	.8479	.3412	5.43	−0.02
							5	.8549	.3482	5.47	+0.01
138	1	193.6840	0.4961	5.36	0.00		6	.8618	.3551	5.43	−0.03
	2	.6910	.5031	5.35	0.00		7	.8688	.3621	5.44	−0.03
	3	.6979	.5100	5.35	0.00		8	.8757	.3690	5.42	−0.05
	4	.7049	.5170	5.40	+0.05						
	5	.7118	.5239	5.35	0.00	317	1	394.8840	0.3773	5.48	0.00
	6	.7187	.5308	5.37	0.00		2	.8910	.3843	5.49	0.00
	7	.7257	.5378	5.34	−0.01		3	.8979	.3912	5.51	+0.02
	8	.7326	.5447	5.31	−0.06		4	.9049	.3982	5.49	0.00

TABLE V—Continued

PHOTOGRAPHIC OBSERVATIONS OF RT AURIGAE

Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	r	O–C	Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	r	O–C
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days	mag.	mag.			2420	days	mag.	mag.
317	5	394.9118	0.4051	5.47	−0.01	389	2	450.6684	0.2457	5.32	+0.06
	6	.9188	.4121	5.52	+0.04		3	.6788	.2561	5.44	+0.16
	7	.9257	.4190	5.52	+0.04						
	8	.9326	.4259	5.52	+0.05	396	1	457.6861	3.4558	5.46	−0.06
							2	.6944	.4641	5.54	+0.02
338	1	401.8417	3.6076	5.20	+0.04		3	.7028	.4725	5.54	+0.04
	2	.8486	.6145	5.25	+0.10		4	.7111	.4808	5.46	0.00
	3	.8556	.6215	5.24	+0.10		5	.7194	.4891	5.44	0.00
	4	.8625	.6284	5.21	+0.09		6	.7278	.4975	5.38	0.00
	5	.8694	.6353	5.20	+0.09						
	6	.8764	.6423	5.19	+0.09	397	1	457.7375	3.5072	5.32	−0.03
	7	.8833	.6492	5.17	+0.08		2	.7458	.5155	5.32	−0.02
							3	.7542	.5239	5.32	0.00
339	1	401.8917	3.6576	5.06	−0.02		4	.7625	.5322	5.34	+0.02
	2	.8986	.6645	5.10	+0.02		5	.7708	.5405	5.29	0.00
	3	.9056	.6715	5.08	0.00		6	.7792	.5489	5.22	−0.05
	4	.9125	.6784	5.10	+0.03						
	5	.9194	.6853	5.12	+0.05	398	1	457.7889	3.5586	5.25	0.00
	6	.9264	.6923	5.09	+0.03		2	.7972	.5669	5.19	−0.03
	7	.9333	.6992	5.09	+0.03		3	.8056	.5753	5.21	+0.01
							4	.8139	.5836	5.23	+0.03
363	1	425.7264	1.3982	5.88	−0.02		5	.8222	.5919	5.18	0.00
	2	.7347	.4065	5.90	+0.01		6	.8306	.6003	5.16	0.00
	3	.7431	.4149	5.92	+0.01						
	4	.7514	.4232	5.92	0.00	399	1	506.5434	0.2011	5.23	−0.01
	5	.7597	.4315	5.90	−0.03		2	.5538	.2115	5.35	+0.10
	6	.7681	.4399	5.88	−0.07		3	.5642	.2219	5.26	0.00
							4	.5747	.2324	5.27	0.00
364	1	425.7861	1.4579	5.96	−0.03		5	.5851	.2428	5.23	−0.05
	2	.7944	.4662	5.92	−0.07						
	3	.8028	.4746	5.89	−0.10	400	1	506.6094	0.2671	5.37	+0.04
	4	.8111	.4829	5.91	−0.07		2	.6198	.2775	5.37	+0.02
	5	.8194	.4912	5.91	−0.06		3	.6302	.2879	5.38	0.00
	6	.8278	.4996	5.88	−0.08		4	.6406	.2983	5.42	+0.03
							5	.6510	.3087	5.39	−0.01
365	1	425.8375	1.5093	5.93	−0.02		6	.6615	.3192	5.36	−0.06
	2	.8458	.5176	5.94	0.00						
	3	.8542	.5260	5.94	0.00	401	1	509.5309	3.1887	6.02	0.00
	4	.8625	.5343	5.94	0.00		2	.5413	.1991	6.01	0.00
	5	.8708	.5426	5.93	−0.02		3	.5517	.2095	6.00	0.00
	6	.8792	.5510	5.91	−0.05		4	.5622	.2200	5.93	−0.05
							5	.5726	.2304	5.93	−0.03
379	1	429.7229	1.6670	6.10	0.00						
	2	.7313	.6754	6.10	0.00	402	1	509.5990	3.2568	5.94	0.00
	3	.7396	.6837	6.09	0.00		2	.6094	.2672	5.91	0.00
	4	.7479	.6920	6.08	0.00		3	.6198	.2776	5.93	+0.03
	5	.7563	.7004	6.08	0.00		4	.6302	.2880	5.87	0.00
	6	.7646	.7087	6.03	−0.05		5	.6406	.2984	5.82	−0.04
380	1	429.7958	1.7399	6.11	+0.01	403	1	509.6514	3.3092	5.86	+0.01
	2	.8042	.7483	6.07	0.00		2	.6597	.3175	5.85	+0.01
	3	.8125	.7566	6.05	0.00		3	.6681	.3259	5.86	+0.04
	4	.8208	.7649	6.03	0.00		4	.6764	.3342	5.83	+0.03
	5	.8292	.7733	6.05	+0.02		5	.6847	.3425	5.82	+0.03
	6	.8375	.7816	5.99	−0.01		6	.6931	.3509	5.77	0.00
389	1	450.6267	0.2040	5.46	+0.21	404	1	509.7195	3.3773	5.72	0.00

TABLE V—Continued

PHOTOGRAPHIC OBSERVATIONS OF RT AURIGAE

Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	v	O–C	Plate No.	Exp. No.	Julian Day G. M. T.	Helioc. Phase	v	O–C
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days	mag.	mag.			2420	days	mag.	mag.
404	2	509.7278	3.3856	5.69	0.00	414	1	544.6899	1.0653	5.79	0.00
	3	.7361	.3939	5.68	+0.01		2	.7004	.0758	5.80	0.00
	4	.7444	.4022	5.68	+0.04		3	.7108	.0862	5.81	+0.01
	5	.7528	.4106	5.66	+0.04		4	.7212	.0966	5.77	−0.02
	6	.7611	.4189	5.60	+0.02		5	.7316	.1070	5.79	0.00
	7	.7695	.4273	5.57	+0.02						
						415	1	545.6038	1.9791	6.10	0.00
406	1	512.5767	2.5063	6.22	0.00		2	.6143	.9896	6.11	0.00
	2	.5871	.5167	6.21	0.00		3	.6247	2.0000	6.16	+0.01
	3	.5976	.5272	6.20	0.00		4	.6351	.0104	6.14	0.00
	4	.6080	.5376	6.16	−0.04		5	.6455	.0208	6.12	+0.01
	5	.6184	.5480	6.20	0.00		6	.6559	.0312	6.13	0.00
407	1	512.6292	2.5588	6.22	0.00	416	1	545.6684	2.0437	6.15	0.00
	2	.6375	.5671	6.21	−0.01		2	.6788	.0541	6.17	0.00
	3	.6458	.5754	6.22	−0.02		3	.6892	.0645	6.14	0.00
	4	.6542	.5838	6.25	0.00		4	.6997	.0750	6.15	0.00
	5	.6625	.5921	6.27	+0.01		5	.7101	.0854	6.16	0.00
	6	.6708	.6004	6.25	0.00						
	7	.6792	.6088	6.20	−0.03	417	1	591.5976	3.2318	5.95	−0.02
							2	.6080	.2422	5.93	−0.04
409	1	539.5778	3.4095	5.64	−0.04		3	.6184	.2526	5.92	−0.02
	2	.5861	.4178	5.62	−0.03		4	.6288	.2630	5.92	0.00
	3	.5944	.4261	5.62	−0.01		5	.6392	.2734	5.90	−0.01
	4	.6028	.4345	5.62	0.00						
	5	.6111	.4428	5.52	−0.05	418	1	598.5885	2.7658	6.20	0.00
	6	.6195	.4512	5.50	−0.05		2	.5990	.7763	6.19	0.00
							3	.6094	.7867	6.21	0.00
410	1	539.6333	3.4650	5.51	0.00		4	.6198	.7971	6.25	+0.01
	2	.6417	.4734	5.50	0.00						
	3	.6500	.4817	5.46	0.00	419	1	598.6351	2.8124	6.19	0.00
	4	.6583	.4900	5.45	+0.01		2	.6455	.8228	6.18	−0.01
	5	.6667	.4984	5.40	0.00		3	.6559	.8332	6.18	0.00
	6	.6750	.5067	5.39	0.00		4	.6663	.8436	6.26	+0.06
411	1	539.6847	3.5164	5.38	0.00	420	1	599.5927	0.0419	5.06	0.00
	2	.6931	.5248	5.33	−0.01		2	.6031	.0523	5.07	0.00
	3	.7014	.5331	5.35	+0.01		3	.6135	.0627	5.06	−0.02
	4	.7097	.5414	5.31	0.00		4	.6240	.0732	5.08	−0.01
	5	.7181	.5498	5.29	0.00						
						421	1	599.6420	0.0912	5.17	+0.02
412	1	544.5684	0.9438	5.72	+0.01		2	.6524	.1016	5.16	0.00
	2	.5788	.9542	5.71	+0.02		3	.6629	.1121	5.17	0.00
	3	.5892	.9646	5.68	0.00		4	.6733	.1225	5.18	0.00
	4	.5996	.9750	5.68	0.00						
	5	.6101	.9855	5.66	−0.02	422	1	602.5990	3.0480	6.17	+0.03
							2	.6094	.0584	6.15	+0.01
413	1	544.6267	1.0021	5.75	0.00		3	.6198	.0688	6.14	0.00
	2	.6372	.0126	5.74	0.00		4	.6302	.0792	6.15	+0 01
	3	.6476	.0230	5.72	0.00		5	.6406	.0896	6.08	−0.05
	4	.6580	.0334	5.71	0.00		6	.6510	.1000	6.13	0.00
	5	.6684	.0438	5.77	+0.02						

In his discussion of the radial velocity observations of RT Aurigae, Duncan remarks[1] that the variable velocity is not satisfactorily represented on the assumption of elliptic orbital motion. Although only five of Duncan's observations occur at, or near, phases 0.5 day and 1.0 day after maximum light, yet these few observations indicate the same rise and fall in the velocity curve that is characteristic of the light curve. In view of the recent interesting criticisms[2] of the idea, at present generally accepted, that variables of the δ-Cephei type are binaries, it would seem desirable that observers who have the necessary apparatus at their disposal make a few well-placed spectroscopic observations to determine whether the velocity curves of these stars show secondary oscillations of the kind exhibited by the light curves.

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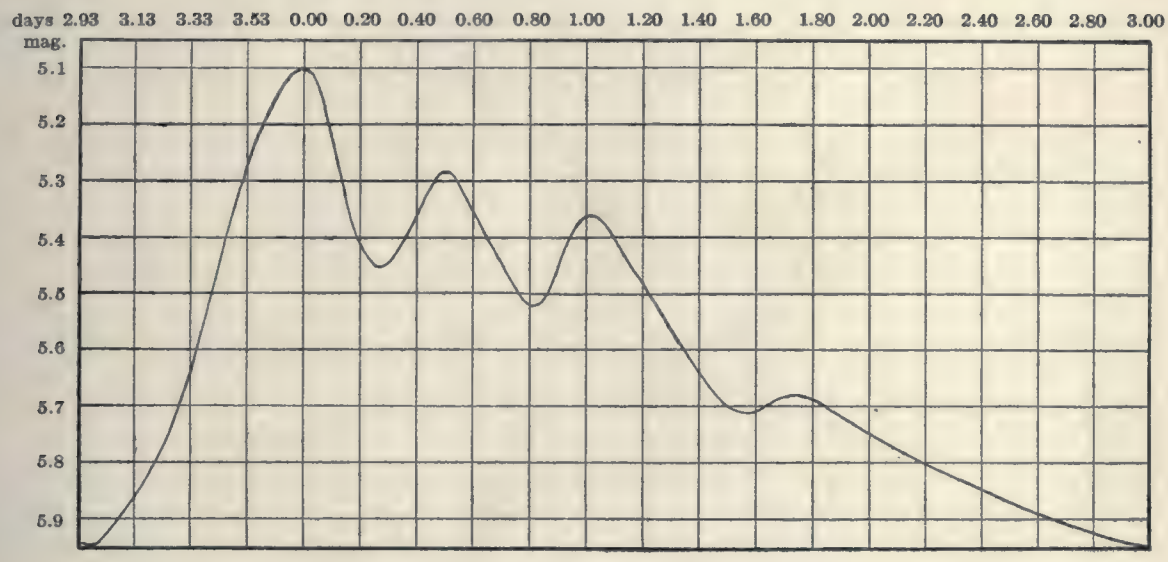

Figure 1. VISUAL LIGHT CURVE OF RT AURIGAE

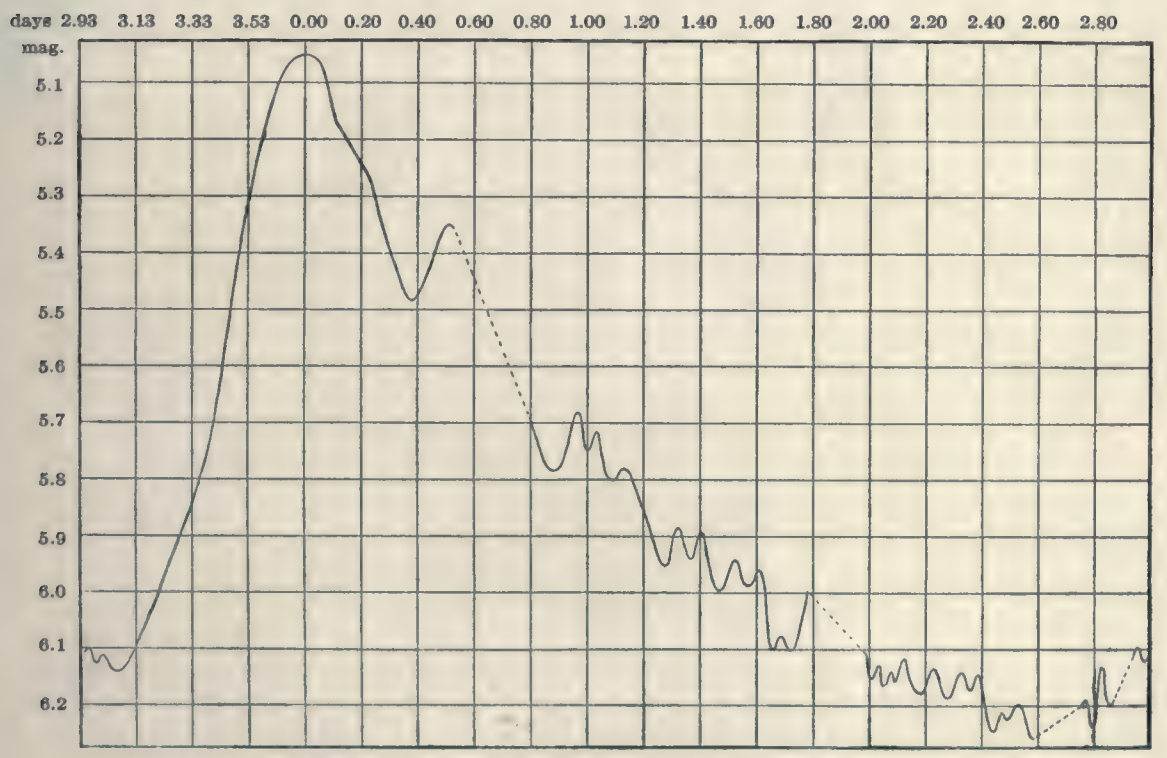

Figure 2. PHOTOGRAPHIC LIGHT CURVE OF RT AURIGAE

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INVESTIGATIONS IN EXTRAFOCAL PHOTOMETRY

By Robert H. Baker and Edith E. Cummings

The application of photography to the precise measurement of star magnitudes has opened an attractive field of investigation that may be entered profitably by an observatory having limited equipment. Recent contributions to the subject from Harvard[1], Yerkes[2], Mount Wilson[3] and other observatories indicate the value of these investigations and the difficulties to be overcome. Consistency among the various photographic scales is especially to be desired and greater surety in the control of systematic errors.

Among the many methods employed in photographic photometry, the extrafocal method developed by Professor Parkhurst at the Yerkes Observatory deserves special consideration. The plate is exposed in the camera beyond focus; and the resulting images have relative densities which vary with the intensities of the stars producing them. The distance from focus is great enough to confuse the aberrations of the lens and to give nearly uniform illumination to the star images. The images on the Yerkes plates have diameters somewhat greater than 1 mm. A subsequent exposure in a sensitometer, directed toward the northern sky in the daytime, impresses on the plate artificial star-disks of known relative magnitudes. A Hartmann microphotometer is employed to measure the densities of the extrafocal and artificial images. On plotting wedge readings for the sensitometer images against relative magnitudes, a reduction curve is determined; and by means of this curve the wedge readings for the star images are converted to relative magnitudes on an absolute scale.

The method is simple and direct, and it requires a moderate outlay; in addition, it admits easy control of the peculiarities of individual plates. At present, the inaccuracies in the results far exceed errors in measurement, as Parkhurst has pointed out.

The extrafocal method was adopted at the Laws Observatory for the reasons given, and because of its applicability for the photographic study of variable stars. In using

[1]H. S. Leavitt, *Harvard Annals*, **71**, No. 3, 1915.
[2]J. A. Parkhurst, *Astrophysical Journal*, **36**, 169, 1912.
[3]F. H. Seares, *Mount Wilson Contributions*, **97** and **98**, 1915.

this method we have the advantage of Parkhurst's experience, and while generally the procedure is similar, departures will be noted.

During the past three years a number of additions have been made to the equipment of the Laws Observatory. to make possible its participation in photographic photometry. The 5-inch photographic doublet and the Hartmann microphotometer were purchased. The 7½-inch refractor was reconstructed in many of its mechanical parts, in preparation for the camera. A dark room was built and equipped for constant temperature developing. A sensitometer, an instrument for measuring the diameters of star images and other auxiliary apparatus were constructed and tested. The work of the first year was undertaken by Mr. Baker, with the able co-operation of the University mechanician, Mr. Emil Klinkerfuss, who also constructed most of the photometric equipment now in use here. The successful outcome of our undertaking is in large measure due to his skill in meeting the mechanical difficulties we have encountered.

Systematic work on the regular program was begun by the writers in June, 1914. The number of plates secured with the camera now exceeds 600, and they contain a total of more than 7000 exposures. Our object thus far has been, first to investigate the behavior of instruments and plates and to determine the procedure that yields maximum accuracy; second, to accumulate material for the photographic study of a selected number of eclipsing binary stars. The work has now reached the point where reliability in its results is assured with a reasonable degree of safety. Many of the conclusions are provisional and certain of the results are withheld until they can receive further attention.

The present paper contains descriptions of the instruments and methods we have used. These are followed by a summary of the investigations for control and an account of the provisional scale of the Laws Observatory photographic magnitudes. In conclusion we have added a report of progress on the program of variable stars under investigation.

I. EXPOSURE OF THE PLATES

Guiding Telescope. The 7½-inch refractor (189 mm.) carries the camera and serves as the guiding telescope. Its focal length is about 10 feet. The radius of the guiding field represents a distance of 2.5mm. on the plate in the camera.

Many alterations were necessary to prepare the telescope for the present work. A driving clock was secured from the Brashear Company, with motor-driven differential gear for slow motion in right ascension, that is operated by a switch convenient to the observer. Clamp and slow motion in declination of the usual type were also installed. In the present arrangement, the worm and the gear wheel on the polar axis are permanently connected. A divided collar is attached to the wheel. Clamping in right ascension is effected by drawing the two halves together upon the axis by a slight turn of a screw. These and other improvements, made by Mr. Klinkerfuss, have given us a serviceable instrument.

Camera. The camera objective is a doublet of the Petzval type, made by the John A. Brashear Company from designs by Dr. Hastings. The clear aperture is 127 mm. and

the focal length is 657.5 mm. The objective cell is contained in an aluminum tube 672 mm. in length and 198 mm. in diameter; it is moved in the tube toward or away from the plate by a 2 mm. screw with head divided to 0.1 mm. A millimeter scale on the side of the tube, graduated from 0 to 40, represents the range of this motion.

A knife edge, mounted in a frame similar in size to the plate holder, and given rapid motion across the optical axis by rack and pinion, serves to determine the setting for focus with required accuracy. The setting is made by moving the objective in the direction of increasing numbers. It averages 22.7 divisions on the scale, and varies so little with extremes of temperature that the knife edge is seldom used.

At the back of the camera a movable carrier receives the plate holders. The carrier is moved in ways by a 2 mm. screw, which is operated from the floor through a train of four small bevel gears by turning a graduated head. One division of the head represents a motion of 0.1 mm. The range of the carrier's motion is 50 mm. The plate may thus be shifted lengthwise, that is, in declination and receive a series of exposures.

Plate I shows the relation of camera and guiding telescope, and the mechanical parts we have mentioned. The two tubes are separated a distance of 125 mm., that is sufficient to prevent the objective end of the guiding telescope from shading any part of the plate. The width of the dome slit, originally less than two feet, was enlarged to 50 inches to give the camera greater freedom.

The aberrations in the camera objective have been partly studied by Hartmann's method of extrafocal measurements. A detailed description of the method is given by its author[1]. It consists in determining the focal length from small areas of the lens taken in pairs that are opposite and equidistant from the center.

A perforated zone plate was placed over the objective. The circular apertures, 5 mm. in diameter, are aligned on diameters at 45° intervals; and these openings, four to each zone, lie in eight zones whose radii are respectively 9, 16, 23, 30, 37, 44, 51, 58 mm. Exposures of six minutes were made on Capella, when it was near the zenith, using Seed Process plates. The exposures were in pairs: first, with the plate about 15 mm. inside of focus; second, with the plate the same distance outside of focus. Between pairs of exposures the zone plate was turned $22\frac{1}{2}°$. For the different exposures the positions were: $\phi = 0°$, $22\frac{1}{2}°$, 45°, $67\frac{1}{2}°$, and in this way many parts of the objective came under investigation. Each exposure impressed on the plate the pattern of the zone plate.

The plates were measured and reduced by Mr. F. M. Walters, Jr., then Assistant in the Department of Physics of the University. Figure 1 exhibits local variations from the mean focal length of the objective. The numbers are differences, local *minus* mean focal length. The unit is 0.01 mm., and a negative difference, i. e. a focus shorter than the mean, is enclosed in a lighter circle. Opposite points have the same value, from the nature of the method. The differences for the zone nearest the center are, of course, subject to the greatest inaccuracies; and this probably explains the rapid changes of sign around

[1] *J. Hartmann, Zeitschrift für Instrumentenkunde*, **24**, 1, 33, 97, 1904.

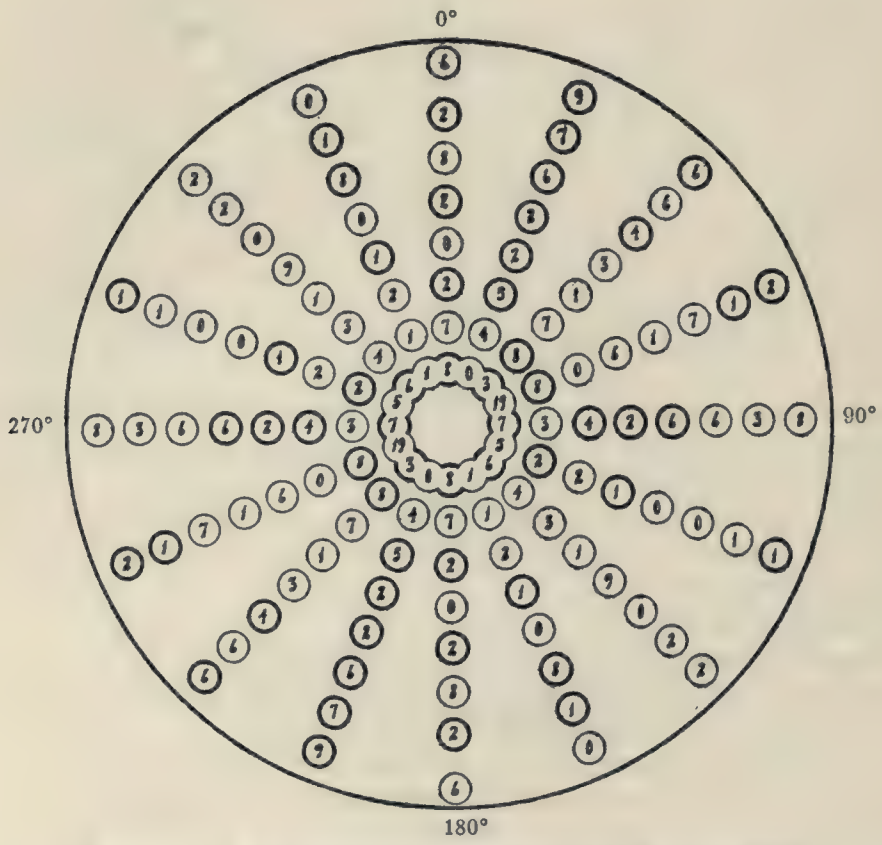

FIGURE 1. LOCAL DIFFERENCES IN FOCUS OF CAMERA OBJECTIVE
Heavy circle indicates focus longer than the mean; light circle, focus shorter than the mean. The unit is 0.01 mm.

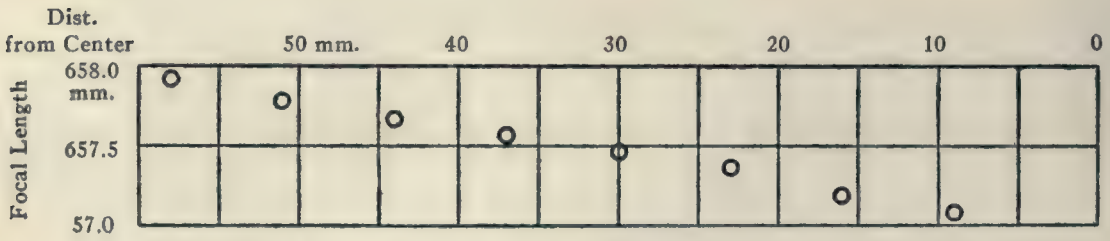

FIGURE 2. ZONAL FOCI OF CAMERA OBJECTIVE

the inner circle. In Figure 2, the mean focal length of each zone is plotted against its radius. The relation is linear and the focal length near the center is nearly 1 mm. shorter than near the edge of the lens.

The color curve of the camera objective could not be readily determined with the means at hand. A preliminary investigation indicated a remarkably flat curve for that part of the spectrum to which the Cramer Spectrum plate is sensitive. This conclusion is strengthened, in Section V of this paper, by the small variation in the diameters of the star images between blue and red stars; in fact, we have detected no change in the diameters between classes A0 and K0.

The camera field is reasonably flat. When the center of the plate is in focus, a point 45 mm. from the center is 1.0 mm. beyond focus. The character of the extrafocal images has been carefully investigated at small intervals as far as 5 mm. from focus in both directions. Inside of focus the images are unsatisfactory. Outside of focus they are generally not uniform, due to a marked diffraction pattern. It is fair to mention that Mr. James B. McDowell of the Brashear Company was willing to alter the objective to make it more suitable for this special purpose. But as the early results with the camera came out well, it was decided to make no change in the lens. The character of the star disks varies evidently with temperature. It is certain, that on plates exposed in the winter the illumination is more nearly uniform than on summer plates.

A ray filter for photo-visual work with the camera has been secured from Mr. R. J. Wallace. It is similar to the "visual luminosity" filter employed by Professor Parkhurst in the determination of "visual" magnitudes.

Sensitometer. In general design the sensitometer resembles the device used at the Yerkes Observatory.[1] It consists of a nest of 24 thin brass tubes six in a row, each tube 76 mm. long and 14.3 mm. inside diameter. These are set in a brass box whose dimensions are 209x159x76 mm. Two thin brass plates that close the box at the front and back, are pierced with circular holes opposite the center of each tube. The front plate contains apertures whose diameters range from 1.0 to 5.5 mm. Three back plates have been used at different times. Plate A has apertures all 1 mm. in diameter; in plates B and C the diameters are all about one-third millimeter. We have used back plate B generally.

In front of the sensitometer box is a fixed frame containing four plates of opal glass, size 10x12 inches, set close together. We have experienced much difficulty in getting opal glass of this size, which has satisfactory uniformity of thickness and density over the plate. The ones now in use are far from plane-parallel; but they are matched in such a way that their performance is very good. The sensitometer box is attached to a shaft and is rotated by turning a crank, so that each cell may be brought opposite different parts of the opal glass. Irregularities in the glass thus have less effect on the results.

[1] J. A. Parkhurst and F. C. Jordan, *Astrophysical Journal*, **26**, 244, 1907; J. A. Parkhurst, *Astrophysical Journal*, **36**, 173, 1912.

Care has been taken to have the metal and glass plates parallel, and perpendicular to the cells and to the axis of rotation.

A special plate holder fits into ways at the back of the sensitometer box. When the slide is drawn, the photographic plate is pressed forward by springs against the metal back plate of the sensitometer.

Some of these points will be made clearer by consulting Plate 2, which shows the sensitometer inclined for pointing toward the pole. Light from the northern sky,

TABLE I

CALIBRATION OF THE SENSITOMETER

No.	Diam. of Aperture	Relative Area	Δ Log. Area	Computed ΔMag.	Correction	Adopted ΔMag.
	mm.				mag.	
1	1.003	1.006	0.000	0.000	−.09	0.00
2	1.090	1.188	0.072	0.181	.00	0.27
3	1.252	1.568	0.193	0.482	+.04	0.61
4	1.347	1.814	0.256	0.640	+.03	0.76
5	1.402	1.966	0.291	0.727	+.03	0.85
6	1.540	2.372	0.373	0.931	−.03	0.99
7	1.590	2.528	0.400	1.000	.00	1.09
8	1.702	2.897	0.459	1.148	+.03	1.27
9	1.865	3.478	0.539	1.347	+.02	1.46
10	2.005	4.020	0.602	1.504	+.02	1.61
11	2.092	4.377	0.639	1.596	−.01	1.68
12	2.284	5.217	0.715	1.787	−.08	1.80
13	2.434	5.924	0.770	1.925	−.02	1.99
14	2.687	7.220	0.856	2.140	−.02	2.21
15	2.809	7.891	0.895	2.236	+.04	2.37
16	3.058	9.351	0.968	2.421	+.02	2.53
17	3.367	11.337	1.052	2.630	+.02	2.74
18	3.512	12.334	1.088	2.721	−.01	2.80
19	3.741	13.995	1.143	2.858	−.07	2.88
20	4.061	16.492	1.215	3.037	−.02	3.11
21	4.283	18.344	1.261	3.152	−.02	3.22
22	4.723	22.307	1.346	3.365	.00	3.45
23	5.050	25.502	1.404	3.510	.00	3.60
24	5.445	29.648	1.469	3.673	.00	3.76

after diffusion in the opal glass, is admitted into the cells through the apertures of the front plate. Each cell contains two diaphragms and its interior is blackened to minimize reflections. The light passes through the apertures in the back plate and impresses on the photographic plate circular images of nearly uniform size. The relative densities of the images depend on the relative sizes of the apertures in the front plate; and their size is restricted by the apertures in the back plate. A two-piece shutter, between the opal plates and the sensitometer box, is operated quickly by a lever in starting and ending the exposure.

The calibration of the sensitometer is summarized in Table I. Thus far a single front plate has been used. Diameters of the apertures were measured in two directions on both sides of the plate. The mean for each aperture is given in the second column of the table. Squaring, we have the relative area and so the relative intensity of the light; and this is converted to relative magnitude in the usual way.

The Δmagnitudes, so computed, appear in the fifth column of the table. A set of empirical corrections, column (6), is the result of experience with sensitometer curves. We have adopted the Δmagnitudes in the last column.

Procedure. Seed 27 plates, size 4x5 inches, are generally used. The plate is exposed 1.5 or 1.6 mm. outside of focus, this side giving more nearly uniform images. The central image on the plates averages one-third millimeter in diameter and varies a few hundredths of a millimeter, due to differences in setting, temperature changes and other causes.

An exposure of one minute gives appropriate density for a star whose photographic magnitude is 5.5. Six minutes is the normal exposure for magnitude 7.5, and 12 minutes for magnitude 9.0, with some loss of density in the latter case. For stars brighter than 5.5, we have increased the distance from focus to 5.0 mm., giving the central image a diameter of 1.0 mm. and a 9-fold decrease in intensity. More recently Seed Process plates have been used in connection with small images, to give appropriate density for the brighter stars. A combination of the two expedients permits observation of still brighter stars. Thus we may obtain suitable images of stars as faint as photographic magnitude 9.5 with conveniently short exposures.

The relatively long focus of the guiding telescope increases the ease of guiding. However, during exposures one of us has been constantly at the telescope, and the results appear to justify most careful guiding. Close watch is kept for clouds and for smoke that sometimes terminates observations. Unevenness of the sky is a signal to stop work. Exposures are not often undertaken more than four hours from the meridian, except on stars having high declinations.

The number of exposures on a plate seldom exceeds twenty and is usually less. Between exposures the plate is shifted quickly without capping the camera; and care is taken to time the separate exposures accurately and to space them evenly, in order to maintain the same system where images overlap. For the same reason there is no extra spacing between sets of five, as practiced elsewhere. The usual spacing is 0.5 mm. The total exposure is rarely longer than 90 minutes; for excessive sky-fog is to be avoided, and artificial lights abound in the vicinity of the Observatory. Often the length of exposure is limited by the number of individual exposures that are possible without overlapping of the stars to be measured. Whenever the safety of the measurements is endangered by halation from a bright star in the field, the plates are backed before exposure, by application of a caramel and burnt sienna mixture.

A subsequent exposure of the plate in the sensitometer is made in the daytime. Light from the northern sky, after passing through the cells of the sensitometer, im-

presses on the plate a sequence of artificial star-disks whose relative magnitudes we have given in Table I. It is the practice to sensitometrize every star-plate, usually immediately before developing. Exposures in the sensitometer average 25 or 30 seconds, varying considerably according to the densities desired and the condition of the sky.

Plate V is intended to illustrate the above description. It is a reproduction of Plate 494, one of the set taken for the study of the variable star Z Vulpeculae. The variable, here shown near primary minimum, and the four comparison stars are denoted respectively by the letters *v*, *a*, *b*, *c* and *d*. A number of sensitometer images are confused with star images on this plate and, of course, are useless for measurement.

II. DEVELOPMENT OF THE PLATES

Developing Tank. After experimenting with a number of devices for the development of plates at constant temperature, the developing tank, shown in Plate IV, was designed by Mr. Baker and Mr. Klinkerfuss and was constructed by the latter. It is a metal box, $16\frac{3}{4}$x$13\frac{1}{4}$x$5\frac{1}{2}$ inches outside dimensions, with double bottom and walls, and removable cover. Running water from the University supply is the cooling element. Entering from below, it flows through the $\frac{3}{8}$-inch space between the walls and bottoms of the tank and is piped from the top into the sink for additional service in washing plates The flow of water is regulated by a valve above the sink.

The tank is partly filled with water, and in this the developing tray is immersed nearly to the top. The tray is set between clips on a grid, so that water is in contact with its sides and bottom. The water is kept in motion by a motor-driven stirrer, and is heated by an "El-Boilo" element in connection with 110-volt alternating current.

A sensitive mercury thermostat was originally used, and soon discarded as troublesome and superfluous. A room thermostat was substituted. This is partly immersed in the tank. It is connected to the binding posts on the top and thence to dry cells and to the relay. The thermostat is set for the required temperature by altering the distance between its terminals, which is done by moving the indicator on the front of the tank. The flow of water and the amount of resistance in the heating circuit are regulated according to the temperature of the water and of the room.

The water in the tank quickly assumes the required temperature and thereafter fluctuates slightly from it. The developer in the tray requires a longer time, often 20 or 30 minutes, to take up the water temperature. Having once reached this point, its temperature remains practically constant. Care is taken to quickly close the lid after putting in the plate, and to have the plate near the proper temperature before development is begun. As the tank is light-tight, the dark room is often moderately illuminated while developing is proceeding.

In summer the water in the pipes is often above 20°C. The box above the tank is then filled with ice and water, which flows between the walls of the tank and is drained below at the rate required for cooling.

Procedure. The plates are developed at 20.0C. At the outset 10 minutes was given, following Professor Parkhurst's practice. Beginning in 1915 with the use of emulsion 15194, the time of development was reduced to 8 minutes. The loss in density is inconsiderable and the plates appear clearer and "snappier". Also the saving of time is appreciable.

The developer is similar to that used and described by Parkhurst. For completeness the formula follows:

A		B	
Water	32 oz.	Water	32 oz.
Sodium sulphite, dry	1 oz.	Sodium sulphite, dry	1 oz.
Hydrochinon	1 oz. 2 dr.	Potassium carbonate	4 oz.
Sulphuric acid, c. p.	1 dr.	Potassium bromide	1 dr. 2 sc.

Equal parts of A and B.

What effects age and weakening of the developer have on the reduction curve has not been studied; if any, they are avoided by compounding the developer in small quantities, and by throwing away the 8 ounces used at one time, after not more than a dozen 4x5 plates have been developed in it.

At first a fixing box was used, in which the plates stand on edge; and they remained in it until it was convenient to remove them. The plates are now fixed while lying flat in a standard acid-hypo bath for between 18 and 20 minutes. While the comparison is not conclusive, the latter procedure appears to give greater concordance in the reduction curves.

III. MEASUREMENT OF THE PLATES

Microphotometer. The Hartmann microphotometer was made by the John A. Brashear Company, and is similar to the one they made for the Allegheny Observatory. The instrument has been fully described by its inventor[1]. Plate III illustrates the details clearly. Light from a 25-watt mazda lamp falls upon an opal plate inclined 45°; thence it divides into two beams. One part of the light passes through the vertical tube at the back, and after reflection behind the wedge, passes through it to the horizontal microscope. The intensity of this beam is controlled by moving the wedge, and its amount is read from the scale graduated in millimeters from 0 to 110. The second part of the light passes forward under the platform and is reflected through a hole in the platform and through the photographic plate to the vertical microscope.

At the juncture of the two microscopes a small glass cube is mounted, made of two 45° prisms cemented together. A minute dot of silver is deposited on one of the faces to be joined, so that the finished cube contains a little 45° mirror near its center. On looking into the lower eyepiece, a small portion of the wedge is seen, obstructed at the center by the nearly circular silver dot; and this central portion is illuminated by light that has

[1]J. Hartmann, *Astrophysical Journal*, **10**, 321, 1899.

passed through the photographic plate. The density of that part of the plate under the vertical microscope is determined by the setting of the wedge required to make the two parts of the field equally illuminated. This scale reading we have called wedge reading to avoid possible ambiguity.

The wedge is part of a Harvard plate, numbered E 7741 and was secured through the kindness of Professor E. C. Pickering. The emulsion is Seed 23, while our plates to be measured are Seed 27. When wedge and plate are in focus, the difference in grain introduces much uncertainty in setting the wedge. Accordingly a second wedge, part of a Seed 27 plate and numbered 210036, was furnished by Professor Pickering. But again it was found impossible to make reliable measures in focus. Finally the 23 wedge was adopted and both wedge and plate put out of focus 1.5 and 1.0 mm. respectively. In this way the granulation is not seen, and the uniformly illuminated areas are readily matched.

The instrument used in the Laws Observatory differs from previous ones in the greater ease of setting the images in position for measurement. For this improvement we are indebted to Professor Schlesinger for passing along the suggestion made by Hartmann. Above the eyepiece used for measurement is the finder; this, in connection with the 45° prism over the cube and the vertical objective, permits a view of the plate, obscured only by the silver dot. The dot is thus readily set on the desired image.

Procedure. After the lower eyepiece is focused sharply on the edge of the silver dot, plate and wedge are brought into focus; then the settings of the vertical and horizontal microscope objectives are increased by 1.0 and 1.5 mm. respectively. The silver dot is set on a star image and its illumination matched with the wedge. The border of the silver dot does not disappear with the correct setting of the wedge, but the observer soon comes to disregard it.

In measuring the small images, the dot is placed centrally on the image and the mean of four settings of the wedge is the wedge reading taken. The individual readings are usually in agreement well within one division of the wedge. Measurement of the millimeter star images is more difficult, since their centers are far from uniform. Four settings of the dot are made around the image between its center and edge. For plates of variable stars it is the practice to measure the entire set of images of one star before moving to another. Readings of the film are frequently taken for the control of sky-fog and of irregularities in the plate.

There is considerable personal equation between the two observers, and its character has probably undergone changes. In general, Cummings' wedge readings are higher than Baker's, especially for faint images.

IV. REDUCTION OF THE MEASURES

Reduction to ΔMagnitude. When the densities of the sensitometer images have been measured, the wedge readings are plotted against the adopted Δmagnitudes in the

last column of Table I. A curve drawn to represent the plotted points is the absorption curve of the wedge for the plate. This is called simply the reduction curve. Wedge readings for the extrafocal star images may now be converted to relative magnitudes on an absolute scale, in the sense that it is derived from laboratory experiments, as Professor Parkhurst has pointed out.

In this part of the work we have encountered the most difficulties. The sensitometer has proven to be a veritable Pandora's box. The sensitometer images have given satisfactory reduction curves, only after much experimenting and adjusting; and even now they are not always under control. A lack of co-ordination between the four lines of images indicates trouble and sometimes its nature.

The variations are generally progressive, either radial on the plate or across-plate. The first kind results in bowing of the plotted points for each line of images, the effect being greater for the first and fourth lines. If the bows are concave upward, they may indicate lack of parallelism or progressive change of density in the opal sheets, lack of parallelism between the opal sheets and the perforated plates of the rotating box, or curvature of the photographic plate; if convex upward, they suggest imperfect illumination at the edges of the opal sheets. The second kind of variation, namely across-plate, results in increased or decreased slope in the plotted points for each line of images, if the progression is lengthwise of the plate; and in upward or downward displacement of each line, if the progression is across the width of the plate. These effects may arise from lack of parallelism of the back plate and the photographic plate, from progressive variation in the sensitiveness of the latter, or from non-uniform development or fixing of the plate. Other effects of an irregular character occur frequently and may be ascribed, among other things, to variable sensitiveness of the film, to the superposition of artificial and actual star images, or to irregularities in the sensitometer itself.

At present the majority of the plates yield satisfactory reduction curves. This fact eliminated from the possibilities a number of the errors above mentioned. The sensitometer images admit such accurate measurement, that an anomalous image or set of images is at once detected and rejected.

A mean reduction curve is established for each emulsion. This is based on satisfactory curves from a number of clear-film plates. A comparison for the same plates clearly shows a slight difference between the reduction curves of the two observers, especially for faint images; and a possible variation in the difference is left in doubt. Independent reduction curves have been used.

The character and slope of the reduction curve varies considerably with the emulsion of the Seed 27 plates. Thus the emulsion number becomes an important item in the records. Emulsion 14410 gives the steepest reduction curve, of those we have used, and 14540 the least slope. In a letter, Dr. C. E. K. Mees of the Eastman Company has kindly pointed out that the ultimate gradation is nearly constant for different batches of emulsion, but the velocity constant of development varies appreciably. He advises an adjustment in the time of development corresponding to the velocity constant of each

emulsion. It follows that in a number of cases we have failed to secure the greatest accuracy obtainable from the plate.

The mean reduction curve, once well established, may be used for all clear-film plates of the same emulsion. Variations in the curve are possible, but more probable beyond the limits adopted for use. At frequent intervals sensitometer images are measured and curves are derived for control. Before the mean reduction curve is used, it is corrected by Table IX, and is then called the corrected reduction curve.

TABLE II

CORRECTIONS TO CONSISTENT REDUCTION CURVES

Wedge	Baker			Cummings			Mean Correction
	14794	15194	14410	14794	15194	14447	
	mag.	mag.	mag.	mag.	mag.	mag.	mag.
20	−.10	−.11	−.10	−.09		−.10	−.10
25	−.09	−.08	−.08	−.08		−.09	−.08
30	−.05	−.06	−.04	−.05	−.05	−.03	−.05
35	−.02	−.03	−.02	−.02	−.02	−.02	−.02
40	−.01	−.01	.00	.00	−.01	.00	−.01
45	+.02	+.01	+.01	.00	.00	.00	+.01
50	+.02	+.02	+.03	−.01	+.01	.00	+.01
55	+.01	+.02	+.01	−.03	−.01	−.03	.00
60	−.03	−.02	−.03	−.04	−.02	−.06	−.03
65	−.07	−.05	−.05	−.06	−.05	−.09	−.06
70	−.09	−.08	−.08	−.07	−.08	−.13	−.09
75	−.11	−.11	−.11	−.09	−.14	−.16	−.12
80	−.15	−.14	−.14				−.15

Consistent Reduction Curve. When pairs of star images, resulting from two exposures of different durations on the same region of the sky, are measured and their differences in magnitude derived from the mean reduction curve, these differences are not in agreement; they increase as the average density of the pair increases. We have tried the experiment with emulsions 14410, 14447, 14794 and 15194, with the same result. It is necessary to inquire further into this matter. In each case, taking as the normal magnitude-difference the average for pairs whose densities are between wedge-readings 25 and 70, the per cent of excess or deficit relative to the normal is computed for each pair and is plotted against average Δmagnitude for the pair. The curve drawn to represent the plotted points is integrated for intervals of Δ magnitude, and the results are corrections to be applied to the mean reduction curve in order to obtain the curve that yields consistent magnitude differences for all the pairs of images. The curve, so corrected, is named the consistent reduction curve.

Table II gives the differences, consistent *minus* mean reduction curve, for intervals of wedge reading. They are in excellent agreement, and the means are evidently well

established. Comparing with Table IX, we see that consistent and corrected reduction curves agree exactly between wedge readings 20 and 40, and very nearly up to 60; beyond 60 the two curves draw apart, differing by 0.09 magnitude at 70. Further reference to this discrepancy is made in Sections V and VI.

Effect of Sky-fog. The effect of general illumination of the plate by sky light is very marked on extra-focal plates taken with a short-focus camera, as Parkhurst has shown.[1] The location of the Laws Observatory in the midst of artificial lights makes the control of sky-fog a matter of great importance in our work. As the effect is much greater for faint images than for dense ones, the reduction curve for clear-film plates cannot be used when the plates are fogged. Individual reduction curves must be employed, and we have to assume only that the effect is similar for sensitometer and star

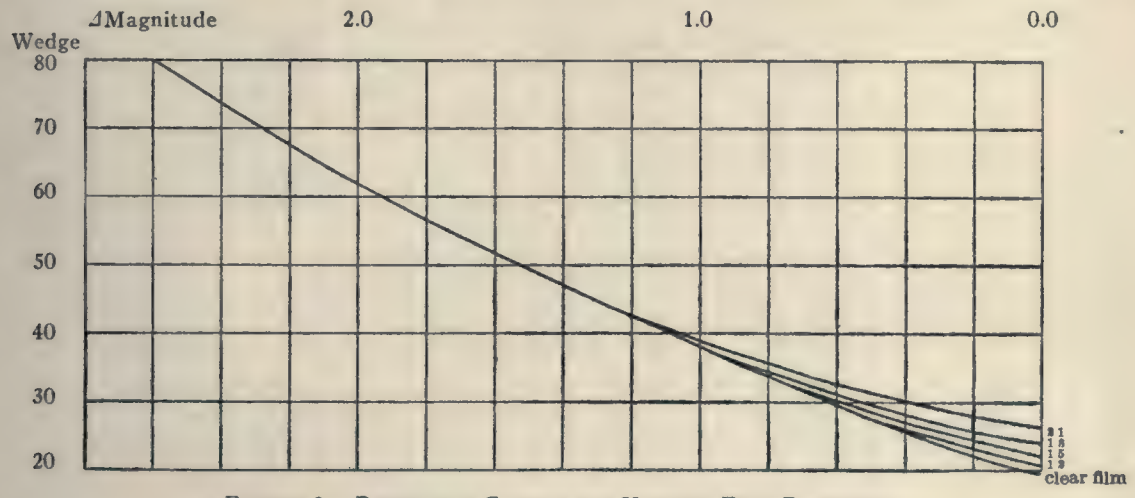

FIGURE 3. REDUCTION CURVES FOR VARIOUS FILM READINGS.
SEED 27 PLATES, EMULSION 14447

images. Herein lies one of the greatest advantages of the method, namely, the control of sky-fog.

Figure 3 shows reduction curves for emulsion 14447 for various degrees of sky-fog which are indicated by the wedge reading of the film. In practice the fogged plate is reduced by means of its individual curve, up to the point of its divergence from the mean clear-film curve. All curves receive correction by Table IX. In case the sensitometer images on the plate are unsatisfactory, the reduction is effected from the curve of another plate of the same emulsion having the same film-reading.

The chief difficulty in these reductions is that the fog-curve is determined by faint images that are especially sensitive to changing conditions. It is not now expedient to establish a standard set of fog curves for each emulsion. The best procedure is to minimize fogging by limiting the duration of total exposure, and to time the separate ex-

[1] J. A. Parkhurst, *Astrophysical Journal*, 31, 17, 1910.

TABLE III

REDUCTION TO CENTER

Dist. from Center	Diameter of Central Image (mm.)											
	0.28	0.29	0.30	0.31	0.32	0.33	0.34	0.35	0.36	0.37	0.38	1.00
mm.	mag.	mag.	mag.	mag.	mag.	mag.	mag.	mag.	mag.	mag.	mag.	mag.
1	+0.01	+0.00	+0.00	+0.00	+0.00	+0.00	+0.00	+0.00	+0.00	+0.00	+0.00	+0.00
2	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00
3	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.00
4	0.03	0.03	0.03	0.03	0.03	0.02	0.02	0.02	0.02	0.01	0.01	0.00
5	0.05	0.04	0.04	0.04	0.04	0.03	0.03	0.03	0.02	0.02	0.02	0.01
6	0.06	0.06	0.06	0.05	0.05	0.04	0.04	0.04	0.03	0.03	0.02	0.01
7	0.08	0.07	0.07	0.06	0.06	0.06	0.05	0.05	0.04	0.04	0.03	0.01
8	0.10	0.09	0.09	0.08	0.08	0.07	0.06	0.06	0.05	0.05	0.04	0.02
9	0.12	0.11	0.10	0.10	0.09	0.08	0.08	0.07	0.07	0.06	0.05	0.03
10	0.13	0.13	0.12	0.11	0.11	0.10	0.09	0.09	0.08	0.07	0.07	0.03
11	0.15	0.14	0.14	0.13	0.12	0.12	0.11	0.10	0.10	0.09	0.08	0.04
12	0.17	0.16	0.15	0.15	0.14	0.13	0.12	0.12	0.11	0.10	0.10	0.05
13	0.19	0.18	0.17	0.17	0.16	0.15	0.14	0.13	0.13	0.12	0.11	0.05
14	0.21	0.20	0.20	0.19	0.18	0.17	0.16	0.15	0.14	0.14	0.13	0.06
15	0.24	0.23	0.22	0.21	0.20	0.19	0.18	0.17	0.16	0.15	0.14	0.07
16	0.27	0.26	0.24	0.24	0.22	0.21	0.20	0.19	0.18	0.17	0.16	0.08
17	0.30	0.29	0.27	0.26	0.25	0.24	0.22	0.21	0.20	0.19	0.18	0.09
18	0.33	0.32	0.30	0.29	0.28	0.26	0.25	0.24	0.22	0.21	0.20	0.10
19	0.36	0.35	0.33	0.32	0.30	0.29	0.28	0.26	0.25	0.23	0.22	0.11
20	0.40	0.38	0.37	0.35	0.33	0.32	0.30	0.29	0.27	0.25	0.24	0.12
21	0.43	0.41	0.40	0.38	0.36	0.34	0.33	0.31	0.29	0.28	0.26	0.13
22	0.47	0.45	0.43	0.41	0.39	0.37	0.35	0.34	0.32	0.30	0.28	0.14
23	0.50	0.48	0.46	0.44	0.42	0.40	0.38	0.36	0.34	0.32	0.30	0.15
24	0.54	0.52	0.49	0.47	0.45	0.43	0.41	0.39	0.37	0.34	0.32	0.16
25	0.57	0.55	0.53	0.51	0.48	0.46	0.44	0.42	0.39	0.37	0.35	0.17
26	0.61	0.59	0.56	0.54	0.52	0.49	0.47	0.44	0.42	0.40	0.37	0.18
27	0.64	0.62	0.60	0.57	0.55	0.52	0.50	0.47	0.45	0.42	0.40	0.19
28	0.68	0.65	0.63	0.60	0.58	0.55	0.53	0.50	0.48	0.45	0.43	0.20
29	0.72	0.69	0.66	0.64	0.61	0.58	0.56	0.53	0.50	0.48	0.45	0.21
30	0.75	0.72	0.70	0.67	0.64	0.62	0.59	0.56	0.53	0.51	0.48	0.22
31	0.79	0.76	0.73	0.70	0.68	0.65	0.62	0.59	0.56	0.54	0.51	0.24
32	0.82	0.79	0.76	0.74	0.71	0.68	0.65	0.62	0.60	0.57	0.54	0.25
33	0.86	0.83	0.80	0.77	0.74	0.71	0.68	0.65	0.63	0.60	0.57	0.26
34	0.89	0.86	0.83	0.80	0.77	0.74	0.71	0.68	0.66	0.62	0.60	0.28
35	0.93	0.90	0.87	0.84	0.81	0.78	0.75	0.72	0.68	0.66	0.62	0.29
36	0.97	0.94	0.90	0.87	0.84	0.81	0.78	0.75	0.72	0.68	0.65	0.30
37	1.01	0.97	0.94	0.91	0.88	0.84	0.81	0.78	0.75	0.71	0.68	0.31
38	1.04	1.01	0.98	0.94	0.91	0.88	0.85	0.81	0.78	0.74	0.71	0.33
39	1.08	1.05	1.02	0.98	0.95	0.91	0.88	0.84	0.81	0.78	0.74	0.34
40	1.12	1.09	1.05	1.02	0.98	0.95	0.91	0.88	0.84	0.81	0.77	0.36
41	1.16	1.13	1.09	1.06	1.02	0.98	0.95	0.91	0.87	0.84	0.80	0.37
42	1.21	1.17	1.13	1.09	1.06	1.02	0.98	0.94	0.91	0.87	0.83	0.39
43	1.25	1.21	1.17	1.13	1.09	1.06	1.02	0.98	0.94	0.90	0.86	0.40
44	1.29	1.25	1.21	1.17	1.13	1.09	1.05	1.01	0.97	0.93	0.89	0.42
45	1.33	1.29	1.25	1.21	1.17	1.12	1.08	1.04	1.00	0.96	0.92	0.44

posures so as to bring most of the images to be measured above the point of divergence from the clear-film curve.

Correction for Atmospheric Extinction. The observed Δmagnitudes of the stars measured on the plate must receive correction for differential atmospheric extinction of their light. It is the practice to correct the Δmagnitudes to the zenith distance of the guiding star. Inasmuch as measures are not undertaken more than 2° from the center of the plate, and generally the distance is less than 1°, and since in addition the zenith distance is usually less than 50°, the corrections are never more than a few hundredths of a magnitude. Thus the choice of extinction curve is not important. Following Parkhurst's example the curve determined by Wirtz[1] is used. The corrections for atmospheric extinction are given in Table IV.

TABLE IV

CORRECTION FOR ATMOSPHERIC EXTINCTION

z	Corr.	z	Corr.	z	Corr.	z	Corr.	z	Corr.	z	Corr.	z	Corr.
	mag.		mag.		mag.		mag.		mag.		mag.		mag.
0°	.00	10°	.01	20°	.03	30°	.06	40°	.12	50°	.22	60°	.40
1	.00	11	.01	21	.03	31	.06	41	.13	51	.24	61	.43
2	.00	12	.01	22	.03	32	.07	42	.14	52	.25	62	.45
3	.00	13	.02	23	.04	33	.07	43	.15	53	.27	63	.48
4	.00	14	.02	24	.04	34	.08	44	.16	54	.28	64	.51
5	.00	15	.02	25	.04	35	.09	45	.17	55	.30	65	.55
6	.00	16	.02	26	.04	36	.09	46	.18	56	.32	66	.58
7	.01	17	.02	27	.05	37	.10	47	.19	57	.34	67	.62
8	.01	18	.03	28	.05	38	.10	48	.20	58	.36	68	.66
9	.01	19	.03	29	.06	39	.11	49	.21	59	.38	69	.71
10	.01	20	.03	30	.06	40	.12	50	.22	60	.40	70	.76

Reduction to Center. Curvature of the field causes the diameters of the star images to increase as their distance from the optical center of the plate increases. The Δmagnitudes, which are derived from measured densities of star images may not be taken as the relative magnitudes of the stars, until they receive corrections. The addition of the correction to an observed Δmagnitude gives the value that would be obtained directly if the star image were at the optical center of the plate. The correction takes up the loss in intensity due to the enlarged image, and is called the reduction to center.

The investigation of reduction to center involves 21 plates that were secured for this purpose. They are divided as follows: seven plates of 32H.Cephei, eight of 43H.Cephei, two of Polaris and four of stars near the zenith. Three additional plates were finally rejected because of evident change in the transparency of the sky during their exposure. The images at the centers of the plates vary in diameter between 0.3 and 0.4 mm., except those of Polaris, whose diameters are about 1.0 mm. For the circumpolar stars a series

[1]C. W. Wirtz, *Astronomische Nachrichten*, **154**, 349, 1900.

of exposures were made, moving the telescope 20′ in declination between exposures, up to 4° on either side of the center. For the zenith stars the intervals are 30′ in right ascension. The duration for all the exposures was 1 minute, and great care was exercised in timing, so that in no case is the error greater than 1 second. Thus on each plate we have a series of images of varying distances from center, whose densities would be nearly equal except for this reason.

The measured wedge readings are converted to Δ magnitudes by use of individual reduction curves. After subtraction from the Δ magnitude of the central image, the differences are plotted against distance from center for each plate, giving the reduction to center curve for the plate. These curves are then assembled according to diameter of central image. By interpolation, curves are drawn for intervals of 0.01 mm. change in the diameter, and from these curves Table III is formed. The quantities in the table are corrections to the Δ magnitudes, for images within 45 mm. of the center; and for plates whose central images have diameters between 0.28 and 0.38 mm. The last column serves for plates exposed 5 mm. from focus, in which case the central image is 1 mm. in diameter.

Reduction to center is, then, a function of the diameter of the central image, and the function is nearly linear for the small images within the range of the table. But the diameter of the star image is dependent on its density, as shown in Figure 7 and Table X, and receives correction from this table before its use as an argument in Table III. Unless the star is redder than class K0 (see Figure 5) no correction for color is here required.

The variation in the accuracy of the reduction to center with change in distance from center is indicated by taking residuals: observed correction *minus* tabular. When the residuals are grouped according to distance, they yield for each group the following probable error and weight of a single correction:

Distance from Center				Probable Error	Weight
0.0	to 11.5	mm. = 0°	to 1°	±0.026 mag.	6.12
11.5	23.0	1	2	±0.027	5.43
23.0	34.5	2	3	±0.043	2.16
34.5	45.0	3	4	±0.064	1.00

The weights are in close agreement with a similar system of weighting[1] used by Parkhurst and based on the progressive loss in uniformity of the images at increasing distances from center.

Conditions Affecting Accuracy of the Reductions. The greatest known source of error in extrafocal photometry appears to be the effect of sky-fog. The measures we have taken to keep this factor under control are mentioned above. It remains to mention additional sources of error in the reductions.

1. Time of Exposure. A fundamental assumption in the method is that the relation between relative magnitude and density of the images is not altered by changing the exposure time. Otherwise the reduction curve derived from a half-minute exposure in the

[1]J. A. Parkhurst, *Astrophysical Journal*, **36**, 181, 1912.

sensitometer will not serve for star images obtained by several minutes' exposure. The earlier experiments at the Yerkes Observatory are reported as follows[1]: "Changes in the exposure time between ten seconds and thirty minutes had no effect on the curve. Had this test given a different result the method could not be used on the stars, since the exposure times must vary according to the faintness of the stars required."

In his later investigations Professor Parkhurst[2] varied the exposure time from 5 seconds to 34 minutes, and he finds that the gradation is steeper for exposures of 5 minutes than for exposures in the neighborhood of 10 seconds. He detects no effect between 5 and 25 minutes. Schwarzschild[3] has called attention to the similarity in direction of the effect found by Parkhurst, and that shown by Kron and Eberhard at Potsdam.

The Laws Observatory investigations indicate that the effect of exposure time is confined mostly to short exposures. For this reason enough opal glass is interposed before the sensitometer to make the subsequent exposures about 30 seconds. Thus it is our opinion that this effect is of no serious consequence in the results.

2. Temperature, or Humidity, during Exposures. The temperature at which the plates are exposed varies between $-5°$ and $+30°$C. during the observing season. When each plate is reduced independently, the important point is the effect of temperature change between exposures in the camera and sensitometer. A corresponding difference in gradation for star images and sensitometer images may occur. Parkhurst has shown[4] that the effect of temperature change, ranging from $-2°$ to $+17°$C., is not evident within suitable limits of density. But for faint images he infers an appreciable effect; and we suspect this to be the case also for dense images. Confusion may arise between the effects of temperature and of humidity, as Dr. Mees has pointed out.[5] Inasmuch as the practice is to neglect very dense and faint images, the effect of temperature is not a serious source of error.

3. Order of Exposure. In this connection the phenomenon, noted by Professor E. C. Pickering, requires mention. The statement is[6] that when several exposures are impressed on the same plate, the first and last being short and equal, the images of the first are often systematically brighter than those of the last, the differences amounting on the average to a quarter of a magnitude. The Harvard observations show[7] that the effect varies with the brightness of the star. This phase of the effect concerns our work. We have noted from time to time the apparent greater uncertainty of the first exposure on the plate. Ample material is now here at hand for the investigation, the results of which will appear at another time.

[1] J. A. Parkhurst and F. C. Jordan, *Astrophysical Journal*, **26**, 247, 1907.
[2] J. A. Parkhurst, *Publication of the Astronomical Society*, **2**, 151, 1913.
[3] K. Schwarzschild, *Vierteljahrsschrift der Astron. Gesellschaft*, **47**, 357, 1913.
[4] J. A. Parkhurst, *Astrophysical Journal*, **31**, 15 *et seq.*, 1910.
[5] C. E. K. Mees, *Astrophysical Journal*, **40**, 236, 1914.
[6] *Astrophysical Journal*, **36**, 374, 1912.
[7] H. S. Leavitt, *Harvard Annals*, **71**, 121, 1915

4. Outstanding Errors in the Reduction Curves. The mean reduction curves require correction by amounts not certainly established. At present there is hesitation in the choice between two curves that we have termed the corrected curve and the consistent curve. Fortunately the two curves agree closely except near the upper limit of density.

5. Curvature of the Plate. Unless coated plate glass is used, curvature of the plate is to be expected. In a letter, Dr. Mees expresses the opinion that this is the greatest source of general errors in the use of photographic plates for photometric work, the errors in density which can arise from this cause being as much as 15 per cent. However, for extrafocal work, Professor Parkhurst concludes (*loc. cit.*) that the effect of curvature is inappreciable; and our results from eight control plates, Section V, show little effect of this sort. In practice the chance of error due to curvature is minimized by choosing for measurement images as close to center as possible.

TABLE V

PLATES OF CIRCUMPOLAR REGIONS

Plate No.	Central Star			Date, G.M.T.	Exposure		Diam. Central Image	Telescope	Emulsion Seed 27
	B. D.	R.A. 1900	Decl. 1900		*a*	*b*			
		h. m.		d. h. m.	m.	m.	mm.		
384	+79° 675	20 30.6	+79° 53′	1914 Nov. 11 13 58	7	4	0.376	E	14794
385	77 871	22 42.2	78 00	14 16	7	4	.330	E	14794
386	80 50	1 29.6	80 55	14 36	7	4	.370	W	14794
387	78 142	3 54.3	78 40	14 54	7	4	.327	W	14794
496	81 402	12 41.9	81 10	1915 July 5 15 18	10	4	.324	E	15194
497	80 480	15 35.1	80 47	15 39	10	4	.306	E	15194
500	83 536	18 37.4	83 06	16 43	10	4	.337	W	15194
501	76 717	19 12.8	76 24	17 02	10	4	.330	W	15194

6. Other Conditions. Errors in measurement of the images are inconsiderable. The manipulation of the plate in the darkroom is uniform as regards temperature of development, density of fixing bath and time of fixing. Any effect increasing with age of the plate would be detected and proper allowance made. Irregularities of the film enter as accidental errors, except insofar as they can be detected by changes in the film-readings. Change in illumination of the image with distance from center is taken up in the reduction to center. However a variation in the character of the image, evidently with change in the temperature of the camera objective, may be the source of considerable error.

V. CONTROL INVESTIGATIONS

In order to test the behavior of the camera and the reliability of the extrafocal images under the different conditions encountered in practice, eight plates were secured, containing double exposures of selected circumpolar regions. Their description is found in Table V. Conditions of exposure, manipulation and measurement of the plates are

varied within the limits experienced in our work. The plates were sensitometrized and developed in the usual manner at 20°C; those of 1914 for 10 minutes, those of 1915 for 8 minutes. Measures were confined to stars whose photographic magnitudes are given in *Yerkes Actinometry* (*loc. cit.*).

The reduction of the control plates will be readily understood from Table VI, that shows in detail the procedure for five stars of Plate 501.

TABLE VI

SAMPLE REDUCTION (PART OF PLATE 501)

Star B. D.	Parkhurst		Wedge				Δ Magnitude				$z-z_0$	Dist. from Center	Δ Reduction (magnitude)			
	Phot. Mag.	Spectrum	Baker		Cummings		Baker		Cummings				Baker		Cummings	
(1)	(2)	(3)	(4)		(5)		(6)		(7)		(8)	(9)	(10)		(11)	
			a	*b*	*a*	*b*	*a*	*b*	*a*	*b*		mm.	*a*	*b*	*a*	*b*
77°699	6.92	K2	55.3	39.1	56.9	40.7	1.96	1.22	2.03	1.28	+1°9	33.2	.00	.00	.00	.00
77 702	7.25	A7	51.9	36.4	53.4	37.8	1.81	1.08	1.88	1.14	+1.8	29.8	0	−.01	0	.00
77 734	7.44	A6 ±	51.3	35.0	51.6	36.3	1.78	1.01	1.80	1.06	+1.0	23.5	0	−.02	0	−.01
79 604	6.71	A4	50.9	35.5	52.1	37.4	1.77	1.04	1.82	1.12	+3.5	45.0	0	−.02	0	−.01
79 628	6.05	A2	72.9	51.1	72.8	52.7	2.73	1.77	2.73	1.85	+2.5	37.5	0	.00	0	.00

Star B. D.	Extinction Correction	Reduction to Center	Color Correction	Corrected Δ Magnitude				Δ Mag. + Phot. Mag.				Observed − Parkhurst (magnitude)			
				Baker		Cummings		Baker		Cummings		Baker		Cummings	
(12)	(13)	(14)	(15)	(16)		(17)		(18)		(19)		(20)		(21)	
	mag.	mag.	mag.	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*
77°699	+.02	+ .72	−.08	2.62	1.88	2.69	1.94	9.54	8.80	9.61	8.86	+.08	+.04	+.06	+.04
77 702	+.02	+ .61	−.02	2.42	1.68	2.49	1.75	9.67	8.93	9.74	9.00	−.05	−.09	−.07	−.10
77 734	+.01	+ .42	−.01	2.20	1.41	2.22	1.47	9.64	8.85	9.66	8.91	−.02	−.01	+.01	−.01
79 604	+.03	+ 1.12	−.01	2.91	2.16	2.96	2.25	9.62	8.87	9.67	8.96	.00	−.03	.00	−.06
79 628	+.02	+ .86	.00	3.61	2.65	3.61	2.73	9.66	8.70	9.66	8.78	−.04	+.14	+.01	+.12
				Mean =				9.62	8.84	9.67	8.90				

Column (1) gives the number of the star in the *Bonn Durchmusterung*.

Column (2) gives the photographic magnitude, from *Yerkes Actinometry*.

Column (3) gives the class of spectrum, as determined by Parkhurst on the "earlier" Harvard system.[1]

Columns (4) and (5) give the densities of the extrafocal star images in terms of the microphotometer wedge-scale. Each number is the mean of four settings

[1] J. A. Parkhurst, *Astrophysical Journal*, 36, 188, 1912.

of the wedge. Duplicate measures were made throughout. The letters *a* and *b* denote respectively the denser and fainter image of the pair for each star.

Columns (6) and (7) give the Δ magnitude or relative magnitude of the star for each measure, the number increasing with increasing brightness. The eight plates are all clear-film. The Δ magnitude corresponding to the given wedge reading is read from the mean reduction curve for the emulsion.

Column (8) gives the difference in zenith distance: star *minus* central star.

Column (9) gives the linear distance of the image from the adopted optical center of the plate. This point does not coincide exactly with the mechanical center.

Columns (10) and (11) give corrections to the Δ magnitudes occasioned by a change in the reduction curve, to be explained later.

Column (13) gives the corrections for differential atmospheric extinction, applied to the Δ magnitudes to obtain their values at the zenith distance of the central star. The corrections are taken from Table IV, given the distances z-z_o in column (8) and z_o, the zenith distance of the central star. For plate 501 $z_o = 38° 44'$.

Column (14) gives the reduction to center, applied to the Δ magnitudes to obtain their values at the optical center of the plate. The corrections are found by double interpolation in Table III, taking as arguments the distance from center, column (9), and the diameter of the central image corrected for density, Table X, as given in Table V.

Column (15) gives the color corrections, applied to the Δ magnitudes to obtain their values on Parkhurst's system. The corrections are scaled from the curve, Figure 5, given the spectrum of the star in column (3) of this table.

Columns (16) and (17) give the final Δ magnitudes after correction.

Columns (18) and (19) give for each observation the sum of the corrected Δ magnitude and Parkhurst's photographic magnitude. Mean values for the entire plate appear below.

Columns (20) and (21) give the residuals for each star in the sense: observed magnitude *minus* Parkhurst's magnitude. These residuals are obtained from columns (18) and (19) by subtracting individual values from the mean of the column.

The final results derived from our observations of circumpolar stars appear in Table VII. On the eight plates, images of 196 stars, nearly one-third of the number contained in *Yerkes Actinometry*, are suitable for measurement; three stars are duplicated. The entire list of 199 is entered in the table. Star images have been rejected whose distances from center exceed 45 mm. and generally those for which the wedge reading is less than 20 or greater than 80. The residuals in the last two columns result from a final approximation, preliminary residuals having been used to determine the various corrections applied.

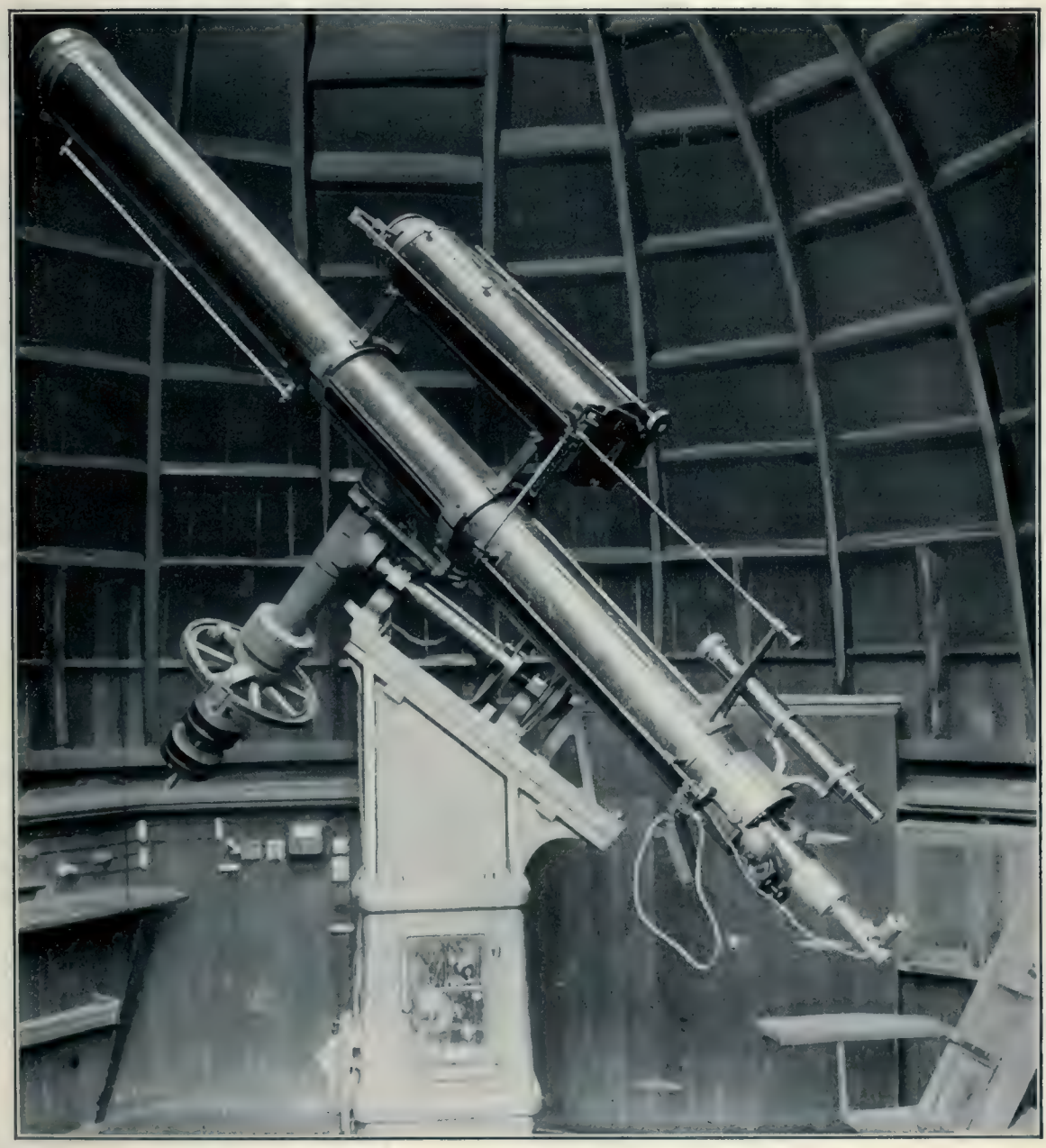

PLATE I. 5-INCH CAMERA AND 7½-INCH REFRACTOR

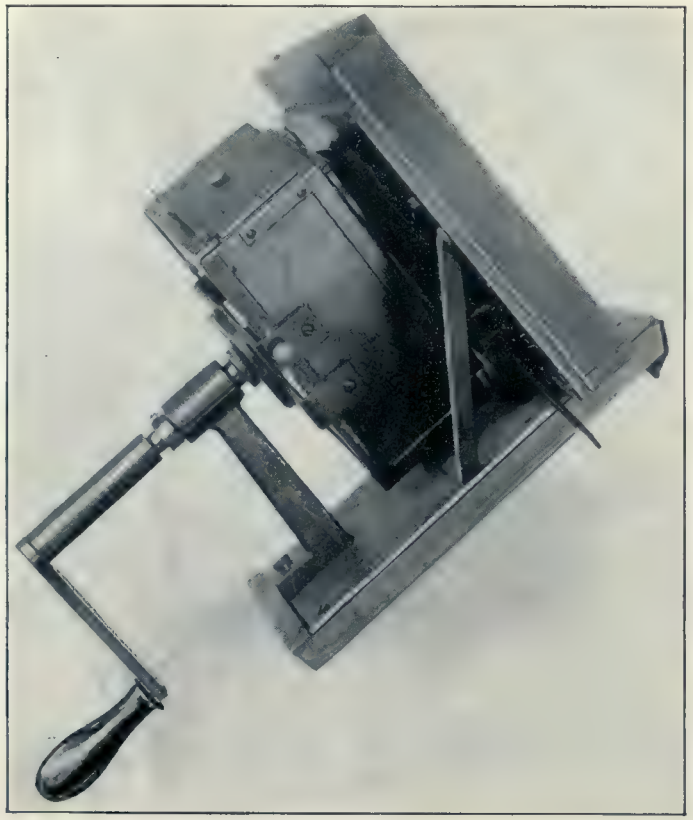

PLATE II. SENSITOMETER

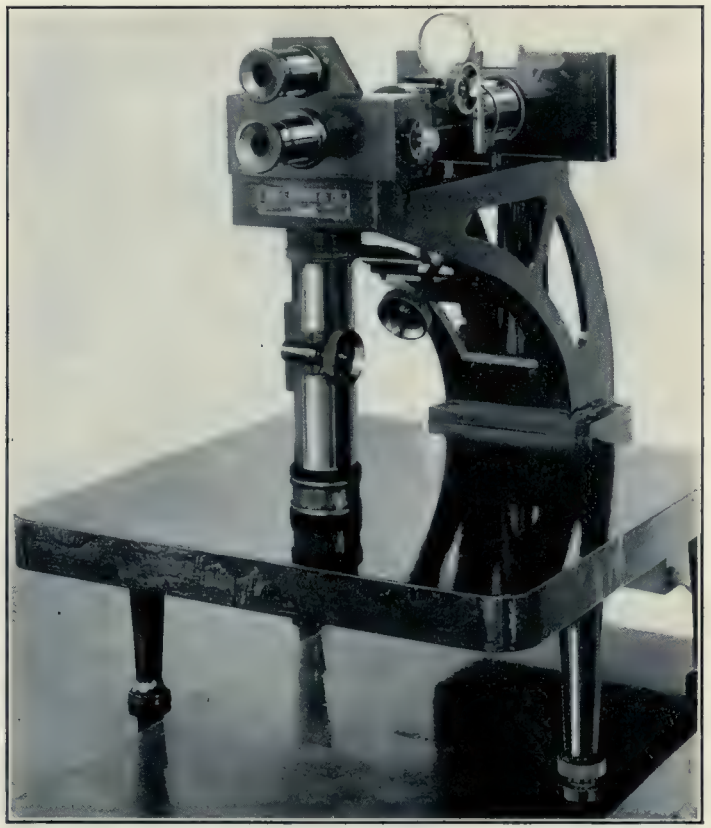

PLATE III. MICROPHOTOMETER

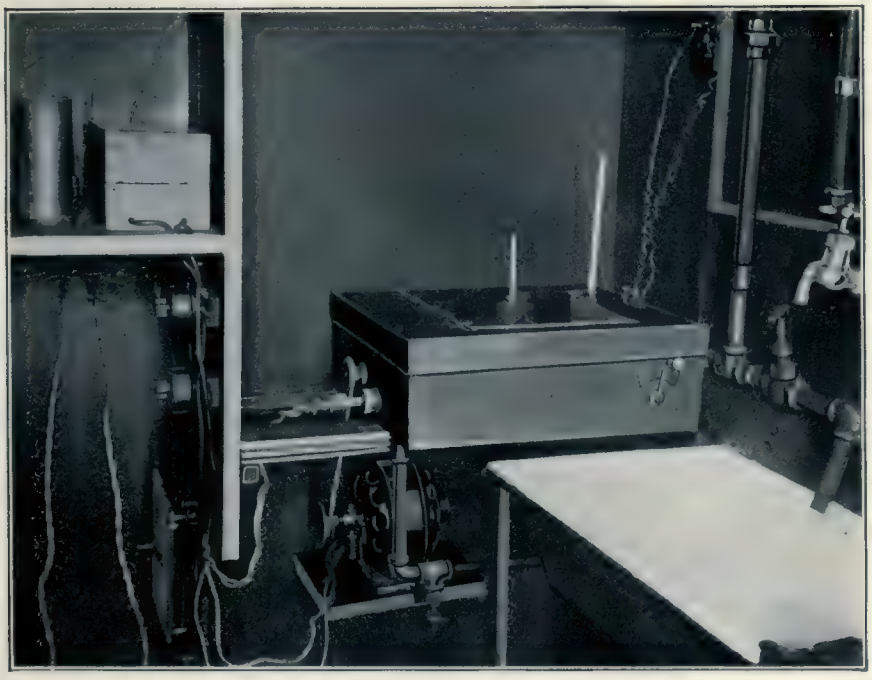

PLATE IV. DEVELOPING TANK

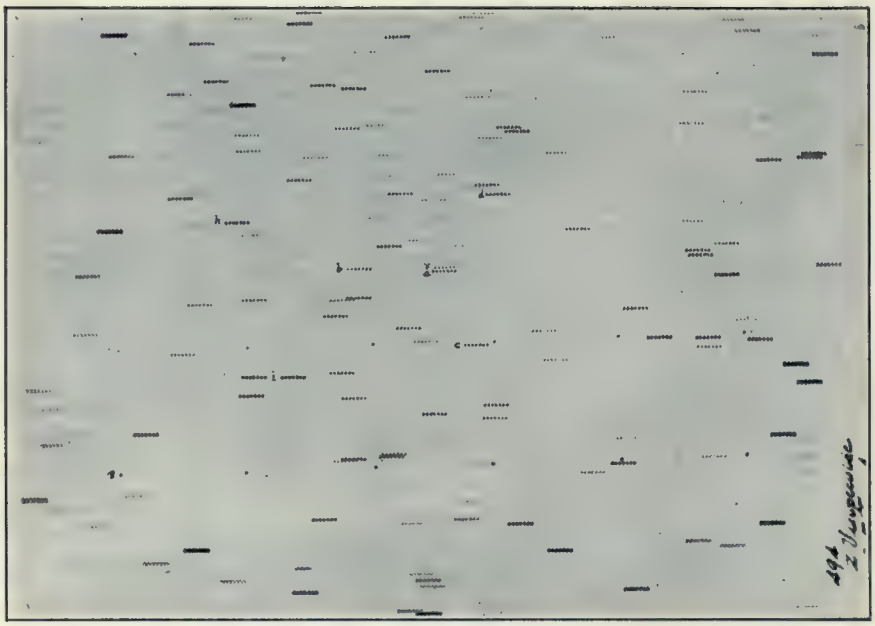

PLATE V. PLATE OF Z VULPECULAE

The illustration fails to do justice to the original plate. The irregularities in the star images are due to limitations of the method of reproduction.

TABLE VII

OBSERVATIONS OF CIRCUMPOLAR REGIONS

Star B. D.	Plate No.	Position, 1900		Spectrum	Dist. from Center	Mean Diam.	Approx Wedge	Photographic Magnitude (Parkhurst)	Observed − Parkhurst (magnitude)	
		R. A.	Decl.						Baker	Cummings
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		h. m.			mm.	mm.	*a* *b*		*a* *b*	*a* *b*
73° 835	501	18 48.3	73° 58′	G ±	34.3	.424	73 52	6.21 ± .02	+.01 +.17	−.03 +.13
845	501	19 01.0	74 00	A5	28.5	.389	51 37	7.31 .02	−.01 −.02	+.01 −.13
857	501	19 17.5	73 10	K0	34.3	.430	83 61	5.84 .02	.00 +.14	+.20
860	501	19 20.8	73 22	A5	32.0	.418	51 36	7.09 .05	+.18 +.04	+.13 +.10
863	501	19 28.7	73 09	A5 ±	35.0	.434	34 21	7.87 .06	+.11 +.10	+.02 +.01
74 792	501	18 49.8	74 36	G ±	28.1	.382	40 27	7.94 .02	−.08 −.14	−.07 −.14
831	501	19 37.3	74 09	G5	26.2	.384	36 24	8.11 .06	+.09 −.06	+.09 −.03
978	385	22 35.1	74 51	K5+	38.8	.476	29 22	7.55 .04	−.08 −.19	−.13 −.07
1002	385	23 03.2	75 02	A0	37.4	.488	42 32	6.64 .03	+.07 +.12	+.08 +.13
1006	385	23 04.7	74 51	F8	40.0	.472	76 60	5.11 .02	−.07 +.11	−.03 +.16
74 1016	385	23 13.8	74 45	A0	43.1	.520	45 35	6.32 .03	+.03 +.06	−.03 +.02
75 160	387	3 53.1	76 08	A8	27.0	.404	26 23	7.94 .03	+.05 +.03	+.01 −.03
173	387	4 08.2	75 52	A0	32.5	.434	49 45	6.56 .02	−.07 −.08	−.15 −.12
189	387	4 35.4	75 46	A4	43.0	.507	49 44	6.22 .02	−.11 −.03	−.10 −.09
678	501	18 44.6	75 12	A6 ±	27.6	.378	44 29	7.73 .04	−.07 −.08	−.03 −.05
75 682	501	18 49.6	75 19	B8	23.8	.363	81	5.28 .01	+.08	
683	501	18 56.9	75 39	A0	17.1	.342	82 63	6.24 .02	+.04 +.08	+.12
718	501	19 59.4	75 26	K5 ±	29.0	.390	22	8.72 .15	+.11	+.22
820	385	22 17.1	75 59	B7	31.0	.428	49 39	6.50 .02	+.03 +.02	+.03 +.10
822	385	22 17.7	75 31	A1	35.4	.451	30 22	7.44 .07	+.03 +.02	+.01 .00
75 836	385	22 30.5	75 43	B8	29.9	.424	70 56	5.58 .03	+.02 +.15	+.02 +.15
76 128	387	3 29.7	76 51	A1	22.8	.374	27 24	7.87 .03	+.16 +.12	+.14 +.08
174	387	4 32.1	76 25	F5	35.4	.447	39 36	6.93 .02	.00 −.02	−.01 −.06
712	501	19 02.2	76 54	A9	14.7	.330	71 52	7.03 .04	−.21 −.19	−.19 −.10
732	501	19 22.6	76 36	G5	5.3		39 24	8.55 .02	−.09 −.09	−.05 −.03
76 771	501	20 02.4	76 12	K5	29.0	.386	36 23	7.95 .04	+.11 +.07	+.18 +.16
809	384	20 40.2	76 28	F ±	41.0		29 21	7.38 .01	−.11 −.04	−.04 −.17
859	385	22 24.0	76 55	A5	19.8	.372	38 30	7.49 .02	−.03 −.03	−.05 −.07
870	385	22 39.8	77 06	A4	12.8	.349	40 31	7.46 .01	+.04 +.09	+.04 −.03
892	385	22 58.4	76 20	A +	22.2	.376	30 22	7.94 .03	−.08 −.08	−.08 −.11
76 934	385	23 47.1	77 03	F1	39.1	.489	38 29	6.89 .02	+.02 +.05	−.05 .00
77 49	386	1 15.0	78 12	B5?	29.6	.442	51 42	6.31 .01	−.03 −.12	+.01 .00
58	386	1 31.6	77 28	A0	38.0		41 32	6.64 .01	−.06 −.07	−.06 −.07
65	386	1 44.7	77 42	K0	36.5		24 17	7.75 .04	−.06	−.12 −.07
115	387	3 07.6	77 22	A3	28.8	.418	76 69	5.27 .07	+.09 +.15	+.13 +.22
77 119	387	3 14.6	78 07	F5	21.0	.370	28 24	8.16 .03	−.07 −.07	−.23 −.11
123	387	3 18.5	77 40	K0	21.0	.366	20 18	8.58 .01	+.03	−.07 −.10
133	387	3 34.9	77 48	A0	12.1	.336	51 46	6.88 .02	+.04 +.03	+.09 +.12
138	387	3 49.0	77 55	G5	6.8	.341	33 30	7.95 .03	+.04 +.04	+.09 +.08
592	497	15 34.4	77 41	K5	38.3	.436	65 45	6.63 .04	−.18 −.04	−.11 +.07
77 696	501	18 29.0	77 30	G5	36.9	.432	31 16	8.03 .01	+.04 +.03	+.04
699	501	18 34.6	77 28	K2	33.2	.418	56 40	6.92 .04	+.08 +.04	+.06 +.04
702	501	18 42.3	77 35	A7	29.8	.392	53 37	7.25 .06	−.05 −.09	−.07 −.10
734	501	19 34.1	78 03	A6 ±	23.5	.358	51 36	7.44 .04	−.02 −.01	+.01 −.01
800	384	21 07.5	77 43	A0	31.9	.465	64 51	5.83 .04	−.10 +.03	−.17 .00

TABLE VII—Continued

OBSERVATIONS OF CIRCUMPOLAR REGIONS

Star B. D.	Plate No.	Position, 1900		Spectrum	Dist. from Center	Mean Diam.	Approx. Wedge	Photographic Magnitude (Parkhurst)	Observed − Parkhurst (magnitude)	
		R. A.	Decl.						Baker	Cummings
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		h. m.			mm.	mm.	*a* *b*		*a* *b*	*a* *b*
77° 811	384	21 17.6	78°11′	A0	31.1	.458	35 27	7.13 ±.04	+.14 +.10	+.03 +.06
834	385	21 46.6	77 46	A0	36.0	.438	37 29	7.02 .01	−.04 −.06	−.03 .00
860	385	22 22.9	77 44	B8	14.8	.356	56 44	6.65 .01	−.06 +.06	+.01 +.08
871	385	22 42.2	78 00	B8	3.0	.334	47 38	7.17 .03	+.07 +.06	+.11 +.09
879	385	22 54.6	77 58	G ±	5.2	.339	28 20	8.25 .10	+.17 +.08	+.13 +.14
77 896	385	23 13.2	77 36	A4	17.2	.366	38 .	7.61 .02	−.06	−.08
908	385	23 26.8	77 20	A0	26.1	.404	42 34	7.04 .01	+.03 −.04	−.01 −.04
909	385	23 27.8	77 16	A0	27.1	.406	43 34	7.06 .02	−.08 −.09	−.12 −.16
78 34	386	1 03.6	79 08	B9	21.0	.416	76 63	5.32 .03	+.05 +.12	+.05 +.12
69	386	1 57.9	78 52	A6	27.4	.450	35 27	7.32 .03	−.10 −.09	−.13 −.13
78 73	386	2 04.7	78 43	A1	31.1	.460	36 28	7.06 .03	.00 −.01	−.08 −.11
103	387	2 52.8	79 01	M	32.4	.434	39 36	7.25 .04	−.14 −.19	−.16 −.13
109	387	3 04.0	78 30	A3	26.3	.400	46 42	6.89 .02	−.08 −.12	−.04 +.03
142	387	3 54.3	78 40	A0	3.2	.334	39 35	7.63 .03	+.01 +.05	+.06 +.05
146	387	3 57.1	78 46	G7	5.1	.330	33 29	8.09 .02	−.04 −.02	−.01 +.01
78 157	387	4 22.2	78 46	A6	18.3	.360	36 33	7.48 .01	+.16 +.05	+.05 +.10
406	496	12 05.1	77 57	G1	41.1	.470	33	7.77 .05	+.02	−.03
411	496	12 07.1	78 00	A0	40.2	.462	55 39	6.54 .04	+.17 +.11	+.11 +.12
412	496	12 07.5	78 10	A4	38.2	.450	70	5.40 .03	.00	−.05
501	497	14 55.4	78 35	G5	31.2	.390	49 34	7.52 .03	−.09 −.09	−.10 −.05
78 510	497	15 21.5	78 45	K ±	26.0	.365	28	8.62 .01	+.05	+.05
660	501	18 55.7	78 42	F ±	33.0	.418	26	8.53 .06	−.08	−.12
716	384	20 40.0	79 05	B2	11.4	.392	53 44	6.65 .02	−.03 −.06	−.02 .00
742	384	21 11.4	78 15	A0	28.0	.432	32 24	7.44 .02	+.08 +.08	+.01 −.01
744	384	21 13.8	78 34	B5	26.2	.430	42 33	7.04 .01	−.07 −.11	−.07 −.10
78 796	385	22 26.0	78 17	A2	12.1	.350	74 59	6.02 .01	−.12 −.03	−.11 +.02
801	385	22 29.0	78 19	A2	10.3	.342	72	5.58 .03	−.13	.00
825	385	23 12.8	78 41	B8 ±	16.2	.365	34 24	7.73 .09	+.06 +.15	+.03 +.05
835	385	23 24.0	79 15	A2	24.1	.402	31 22	7.75 .03	+.03 +.06	−.04 −.03
79 10	386	0 20.7	79 30	B8	34.2	.466	50 41	6.20 .02	−.02 −.05	+.03 +.06
79 24	386	0 52.1	80 00	F0	17.5	.397	45 36	6.98 .02	−.08 −.12	−.04 −.07
29	386	1 00.7	79 29	G2	18.6	.400	42 33	7.16 .03	−.08 −.04	−.06 −.07
36	386	1 07.7	79 23	F1	17.6	.401	52 43	6.59 .01	−.04 −.06	−.06 −.09
61	386	1 55.7	80 11	A2	16.0	.394	33 24	7.64 .03	−.03 −.03	−.11 −.03
86	387	2 41.8	79 42	A2	39.2	.376	32 29	7.27 .03	−.12 −.12	−.07 −.09
79 94	387	3 01.4	79 45	G4	29.9	.413	30 29	7.68 .05	.00	−.02 −.11
150	387	4 28.8	79 28	A1	23.9		52 46	6.56 .01	+.03 +.10	+.01 +.13
418	496	13 13.6	79 14	G0	32.1	.415	27	8.46 .04	.00	−.02
422	496	13 26.1	79 10	G1	37.9	.442	64 46	6.42 .02	+.05 +.06	+.03 +.09
431	496	13 50.4	79 29	G8	43.9	.489	33 22	7.72 .03	+.03 −.17	−.02 −.10
79 447	497	14 28.3	78 56	F3	36.6	.410	43 29	7.50 .03	−.07 −.05	−.04 +.02
570,1	500	18 07.6	79 59	F ±	37.9	.447	67	5.39 .02	+.22	+.10
587	500	18 30.6	79 09	K0	43.7	.506	35 21	7.79 .11	−.08 −.11	−.06 −.10
590a	500	18 33.1	79 33	A7	38.8	.471	41 27	7.58 .09	−.10 −.10	−.04 −.16
604	501	18 52.7	79 49	A4	45.0	.496	52 36	6.71 .05	.00 −.03	.00 −.06

TABLE VII—Continued

OBSERVATIONS OF CIRCUMPOLAR REGIONS

Star B. D.	Plate No.	Position, 1900		Spectrum	Dist. from Center	Mean Diam.	Approx. Wedge	Photographic Magnitude (Parkhurst)	Observed − Parkhurst (magnitude)	
		R. A.	Decl.						Baker	Cummings
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		h. m.			mm.	mm.	a b		a b	a b
79° 604	500	18 52.7	79° 49′	A4	34.9	.449	62 43	6.71 ± .05	−.06 −.02	.00 +.03
628	501	19 27.8	79 24	A2	37.5	.443	73 52	6.05 .01	−.04 +.14	+.01 +.12
645	501	19 49.2	79 17	A0	40.0	.454	31 19	7.77 .05	+.16 +.05	+.13 +.07
645	384	19 49.2	79 17	A0	25.6	.430	26 18	7.77 .05	+.14 +.06	+.10 +.08
660	384	20 07.6	79 24	G0	16.4	.400	33 25	7.63 .04	+.07 +.05	+.11 +.08
79 668	384	20 18.2	79 20	K ±	12.2	.392	24	8.24 .03	+.09	+.11
675	384	20 30.6	79 53	A0	3.5	.382	39 30	7.40 .03	+.09 +.08	+.06 +.08
707	384	21 27.8	80 05	G1	25.4	.430	45 35	6.79 .03	+.11 +.13	+.09 +.04
721	385	21 55.9	79 50	M	33.4	.428	24 17	7.88 .02	+.05	+.08 +.03
739	385	22 26.1	80 11	G6	25.6	.393	30 22	7.75 .01	−.03 −.01	+.05 +.03
79 750	385	22 43.2	79 55	G0	20.0	.380	30 22	7.88 .03	+.07 +.10	+.01 −.05
756	385	22 48.4	79 50	G1	19.1	.373	30 21	7.93 .05	+.07 +.13	+.07 +.09
759	385	22 57.4	79 48	G5	19.8	.375	24 17	8.25 .05	+.11	.00 +.04
761	385	22 59.5	80 15	A5	25.0	.400	46 37	6.91 .02	−.04 −.08	−.09 −.08
769	385	23 05.5	80 02	A3	23.9	.397	39 30	7.34 .05	−.11 −.03	−.04 −.12
80 35	386	1 09.7	80 20	F0	8.2	.384	28	7.55 .06	−.02	−.04
36	386	1 10.0	80 22	A1	8.0	.376	52 43	6.63 .02	.00 −.04	+.05 +.04
50	386	1 29.6	80 55	A0	2.9	.378	45 36	7.01 .01	+.03 +.01	+.08 +.06
55	386	1 38.8	80 23	A0	7.8	.376	45 36	6.94 .01	+.10 +.10	+.07 +.08
57	386	1 39.8	80 53	A0	7.0	.378	36 28	7.52 .03	+.01 .00	−.01 +.02
80 64	386	1 57.1	80 49	A2	15.0	.400	62 50	6.03 .01	+.05 +.10	+.09 +.09
65	386	1 57.9	81 00	A4	15.5	.404	43 34	6.96 .02	+.08 +.07	+.08 +.06
97	387	2 56.2	81 05	A4	40.0	.491	54 50	5.87 .02	+.11 +.08	+.05 +.08
125	387	3 53.3	80 25	F4	22.0	.384	78 73	5.44 .04	+.01 .00	+.08 +.09
127	387	4 01.1	80 17	G4	21.0	.380	42 38	7.35 .02	−.14 −.07	−.14 −.11
80 133	387	4 09.6	80 35	K2p	25.9	.378	52 48	6.57 .01	−.03 −.03	.00 +.05
134	387	4 12.0	80 42	A2	27.6	.397	39 34	7.20 .01	−.07 .00	−.07 −.04
140	387	4 19.2	80 40	A5	29.0		33 29	7.49 .01	−.07 +.01	−.07 −.09
155	387	4 41.6	81 02	K4	39.0	.476	46 42	6.47 .01	−.04 −.11	+.01 +.06
370	496	11 54.5	80 09	F +	21.1	.372	38 25	8.22 .11	−.13 −.20	−.08 −.14
80 389	496	12 34.1	79 46	Fp+	18.3	.351	54 38	7.50 .04	.00 −.07	−.01 −.03
395	496	12 42.3	80 28	K0	12.0	.331	33 21	8.80 .11	−.11 −.21	−.12 −.24
404	496	13 11.2	80 11	K0	23.2	.366	31	8.70 .07	−.18	−.21
421	496	13 42.3	80 42	K5	33.0	.422	25	8.53 .05	+.05	+.02
422	496	13 49.9	80 25	G5	37.9	.453	29	8.17 .04	−.02	−.06
80 448	497	14 36.4	80 06	K3	24.9	.354	50 33	7.59 .01	−.03 +.06	−.03 +.09
480,1	497	15 35.1	80 47	G2	5.5		60	6.88 .02	−.10	−.02
487	497	15 45.1	80 18	F0	12.7	.317	68 50	7.20 .02	−.13 −.08	−.12 −.01
519	497	16 37.8	80 00	F +	36.4	.426	43 29	7.53 .01	−.03 −.04	−.06 −.06
555	500	17 50.1	80 19	K5+	38.8		19	8.59 .03	+.21	+.15
80 604	500	19 06.1	80 18	G ±	30.0	.412	20	8.97 .06	+.11	−.10
607	500	19 14.4	80 34	G0	28.0	.406	31 18	8.35 .05	+.07 −.07	+.05 .00
609	500	19 15.5	80 35	A2	27.9	.408	42 28	7.77 .01	−.05 −.10	+.08 −.04
650	384	20 20.2	80 13	B9	8.2	.387	51	6.69 .01	+.13	+.11
657	384	20 33.2	80 1	G4	12.1		37 28	7.59 .04	−.03 −.05	−.03 +.05

TABLE VII—Continued

OBSERVATIONS OF CIRCUMPOLAR REGIONS

Star B. D.	Plate No.	Position, 1900		Spectrum	Dist. from Center	Mean Diam.	Approx. Wedge	Photographic Magnitude (Parkhurst)	Observed − Parkhurst (magnitude)	
		R. A.	Decl.						Baker	Cummings
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		h. m.			mm.	mm.	a b		a b	a b
80° 659	384	20 34.5	81°05′	G4	12.0	.398	58 46	6.53 ±.01	−.09 .00	−.11 +.04
660	384	20 35.2	80 44	G2	8.1		49 39	6.97 .02	−.03 .00	.00 +.02
672	384	20 52.1	80 11	G8	7.9	.386	57 47	6.67 .02	−.08 −.13	−.11 −.03
679	384	21 08.1	80 45	A6	17.0	.403	42 33	7.17 .02	−.01 −.01	−.02 −.02
688	384	21 16.8	80 23	A1	20.1	.410	36 28	7.37 .05	+.09 −.04	+.02 +.02
80 690	384	21 17.5	80 49	A2	21.4	.416	59 48	6.21 .01	−.05 −.01	.00 +.03
81 13	386	0 32.2	81 56	F5	26.2	.434	43 34	6.87 .02	−.05 −.04	−.07 −.05
147	387	4 02.0	81 43	A0	37.2	.460	30 27	7.32 .02	+.01 +.01	−.02 +.02
389	496	11 55.1	81 25	K8	15.0	.350	48 32	7.88 .06	−.03 +.02	−.01 +.05
400	496	12 31.1	80 48	A6	6.5	.328	56 40	7.45 .03	+.13 +.06	+.10 +.11
81 402	496	12 41.9	81 10	A3	5.9	.326	61	6.39 .01	+.19	+.20
412	496	12 58.6	81 25	G0	12.2	.335	48 32	7.91 .03	−.02 +.04	+.01 .00
416	496	13 11.5	81 00	G3	18.9	.350	61 44	7.21 .01	−.07 −.05	−.01 −.01
452	496	13 52.4	81 16	K2	35.6	.440	38 24	7.80 .03	−.06 −.04	.00 −.05
482	497	14 33.0	81 15	G0	22.8	.354	51 36	7.65 .02	−.07 −.07	−.10 −.10
81 495	497	14 55.0	81 09	A1	13.0	.316	66 49	7.16 .01	−.05 −.02	−.06 −.01
517	497	15 36.0	81 06	G2	5.3	.308	53 37	7.92 .03	−.07 −.05	−.05 −.04
523	497	15 42.9	80 56	G5 ±	8.5	.310	54 37	7.83 .08	−.04 +.02	+.02 +.10
531	497	15 51.4	81 14	K5	12.2	.318	40 26	8.30 .04	+.07 +.14	+.12 +.10
541	497	16 07.5	80 54	G ±	19.4	.339	33 20	8.41 .01	+.16 +.25	+.15 +.09
81 568	497	17 04.8	81 00	G5	44.9	.486	34 22	7.70 .02	−.08 −.13	−.07 −.15
622	500	18 22.4	81 26	K ±	19.9	.367	30 17	8.75 .02	.00 −.16	−.03
699	384	20 15.6	81 55	Fp	23.1	.418	28 22	7.83 .05	−.12 −.12	+.07 −.12
706	384	20 28.7	82 02	F2	23.0	.418	35 27	7.43 .02	−.02 −.05	−.03 +.01
718	384	20 49.8	82 10	B8	25.0	.460	67 54	5.78 .02	−.12 .00	+.03 +.08
82 20	386	0 45.5	83 10	A1	31.7		64 52	5.62 .02	−.05 +.02	−.03 +.07
51	386	2 01.4	83 06	G2	30.8	.468	30 22	7.28 .03	+.13 +.12	+.13 +.09
325	496	11 02.2	82 17	F8	37.0	.458	35 23	7.76 .03	+.06 −.12	−.03 −.15
463	497	15 38.1	82 36	F0	18.9	.345	61 44	7.25 .03	−.06 +.01	−.05 −.03
498	497	16 56.2	82 12	G2	40.3	.455	79	5.05 .03	−.06	−.01
82 498	500	16 56.2	82 12	G2	43.1	.500	74	5.05 .03	+.02	+.05
540	500	18 22.9	82 54	B8	10.0	.342	64 46	7.25 .01	−.07 −.08	−.03 −.04
572	500	19 04.7	82 14	B9	9.0	.341	74 53	6.85 .03	−.07 +.06	−.04 +.06
617	384	20 34.4	82 50	G2	32.0	.458	28 18	7.72 .01	−.04 +.02	−.11 −.09
748	386	23 57.6	82 25	A0	41.0	.520	28 19	7.27 .03	−.07 +.02	−.02 −.12
83 20	386	0 52.8	84 04	A4	39.8	.516	34 26	6.87 .01	+.04 −.02	−.05 .00
369	496	12 53.2	83 04	G +	21.2	.374	36 24	8.11 .07	+.16 +.03	+.13 +.01
397	496	13 45.2	83 15	G3	35.7	.436	53 36	6.85 .02	+.16 +.19	+.13 +.13
431	497	14 57.0	82 55	F8	24.5	.366	61	6.20 .01	+.10	+.08
453	497	15 53.8	83 15	A3	31.9	.401	38	7.61 .03	+.23	+.19
83 512	500	17 32.6	83 25	F +	27.4	.402	38 23	8.11 .02	−.10 − 10	−.15 −.19
535	500	18 36.4	83 18	G3	7.2	.340	43 28	8.33 .09		−.04 −.07
536	500	18 37.4	83 06	A0	5.8	.337	70	6.14 .02	−.03	+.08
547	500	19 04.0	83 46	A2	11.1	.345	70 49	6.92 .02	+.01 +.11	−.02 +.12
552	500	19 28.0	83 16	A2	13.6	.346	81 57	6.60 .02	−.20 −.02	−.14 +.07

TABLE VII—Continued

OBSERVATIONS OF CIRCUMPOLAR REGIONS

Star B. D.	Plate No.	Position, 1900		Spectrum	Dist. from Center	Mean Diam.	Approx. Wedge	Photographic Magnitude (Parkhurst)	Observed − Parkhurst (magnitude)	
		R. A.	Decl.						Baker	Cummings
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		h. m.			mm.	mm.	*a* *b*		*a* *b*	*a* *b*
83° 588	384	20 39.1	83°17′	A2	37.0		47 37	6.37 ±.01	−.02 −.01	−.03 −.03
84 269	496	12 08.8	84 04	F ±	31.7	.434	26	8.44 .06	+.01	−.04
286	496	12 37.8	84 12	F5p	32.5	.434	37 25	7.72 .02	+.10 +.01	+.05 −.07
311	496	13 26.7	83 49	F9	35.0	.440	36 23	7.82 .03	+.07 −.02	−.02 −.14
335	497	15 01.7	84 20	K2	39.0	.456	23	8.31 .04	+.15	+.08
84 361	497	16 33.6	83 55	A4	42.0	.470	43 29	7.29 .02	−.10 −.05	−.06 −.09
412	500	18 24.6	84 34	F2	21.7	.378	48 31	7.71 .01	.00 +.04	−.02 −.01
451	500	20 14.0	84 23	A2	31.1	.422	56 39	6.96 .03	+.09 +.03	+.07 +.05
462	500	20 24.5	84 14	A8	33.5	.436	44 28	7.47 .01	−.01 +.07	+.10 +.06
463	500	20 24.6	84 49	F5	35.9	.438	39 23	7.62 .02	+.05 +.14	+.11 +.06
85 294	500	18 07.2	85 41	A7	34.4		34 20	7.84 .03	+.15 +.14	+.10 +.02
86 282	500	18 47.8	86 35	M	42.9		23	8.29 .01	+.12	+.03
76 734	501	19 25.1	76 22	N	4.2	.289	29	9.06 .03	−.05	+.06
77 150	387	4 05.6	77 50	G3	11.9	.340	33 29	7.70 .04	+.24 +.27	+.26 +.27
77 715	501	19 06.4	77 31	K ±	18.0	.333	28	9.13 .08	−.32	−.30
78 661	501	18 56.1	78 50	A6	34.1	.413	33 20	8.22 .07	−.24 −.31	−.24 −.27
79 407	496	12 52.8	79 02	K +	28.9		25	9.20 .04	−.55	−.50
82 356	496	12 06.5	82 16	K2	14.0	.356	50 35	7.29 .05	+.53 +.41	+.45 +.46
84 274	496	12 16.5	83 56	G ±	29.7	.423	22	8.26 .12	+.51	+.47

The seven stars segregated at the end of the table are first to be noticed. The star B.D.+76°734 is the variable UX Draconis. The star B.D.+82°356 is probably blended with B.D.+82°357 on the Yerkes plates; the combined light is nearly a half magnitude brighter than that of the former star measured singly here. For the remaining five stars the residuals are too great to be ascribed to errors of measurement or reduction. In the absence of sufficient reasons for the large differences, the variability of the five stars is strongly suspected. The residuals for the seven stars are accordingly neglected in the discussion to follow.

Residuals vs. Distance from Center. A reliable check on the reductions to center, Table III, is effected by plotting the final residuals for the separate plates against distance from center. The corrections are well distributed over the table. If the adopted reductions to center are satisfactory, the points will be best represented by the zero line; and this is very nearly the outcome for each plate. Normal places for the separate plates are assembled in Figure 4. The agreement is surprisingly close in view of possible discrepancies due to curvature of the plate.

TABLE VIII

COLOR INDEX DIFFERENCES

Referred to Parkhurst's Classification and Photo-visual System

Spectrum	No. Stars	Δ Index	Color Index (Yerkes)	Color Index (Laws)	Spectrum	No. Stars	Δ Index	Color Index (Yerkes)	Color Index (Laws)
		mag.	mag.	mag.			mag.	mag.	mag.
B5	4	+.02	−0.37	−0.35	F8	4	.00	+0.73	+0.73
B8	11	+.09	−0.17	−0.08	G0	11	−.01	+0.84	+0.83
A0	23	+.04	−0.04	0.00	G2	12	−.03	+0.97	+0.94
A2	23	+.01	+0.09	+0.10	G4	7	−.06	+1.07	+1.01
A4	14	+.02	+0.19	+0.21	G5	12	−.05	+1.12	+1.07
A5	12	.00	+0.23	+0.23	K0	10	−.11	+1.32	+1.21
A8	6	−.04	+0.31	+0.27	K2	6	.00	+1.40	+1.40
F0	10	−.06	+0.36	+0.30	K5	8	−.02	+1.55	+1.53
F2	5	−.03	+0.43	+0.40	M	4	−.09	+1.77	+1.68
F5	9	−.04	+0.57	+0.53					

Residuals vs. Spectrum. The final residuals in Table VII are free from systematic differences arising from differences of color index in the photographic magnitudes compared. In order to determine the color corrections required to reduce the Laws Observatory magnitudes to Parkhurst's system, a preliminary reduction was made, neglecting this correction. The resulting residuals for all the plates were plotted against class of spectrum. Figure 5 shows the normal places for the points thus plotted, and a linear representation from which the color corrections are taken. Nevertheless the systematic arrangement of the points suggests a more complex curve.

Mean color index differences from the figure appear in the third column of Table VIII. The number of comparisons yielding each Δindex is found in the preceding column. Column (4) gives the Yerkes index for the corresponding class of spectrum read from a mean curve we have drawn to represent the mean values of color index in Table IV of *Yerkes Actinometry.* Combining columns (3) and (4), we have in the last

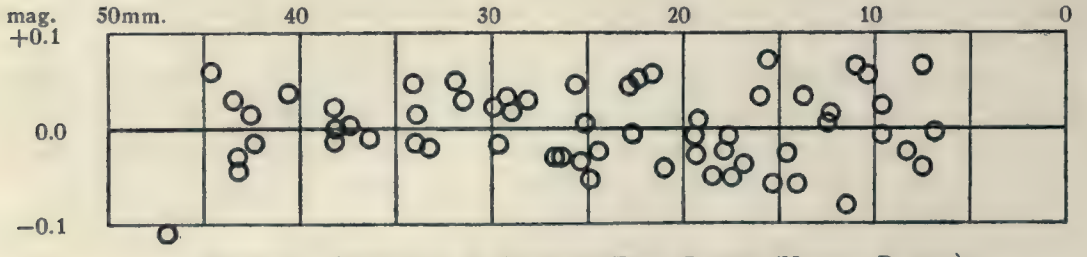

FIGURE 4. RESIDUALS *vs.* DISTANCE FROM CENTER (NORMAL PLACES)

column of our table the color index of the Laws Observatory magnitudes for each class of spectrum tabulated. A graphical representation of the points thus derived shows that the change in color index, relative to spectral class on the decimal system, is not linear, at least between classes B and G. Beyond class G a straight line satisfies the points very well.

At present the discussion of color index cannot be carried with surety beyond this point. The color equation of the Laws Observatory photographic magnitudes, normal scale, is 1.27 magnitudes referred to the Yerkes photo-visual system, and about 0.1 magnitude less than the color equation of Parkhurst's photographic magnitudes on the same system. Relative to the Harvard photometric system, our color equation appears to be about 1.12, similar to or a little less than that of Harvard photographic magnitudes. This value we adopt provisionally.

Residuals vs. Wedge Readings. In the preliminary discussion of our results the conversion from wedge reading to Δ magnitude is effected from the mean reduction curve for the emulsion concerned, that is, the curve established by the sensitometer images on the four plates of each emulsion. However the density gradient of the plate, determined from uniform sensitometer images, may not serve for non-uniform star images without correction. In Section IV of this paper we have shown that the mean reduction curve fails to give consistent magnitude differences between pairs of images having the same difference of exposure. The reliability of the mean reduction curves may now be further investigated.

Having been cleared of color differences, the preliminary residuals are plotted against wedge reading, keeping separate the results for the two observers and for the two emulsions. In each case the normal places follow the zero line closely for stars having densities greater than 40 on the scale; below this point they fall above the zero line. The divergence here is so nearly the same on all four sets that its reality is unquestioned. The Δ magnitude corrections applied to the mean reduction curves appear in Table IX. Figure 6 is intended to show that the mean reduction curves, so corrected, represent the density gradient of the plate for the extrafocal star images, when Parkhurst's photographic magnitudes are taken as standards. Normal places for the final residuals for Baker's measures on four plates of emulsion 15194 are here plotted against wedge reading. It is true, systematic departures of the residuals suggest irregularities in the reduction curve; but residuals from the other sets fail to arrange themselves similarly.

As things now stand, mean reduction curves, derived from the sensitometer images and slightly corrected from Table IX, yield photographic magnitudes in agreement with those of *Yerkes Actinometry*. As a further check we have set down the normal places for all the residuals, observed *minus* Parkhurst, in order of magnitude. As anticipated, the two scales are in exact agreement between magnitudes 5.5 and 8.5.

The corrected curve agrees with the consistent reduction curve exactly between wedge readings 20 and 40, and very nearly between 40 and 60. Above 60 the divergence

is rapid, amounting to .09 magnitude for reading 70. The difference between the two curves is seen by comparing Tables II and IX. In actual practice we have rejected star images having densities greater than 70. Moreover, most of the wedge readings are below 60, and in determining magnitudes of variable stars a number of comparison stars are selected giving images of different densities. Thus far we have used the corrected reduction curves. The uncertainty due to selection of the curve should not exceed .02 magnitude for an individual determination of any variable star on the program.

TABLE IX

CORRECTION OF REDUCTION CURVES

Wedge	Δ Reduction	Wedge	Δ Reduction	Wedge	Δ Reduction
	mag.		mag.		mag.
20	−.10	30	−.05	40	−.01
21	−.10	31	−.04	41	.00
22	−.09	32	−.04		
23	−.09	33	−.03		
24	−.08	34	−.03		
25	−.08	35	−.02		
26	−.07	36	−.02		
27	−.06	37	−.02		
28	−.06	38	−.01		
29	−.05	39	−.01		
30	−.05	40	−.01		

Diameters of Extrafocal Images. The star images used in the present discussion have diameters ranging from 0.3 to 0.5 mm. Mean diameters have been measured, where possible, on the 1914 plates by Cummings; on the 1915 plates by Baker. From a number of duplicate measures a difference in setting between the two observers is shown, that causes the following difference in the measures:

$$\Delta \text{Diameter, Baker } \textit{minus} \text{ Cummings} = -0.007 \text{ mm.}$$

This is taken as a correction to Cummings' diameters.

Diameters have been plotted against distance from center and a mean curve drawn for each plate. Frequent large residuals show that the diameter is not a function of distance from center alone. The effect of density on size of image is noticeable on the plates and may now be studied.

Diameter vs. Wedge Reading. Residuals from the curves above described are plotted against wedge readings. The normal places are shown in Figure 7. As we have anticipated, the images increase in size as their density increases; but there is a definite halt between wedge readings 32.5 and 45.0. Otherwise the function appears to be linear and similar for fainter and denser images. These results appear in Table X.

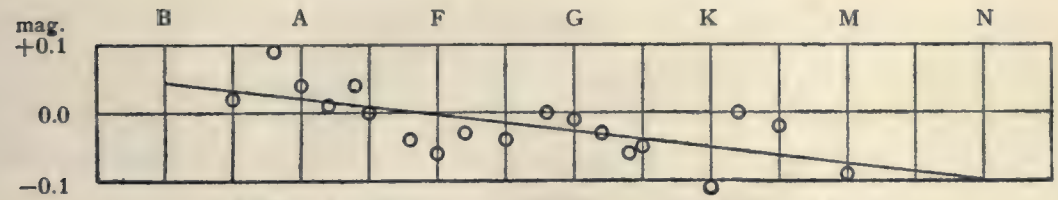

FIGURE 5. RESIDUALS *vs.* SPECTRAL CLASS (NORMAL PLACES)

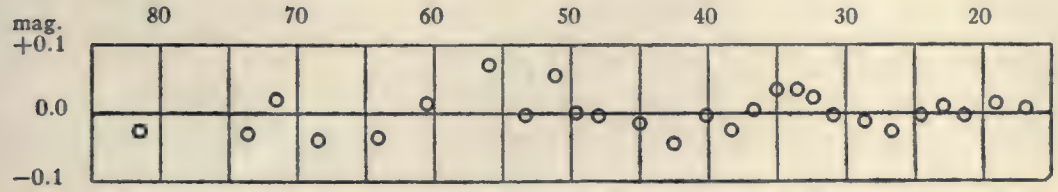

FIGURE 6. RESIDUALS *vs.* WEDGE READING (BAKER, EMULSION 15194)

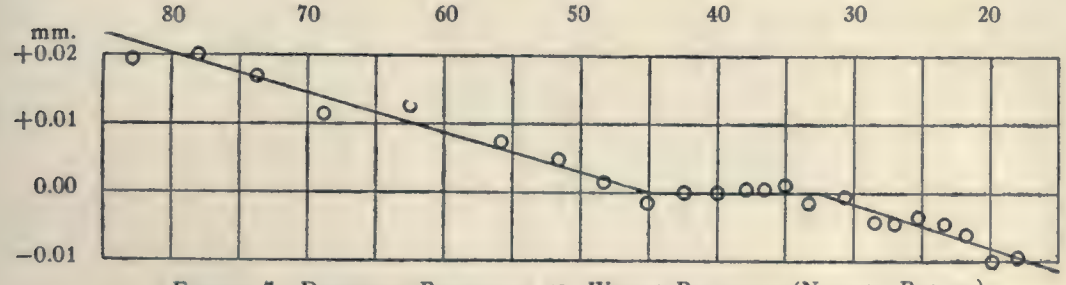

FIGURE 7. DIAMETER RESIDUALS *vs.* WEDGE READINGS (NORMAL PLACES)

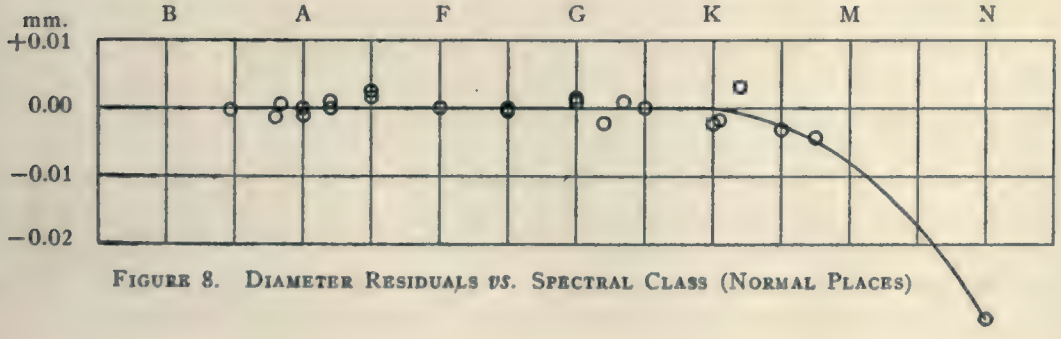

FIGURE 8. DIAMETER RESIDUALS *vs.* SPECTRAL CLASS (NORMAL PLACES)

TABLE X
DIAMETER CORRECTIONS

Wedge	Δ Diam.	Wedge	Δ Diam.	Wedge	Δ Diam.	Wedge	Δ Diam.	Wedge	Δ Diam.	Wedge	Δ Diam.
	mm.		mm.		mm.		mm.		mm.		mm.
20	+.008	30	+.002	40	.000	50	−.003	60	−.009	70	−.015
21	+ 8	31	+ 1	41	0	51	− 4	61	− 9	71	− 15
22	+ 7	32	0	42	0	52	− 4	62	− 10	72	− 16
23	+ 6	33	0	43	0	53	− 5	63	− 10	73	− 16
24	+ 6	34	0	44	0	54	− 5	64	− 11	74	− 17
25	+ 5	35	0	45	0	55	− 6	65	− 12	75	− 18
26	+ 4	36	0	46	− 1	56	− 6	66	− 12	76	− 18
27	+ 4	37	0	47	− 1	57	− 7	67	− 13	77	− 19
28	+ 3	38	0	48	− 2	58	− 8	68	− 13	78	− 19
29	+ 2	39	0	49	− 2	59	− 8	69	− 14	79	− 20
30	+ 2	40	0	50	− 3	60	− 9	70	− 15	80	− 20

TABLE XI
Δ REDUCTION TO CENTER, ADOPTED − COMPUTED

Distance From Center	Plate Number								Mean Δ Reduction
	384	385	386	387	496	497	500	501	
mm.	mag.	mag.	mag.	mag.	mag.	mag.	mag.	mag.	mag.
0	+.00	+.00	+.00	+.00	+.00	+.00	+.00	+.00	.00
5	.00	.00	.00	.02	.01	.03	.02	.03	+.01
10	.03	.03	.03	.05	.03	.06	.05	.06	+.04
15	.04	.04	.04	.07	.04	.09	.07	.10	+.06
20	.08	.07	.05	.09	.05	.11	.10	.14	+.09
25	.09	.07	.04	.10	.07	.14	.13	.16	+.10
30	.10	.09	.07	.12	.10	.15	.15	.16	+.12
35	.11	.10	.09	.12	.12	.15	.16	.17	+.13
40	.10	.09	.11	.12	.19	.18	.17	.19	+.14
45	.10	.07	.11	.12	.22	.20	.18	.21	+.15

Diameter vs. Spectrum. The color curve of the camera has not been studied, but we must suppose that the position of focus varies with the color of the star. What effect this has on the diameters may be learned by plotting the original diameter residuals after correction for density, against class of spectrum. Figure 8 exhibits the normal places for all the residuals so plotted. The result is surprising, in that no change in diameter due to color is evident between classes B5 and K0; beyond K0 the diameters decrease. The point for class N is due to a single image of the star B.D.+76°734, and this is unmistakably smaller than normal. The conclusion is that the color curve of the 5-inch camera is unusually flat. As the plates are exposed outside of focus, the smaller images for red stars indicate greater focal length for light of long wave length.

Parkhurst[1] finds for the 6-inch Zeiss lens a continuous though small variation of diameter with class of spectrum, resulting in a correction of nearly 0.1 magnitude from class K to class A. The Zeiss "UV" triplet of the Potsdam Observatory gives a correction several times greater[2].

Diameter vs. Reduction to Center. In order to coordinate Δ magnitudes resulting from measured densities of star images scattered over the plate, it is necessary to add an amount corresponding to loss of density as the image increases in size with distance from center. Other things being equal, the amount of the correction may be computed from the diameters. It remains to inquire how closely the reductions to center so computed approximate the adopted values taken from Table III. The mean diameters for each plate, corrected for density and color, as given in Table VII, column (7), are plotted against distance from center and mean curves are drawn. Mean diameters are next read from the curves for equal intervals of distance from center. Neglecting the details of the computation, we have in Table XI differences in the reductions to center in the sense: adopted *minus* computed from diameters.

The mean differences in the last column of the table are considered as definitive corrections to the values computed from the diameters. This procedure is to be followed when curvature of the plate is known to be excessive, for in this event the reductions can not be taken safely from the usual table.

In computing the reduction to center from the diameters, we assume uniformly illuminated images. Distribution of light in the extrafocal star images is by no means uniform; the center does not receive a proper proportion of the light. For stars near the center of the plate, the images are but slightly larger than the silver dot of the microphotometer. As distance from center increases, more of the darker part falls outside the dot and has no influence on the measured density. The falling off in density should therefore be greater, and the reduction to center curve should be steeper, for star images than for uniform disks. Accordingly for a given distance from center the non-uniform image demands a larger correction.

The mean differences in Table XI represent corrections due to the peculiar illumination of the star images. The differences are smaller than might be expected. They should vary with any change in pattern of the image, and with change in shape or size of the silver dot. It is known that the character of the image varies considerably on our plates. We have thus far no data on the corresponding change to be anticipated in the reduction to center.

Probable Errors. The lack of exact agreement in individual cases, between the present results and Parkhurst's photographic magnitudes may be attributed to a number of causes. The reductions are subject to errors of greater amount than the inaccuracies in the measurements. Errors in the Yerkes magnitudes contribute, especially where

[1] J. A. Parkhurst, *Astrophysical Journal*, **36**, 181, 1912.
[2] E. Hertzsprung, *Astronomische Nachrichten*, **186**, 180, 1910.

TABLE XII

PROBABLE ERRORS FOR INDIVIDUAL PLATES

Plate No.	ε_{O-P}		ε		ε
	Baker	Cummings	Baker	Cummings	Mean
	mag.	mag.	mag.	mag.	mag.
384	±0.054	±0.050	±0.044	±0.038	±0.041
385	59	58	50	48	49
386	52	58	41	48	44
387	56	63	46	54	50
496	66	58	58	48	53
497	66	57	58	47	52
500	70	57	62	47	54
501	66	71	58	63	60
Mean =	±0.061	±0.058	±0.052	±0.049	±0.050

TABLE XIII

PROBABLE ERROR *vs.* DISTANCE FROM CENTER

Distance from Center	ε_{O-P}		ε		ε
	Baker	Cummings	Baker	Cummings	Mean
mm. mm.	mag.	mag.	mag.	mag.	mag.
0 to 10	±0.053	±0.055	±0.042	±0.047	±0.044
10 15	63	58	54	41	48
15 20	60	55	51	38	50
20 25	58	60	48	54	51
25 30	55	58	45	51	48
30 35	64	64	55	55	55
35 40	65	62	57	50	54
40 45	62	60	53	50	52

TABLE XIV

PROBABLE ERROR *vs.* DENSITY

Wedge	ε_{O-P}		ε		ε
	Baker	Cummings	Baker	Cummings	Mean
	mag.	mag.	mag.	mag.	mag.
15 to 25	±0.071	±0.066	±0.063	±0.058	±0.060
25 35	58	59	48	50	49
35 45	53	54	42	44	43
45 55	53	52	42	41	42
55 65	66	63	58	54	56
65 80	69	66	61	58	60

images overlap. The smaller star disks on our plates minimize blending effects. Variability of the star may well account for some of the differences.

In order to determine the accuracy of our results and the effect of probable sources of inaccuracy, the final residuals in columns (10) and (11), Table VII, are assembled in various ways and probable errors are computed. In the tabulated results the abbreviations are:

ε_{O-P}, the probable error of a single difference, observed *minus* Parkhurst.

ε_{P}, the probable error of a catalog mean in *Yerkes Actinometry*. The average value is ± 0.032 mag.

$\varepsilon = \sqrt{\varepsilon^2_{O-P} - \varepsilon^2_{P}}$ the probable error of a single observation on the Laws Observatory plates, i. e., for a single star image.

Probable Errors for Individual Plates. The probable error of a single observation, derived from the residuals for all measures on the plate, appears for each plate in Table XII. Attention is called to the greater values for the later plates.

Probable Errors vs. Distance from Center. Curvature of the plate is to be expected frequently, with accompanying errors in the reductions to center, that increase with distance from center. The previous discussion of residuals relative to distance from center showed discrepancies smaller than expected. A similar agreement should appear in the probable errors from these residuals. Table XIII shows only a slight increase in the probable error with increasing distance from center. We conclude that curvature is almost negligible on the eight plates here considered. However, in practice we guard against curvature by selecting stars for measurement as close as possible to the center.

Probable Error vs. Density. Star images having densities above wedge reading 70 and below 25, on clear film plates, are difficult to measure. Moreover the reduction curves beyond these limits are sometimes not accordant for the same emulsion. We have measured images on the control plates both fainter and denser, in order to learn what decrease in accuracy follows. As the plates are all clear-film, troublesome sky-fog does not enter the problem. The residuals are assembled into six groups according to wedge reading and the probable error of a single observation is computed for each group. Table XIV shows that maximum accuracy is attained between wedge readings 35 and 55, with rapid increase in the probable error on either side. In practice we have exercised considerable care in exposing plates to keep within these limits the majority of the stars to be measured.

Probable Error vs. Spectrum. Actual fluctuations in the light of the stars observed enter necessarily as accidental errors and increase the probable error. Unless the star is known to vary or yields a photographic magnitude excessively discordant, as in the seven cases isolated in Table VII, no large residual can be ascribed to variability of the star with sufficient confidence to justify its rejection. On the whole we anticipate greater percentage of variability in the case of the late-type stars. With this in mind, we have assembled

the residuals according to spectral class, and have computed the probable error of a single observation for each group. Table XV shows an increase in the probable error for red stars, and appears to prove the point. Accepting this for a moment, we now explain the greater uncertainty of the 1915 plates, the last four in Table XII; for these plates contain the majority of the late-type stars. On the other hand, if we suppose the later plates to be more uncertain from some other cause not understood, we have as a consequence a greater probable error for the red stars.

TABLE XV

PROBABLE ERROR *vs.* SPECTRUM

Spectrum	ε_{O-P}		ε		ε
	Baker	Cummings	Baker	Cummings	Mean
	mag.	mag.	mag.	mag.	mag.
B to A	±0.054	±0.056	±0.044	±0.046	±0.045
A F	57	55	47	45	46
F G	58	62	48	53	50
G K	61	52	52	41	46
K M	72	71	65	63	64

Probable Error in Practice. An additional check on the accuracy of the results is effected by grouping the residuals according to the probable error of Parkhurst's magnitude, as transcribed in Table VII, column (9). In order to make the results more nearly representative of actual practice, we exclude residuals for images whose distances from center exceed 25 mm. and whose densities are below wedge reading 25 or above 70. The resulting probable error of a single observation is ±0.042 magnitude. The probable error of a single photographic observation in the *Yerkes Actinometry* is ±0.069 and in the *Göttingen Aktinometrie* is ±0.046 magnitude.[1]

The comparison is somewhat unfair, since our selection of stars near the center neglects residuals that would increase our probable error. Table XII shows that for all the stars the probable error of a single observation is ±0.050 magnitude. But this cannot be taken as fairly representing the probable error, because residuals for excessively faint and dense images enter in its determination. These images, that an observer would not normally measure, were measured on the control plates for experimental purposes. Thus the probable error of a single observation, for purposes of comparison with others, lies between ±0.042 and ±0.050 magnitude.*

The accuracy of the Laws Observatory results is comparable with that of the *Göttingen Aktinometrie* and is apparently greater than that of the *Yerkes Actinometry*. The conclusion is that no accuracy is lost by exposing the plates close to focus and that the extrafocal method is capable of results which compare favorably with other methods.

[1]K. Schwarzschild, *Vierteljahrsschrift der Astron. Gesellschaft*, **47**, 360, 1912.

*This does not represent the accuracy attained in our work on variable stars; e. g., for RX Herculis the discussion of which will appear in *Laws Observatory Bulletin*, No. 25, the probable error of a single magnitude determinatiou is ± 0^{m}026.

VI. SCALE OF THE LAWS OBSERVATORY MAGNITUDES

So far the photographic magnitudes in the *Yerkes Actinometry* have been taken as standards. We have shown that the mean reduction curves, given slight correction for the fainter images, yield magnitudes in agreement with the normal Yerkes scale. The corrected reduction curve departs from the consistent curve (Section IV) only a little, except for the densest images measured; at the limit the difference is 0.09 magnitude. Fortunately a choice between the two does not seriously affect the results in our studies of variable stars. The corrected reduction curve has been adopted by us up to the present.

The correctness of the Yerkes photographic scale appears to be in doubt. Schwartzchild[1] concludes from correspondence with Parkhurst that the magnitude differences are to be multiplied by 0.94. The Pleiades magnitudes are not involved, as the durations of exposures in the camera and in the sensitometer were made equal, contrary to his usual practice. This correction is accepted by Seares[2] and by Miss Leavitt.[3] As stated directly by Professor Parkhurst the amount of the correction is left somewhat indefinite. He says:[4] "A new determination of the absolute scale for the photographic magnitudes of the actinometry indicates that the published scale may be too extended by an amount not exceeding six per cent. The application of the correction of -6 per cent, brings the magnitudes into better agreement with Harvard and leaves the differences with Göttingen the same in amount but with the sign changed."

In the absence of a more definite correction, we have taken the Yerkes magnitudes as they are published. The correction required to bring the lower part of our mean reduction curves into agreement with these magnitudes has the effect of increasing the Laws Observatory scale, and of taking up in the opposite direction part of the accepted correction to the Yerkes scale. The agreement of the latter with the Mount Wilson scale, that is evident from comparison of the two[5] for the brighter stars of the North Polar Sequence, strengthened our confidence in the choice of standards and in the reliability of the Laws Observatory photographic scale. From these considerations it would appear that our scale is, if anything, somewhat too large.

As the photographic investigations of eclipsing variable stars have taken form, we have been surprised to find instances where the loss of light at both minima is less than that shown by well-established photometric curves. Explanation of these differences in range would be difficult, if we hold to present theories of eclipsing variables. The in-

[1] K. Schwarzschild, *Vierteljahrsschrift der Astron. Gesellschaft*, **47**, 357, 1912.
[2] F. H. Seares, *Mount Wilson Contributions*, **97**, 30, 1915.
[3] H. S. Leavitt, *Harvard Annals*, **71**, 216, 1915.
[4] J. A. Parkhurst, *Astronomical Society Publications*, **2**, 151, 1913.
[5] F. H. Seares, *Mount Wilson Contributions*, **97**, 29,1915.

TABLE XVI

North Polar Sequence

No.	Harvard			Observed Photg. Mag.						O–H	Obsd. ×1.15	$O_{1.15}$–H
	Photg. Mag.	Index	Spectrum	Plate 612	Plate 613	Plate 614	Plate 615	Mean	Wt.			
		mag.								mag.	mag.	mag.
1	4.48	+.04	A0	4.63	4.82	4.86	4.67	4.70	8	+.22	4.50	+.02
2	5.28	−.10	A0	5.28		5.28		5.28	6	.00	5.17	−.11
3	5.81	+.19	F0	5.92	5.87	5.80	5.87	5.86	13	+.05	5.84	+.03
4	5.99	+.13	A3	5.91	6.02	6.02	5.98	5.98	13	−.01	5.98	−.01
5	6.49	+.06	A2	6.53	6.43	6.43	6.45	6.46	15	−.03	6.53	+.04
6	7.11	+.10	A0	7.03	7.06	7.04	7.06	7.05	14	−.06	7.21	+.10
7	7.31	−.13	B8	7.13	7.19	7.22	7.21	7.18	11	−.13	7.36	+.05
8	8.23	+.26	F0	7.87	7.77	7.79	7.92	7.84	12	−.39	8.12	−.11
9	8.83	+.22	A	8.41	8.33	8.35	8.48	8.40	9	−.43	8.76	−.07
10	9.02	+.08	A5	8.69	8.68	8.57	8.65	8.64	7	−.38	9.04	+.02
11	9.55	+.25	A	(8.94)	9.12	9.07	9.12	9.08	3	−.47	9.54	−.01
12	9.86	+.32	A	(9.21)	(9.49)	(9.48)	(9.53)	9.43	2	−.43	9.94	+.08
2s	6.48	0.20	F0	6.36	6.54	6.46	6.52	6 47	15	−.01	6.54	+.06
3s	6.65	0.32	F2	6.46	6.50	6.45	6.43	6.47	12	−.18	6.54	−.11
1r	6.80	1.54	Ma	6.44	6.59	6.65	6.63	6.56	13	−.24	6.64	−.16
2r	7.87	1.32	Mb	7.59	7.69	7.66	7.63	7.65	14	−.22	7.90	+.03
3r	8.75	1.18	K2	8.21	8.46		8.41	8.37	4	−.38	8.73	−.02
4r	9.10	0.84	K0	8.54	8.76	8.70	8.59	8.63	7	−.47	9.02	−.08

TABLE XVII

North Polar Sequence. Comparison of Scales

No.	Observed ×1.15	Harvard re-reduced by Seares	$O_{1.15}$–H_R −0.15	Mount Wilson	$O_{1.15}$–MW	Yerkes	$O_{1.15}$–Y −0.13	O − Y −0.13	Yerkes ×0.94	$O_{1.15}$–$Y_{0.94}$ −0.13
			mag.		mag.		mag.	mag.		mag.
1	4.50	4.22	+.13	4.39	+.11	4.25	+.12	+.32	4.36	+.01
2	5.17	5.01	+.01	5.30	−.13	5.06	−.02	+.09	5.12	−.08
3	5.84	5.62	+.07	5.83	+.01	5.62	+.09	+.11	5.64	+.07
4	5.98	5.79	+.04	5.91	+.07	5.91	−.06	−.06	5.92	−.07
5	6.53	6.43	−.05	6.45	+.08	6.43	−.03	−.10	6.40	.00
6	7.21	7.02	+.04	7.11	+.10	7.12	−.04	−.20	7.05	+.03
7	7.36	7.20	+.01	7.41	−.05	7.27	−.04	−.22	7.19	+.04
8	8.12	8.11	−.14	8.32	−.20					
9	8.76	8.69	−.08	8.88	−.12					
10	9.04	8.87	+.02	9.12	−.08					
11	9.54	9.48	−.09	9.73	−.19					
12	9.94	9.77	+.02	10.09	−.15					
2s	6.54	6.44	−.05	6.45	+.09	6.36	+.05	−.02	6.43	+.07
3s	6.54	6.48	−.09	6.64	−.10	6.54	−.13	−.20	6.51	−.10
1r	6.64	6.55	−.06	6.61	+.03	6.70	−.19	−.27	6.66	−.15
2r	7.90	7.71	+.04	7.94	−.04	8.09	−.32	−.57	7.96	−.19
3r	8.73	8.61	−.03	8.96	−.23					
4r	9.02	8.93	−.06	9.22	−.20					

ference is that the photographic scale is too small. Independent of other considerations, we have desired justification for increasing the scale about 15 per cent. Again, photographic observations on this scale of the short-period variable RR Lyrae[1] give a variation smaller than the visual, a result not certainly paralleled in the case of other stars whose variation is of this character.

A comparison of our scale with Parkhurst's in the study of variable stars is instructive. The Yerkes observations of the eclipsing variable RZ Cassiopeiæ[2] show a range of 1.21 magnitudes at primary minimum. A complete set of plates of this star has been secured here, but definitive measurements are not yet made. Measures of a few images on two plates indicate a range of 1.32 magnitudes at primary minimum. There is reason for supposing that this value is a few hundredths too large. As before, the scale appears to be in fair agreement with Yerkes. The photographic ranges of both are smaller than the visual range of 1.33 magnitudes as given by Graff.[3] The conclusion is that both Yerkes and Laws Observatory scales may be too small. Further evidence is required.

The North Polar Sequence.[4] Four plates of the Harvard North Polar Sequence are now available. Each plate contains several exposures whose durations vary from 30 seconds to 20 minutes. The guiding star is different for each plate; thus any errors in the reductions to center have smaller effects. All the plates are Seed 27 of emulsion 15194. Exposure conditions were below average and the plates themselves are not entirely satis factory; two of them would doubtless be rejected, if a selection were possible.

The measures, by Baker, are converted to magnitude differences by use of the consistent reduction curve for the emulsion. Star images having densities greater than wedge reading 70 are rejected: likewise those falling below 25, except for stars 11 and 12. After applying corrections for atmospheric extinction and for reduction to center, the relative magnitudes for each plate are assembled and the mean taken for each star. The resulting magnitudes are found, according to star and plate, in columns (5) to (8), Table XVI. Magnitudes enclosed in parentheses are determined from images fainter than wedge reading 25. These receive one-half weight. The mean observed photographic magnitude for each star follows, with its weight, i. e. the number of exposures that determine it except in cases of stars 11 and 12. Column (O–H) gives the difference, observed *minus* Harvard. A lack of agreement in the two scales is at once evident. After a few trials the correction: (observed magnitude *minus* 6.00) *times* 1.15, is found appropriate; this, applied to the observed magnitudes, brings them into excellent agreement with the Harvard photographic magnitudes, as shown by the residuals in the last column. Moreover there is little evidence of difference in color equation between the two systems.

[1] C. C. Kiess, *Laws Observatoryr Bulletin*, No. 22, 96, 1915.
[2] J. A. Parkhurst and F. C. Jodan, *Astrophysical Journal* 26, 254, 1907.
[3] K. Graff, *Mitteilungen der Hamburger Sternwarte*, No. 13, 1913.
[4] H. S. Leavitt, *Harvard Annals*, 71, No. 3, 1915.

Table XVII presents a comparison of the revised Laws Observatory magnitudes with other scales. Constant corrections noted in the heading are frequently added to the residuals. Professor Seares[1] has found for these brighter stars of the sequence a serious discrepancy between the Harvard and the Mount Wilson photographic scales. In the endeavor to explain the difference, he re-reduced the former magnitudes and found justification for their revision (See Table X of his memoir). The Harvard magnitudes, as re-reduced by Seares, are first to be noticed in our table. The differences, observed $\times 1.15$ *minus* Harvard re-reduced, in column (4), show very slight difference in scale; a further increase in our scale of one or two per cent is perhaps suggested.

The differences, observed $\times 1.15$ *minus* Mount Wilson, show an abrupt change in sign at magnitude 7.5. The agreement for the fainter stars of the table would be improved by increasing the Laws Observatory scale correction from 1.15 to 1.18.

Column (Yerkes) contains photographic magnitudes taken from the *Yerkes Actinometry* and column (Yerkes $\times 0.94$) gives the same magnitudes on the reduced scale. Of the three comparisons with Yerkes, the last column, observed $\times 1.15$ *minus* Yerkes $\times 0.94$ is most acceptable. We find difficulty in interpreting the last result, in the light of previous comparisons with *Yerkes Actinometry*. The larger residuals for the red stars, lr and 2r, are due to a difference in color equation that we have noted before (Section V).

Pleiades Sequence. Further evidence on the foregoing points is derived from measures of three plates of the Pleiades region. Consistent reduction curves are used again in this case. Differential atmospheric extinction is neglected. Table XVIII is self-explanatory. The differences, observed *minus* Harvard, again exhibit the need of scale adjustment. The differences, observed $\times 1.15$ *minus* Harvard, show excellent agreement as before. There is also a fair degree agreement with the Göttingen series.[2] Only six stars are available for comparison with Parkhurst's Pleiades series. If anything is shown here, it is that the Yerkes scale and the original Laws Observatory scales are concordant; and this recalls the similar inference in Section V.

Provisional Scale and Color Equation. All discussion of scale and color equation in this paper is regarded as preliminary, while further inquiry is pending, and is subject to change. We have shown that an increase of 15 per cent in the magnitude differences is justifiable; that our scale so increased is in accord with the Harvard photographic scale between magnitudes 4.5 and 9.5 for the North Polar and Pleiades sequences; that a further increase in scale, not exceeding three per cent, may be necessary to effect satisfactory agreement with the Mount Wilson scale. The scale of the Yerkes extrafocal results appears to vary. For the circumpolar stars generally and for the Pleiades, the Yerkes and normal Laws Observatory scales agree and both require adjustment. For the North Polar Sequence the two normal scales differ by 20 per cent.

[1] F. H. Seares, *Mount Wilson Contribution*, **98**, 1915.
[2] K. Schwarzschild, *Göttingen Aktinometrie*, **B**, 14, 1912.

For the present, we propose using corrected reduction curves or consistent reduction curves, as the occasion demands and to confine our measures to star images whose densities fall between readings 25 and 70 on the wedge-scale of the microphotometer. *The adopted scale of the Laws Observatory photographic magnitudes equals the normal scale multiplied by 1.15. The color equation equals 1.12 magnitudes.*

TABLE XVIII

PLEIADES SEQUENCE

Harvard				Observed Photg. Mag.					*Harvard* O–H	Obsd. ×1.15	*Harvard* $O_{1.15}$–H	*Göttingen* $O_{1.15}$–G +0.03	*Yerkes* $O_{1.15}$–Y	*Yerkes* O–Y +0.06
No.	Photg. Mag.	Index	Spectrum	Plate 393	Plate 616	Plate 617	Mean	Wt.						
		mag.							mag.		mag.	mag.	mag.	mag.
7	4.90	−.25	B8			5.00	5.00	1	+.10	4.85	−.05	−.06	−.21	.00
8	5.32	−.14	B5	5.45		5.45	5.45	2	+.13	5.37	+.05	+.03	−.12	+.02
9	5.72	−.11	B8	5.67	5.72	5.70	5.70	3	−.02	5.66	−.06	.00		
10	6.31	−.11	B9	6.20	6.32	6.32	6.28	3	−.03	6.32	+.01	.00		
11	6.81	+.07	B9	6.65	6.66	6.77	6.69	3	−.12	6.79	−.02	−.02	.00	−.04
12	7.00	+.16	A1	6.93	6.85	6.92	6.90	3	−.10	7.04	+.04	+.05	+.08	.00
13	7.26		A0	7.13	7.07	7.07	7.09	3	−.17	7.25	−.01	+.03		
14	7.37	+.11	A1	7.29	7.25	7.12	7.22	3	−.15	7.40	+.03	.00	+.14	+.02
15	7.41	+.07	A2		7.26		7.26	1	−.15	7.45	+.04	−.09	+.09	−.04
16	7.56	+.13	A1		7.28		7.28	1	−.28	7.47	−.09	−.10		
17	7.99	+.07	A2		7.74		7.74	1	−.25	8.00	+.01	−.12		
18	8.39		A5		8.10		8.10	1	−.29	8.42	+.03			

VII. THE PROGRAM

The Laws Observatory has undertaken the photographic investigation of twelve eclipsing variable stars. The program appears below, with the number of plates we have secured in each set up to March 1, 1916, and the number of exposures available in each set. An asterisk (*) before the name of the star indicates that the set of plates is complete and a double asterisk (**) that the discussion is nearly ready for publication.

Star	R. A. (1900)	Decl. (1900)	Number of Plates	Number of Exposures
** TV Cassiopeiae	$0^h\ 13^m.9$	+58° 35′	50	496
U Cephei	0 53.4	81 20	28	302
* RZ Cassiopeiae	2 39.9	69 13	31	551
RS Canum Ven.	13 6.0	36 28	14	92
U Coronae	15 14.1	32 01	14	116
TW Draconis	15 32.4	64 14	17	131
** u Herculis	17 13.6	33 12	54	859
TX Herculis	17 15.4	42 00	18	125
** RX Herculis	18 26.0	12 33	46	499
RS Vulpeculae	19 13.4	22 16	5	40
U Sagittae	19 14.4	19 26	14	228
** Z Vulpeculae	19 17.5	25 23	48	499

NOTES ON THE PROGRAM

TV Cassiopeiæ. The range of magnitude is 0.90 at primary minimum and 0.09 at secondary minimum. On rectification the secondary entirely disappears; and the photographic ranges for both minima are smaller than visual ranges determined by Nijland and McDiarmid. The light curve clearly shows the effects of ellipticity and brightening at periastron; and to a smaller degree the effect of reflection.

RZ Cassiopeiæ. The set of plates, now complete for this star, contain also the short-period variable SU Cassiopeiæ.

RS Canum Ven. In *Astronomische Nachrichten*, **200**, 177, 1915, Hoffmeister published elements for this recently discovered eclipsing variable (10.1914) and the light curve for primary minimum from visual observations. He finds no evidence of secondary minimum or of constant phase during primary minimum. The range he gives as 1.08 magnitudes. Our photographic magnitudes for this star show a range of 1.4, and a constant phase at primary minimum whose duration is 0.17 days. Hoffmeister's epoch required in May, 1915, the correction +0.05 days.

U Coronæ. The photographic range is not yet well established. It appears to be equal, at least, to the visual range observed by Wendell.

TW Draconis. The eclipsing system is one component of a visual binary, which is readily separated in the guiding telescope, but produces a single image on the plate. The photographic range for the triple system is 2.0 magnitudes. The light is constant at primary minimum for about 0.05 days.

u Herculis. The photographic ranges are 0.67 and 0.31 magnitudes for primary and secondary minima respectively. Corresponding visual ranges from Wendell's observations are 0.71 and 0.24 magnitudes. The effects of ellipticity and reflection are evident. Orbital eccentricity is shown by the displacement of the secondary minimum and by a slight periastron effect.

TX Herculis. The photographic ranges at the two minima are 0.72 and 0.35 magnitudes.

RX Herculis. The photographic ranges for primary and secondary minima are 0.58 and 0.44 magnitudes respectively. Corresponding visual ranges by Shapley are 0.57 and 0.48. The light curve shows the effects of ellipticity and reflection.

Z Vulpeculæ. The photographic ranges at the two minima are 1.64 and 0.25 magnitudes. The secondary minimum has less depth than in the visual curve published by Baker, and its displacement is in the opposite direction, i. e. to the right. The effects of ellipticity and reflection are marked, and brightening at periastron is indicated by the least-squares solution.

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ASTRONOMICAL SERIES

LAWS OBSERVATORY BULLETIN NO. 25

THE ECLIPSING BINARY RX HERCULIS

By Robert H. Baker and Edith E. Cummings

The variability of RX Herculis (R.A. = 18^h26^m0, Decl. = $+12°\ 32'$) was detected in 1898 by Sawyer[1], who announced it as Algol type. A series of 847 observations of this star by Luizet[2], made by the Argelander method in the years 1899 to 1904, confirmed the character of its variation and established a period of 0.889288 days. The observations are well distributed over the entire interval; their number and accuracy insure confidence in the published light curve, aside from the fact that the determinations are not photometric. The range of variation given by Luizet is 0^m6 and the maximum light is about the seventh magnitude. The duration of eclipse is $4^h\ 35^m$, the time of decreasing light being longer by five minutes than that of increasing light. The normal places for Luizet's observations outside of eclipse are well represented by a straight line, indicating the absence of ellipticity and of a secondary minimum. As the period is to be doubled no estimate of reflection effect can be made without the individual determinations. A systematic arrangement of the normal places at maximum is to be noted, especially after minimum when they rise above the line, and before minimum when they fall below.

The probability that the period of revolution of RX Herculis is twice the period given above was suggested by Miss Clerke[3]. The failure of Luizet's light curve to yield a definite solution at Princeton[4] added to the weight of this conclusion. However, the shape of the curve during eclipse did not justify a solution with the period doubled. To secure additional data Dr. Shapley made a series of observations (*loc. cit.*) of the star in 1912 with the polarizing photometer and derived a light curve suitable for a double period solution with the following elements:

[1]E. F. Sawyer, *Astronomical Journal*, **19**, 120, 144, 1898.
[2]M. Luizet, *Astronomische Nachrichten*, **150**, 357, 1899; **157**, 337, 1901; **168**, 283, 1905; *Bulletin Astronomique*, **22**, 232, 1905. *Cf.* P. S. Yendell, *Astronomical Journal*, **22**, 162, 1902.
[3]A. M. Clerke, *Problems in Astrophysics*, p.315.
[4]H. Shapley, *Princeton Contributions*, **3**, 40, 1915.

Primary Minimum = J.D. 2419658.5882 + 1^{d}.7785740·E,G.M.T. (Shapley)

As additional evidence of the double period he refers to the Yerkes spectrograms of this star which show the spectra of both components. The Princeton observations give no certain indication of difference in depth between alternate minima, or of ellipticity or radiation effects outside of eclipse. Shapley states that the measures were unusually difficult because of the brightness of the stars measured and because of the frequent low altitude of the field.

The reality of the double period was established in 1914, and at about the same time, from the plates of the Laws Observatory and from Shapley's discussion of Professor Wendell's visual observations.[1] In both cases alternate minima differ in depth by about $0^{m}.1$. In the Harvard series only 23 observations occur outside of eclipse; the probable error of a single observation is slightly less than $\pm 0^{m}.03$.

A new solution for the orbital elements has been made by Shapley[2] after combining the Harvard and Princeton observations and having now available the spectroscopic results of the Yerkes Observatory. The star eclipsed at primary minimum he supposes to have less than half the light of the system, because the lines in its spectrum are estimated slightly the fainter of the two components. The photometric orbital elements, otherwise highly indeterminate by reason of the shallow minima, are now fixed within narrow limits. Shapley has computed the elements of the spectroscopic orbit from the Yerkes radial velocities and has finally derived the absolute dimensions of the system. The masses of the two components are nearly the same and have been assumed equal, but it is added that the star eclipsed at primary minimum is probably the more massive.

The system of RX Herculis thus appears to be a rare exception to the general rules of binaries, as Shapley has pointed out. The star eclipsed at primary minimum is assumed by him to be the fainter and, having the greater surface intensity, is therefore smaller. This star is probably the most massive. But, we quote Professor Russell,[3] "among all the binaries visual and spectroscopic in which this ratio has been determined, the brighter star is found to be the more massive (with the doubtful exception of 85 Pegasi)."

DISCUSSION OF YERKES RADIAL VELOCITIES

Our knowledge of the spectrum of RX Herculis is based on the description[4] of a few spectrograms of the star secured at the Yerkes Observatory. The spectrum is evidently of class A, with diffuse lines, and both components visible. In Table I we have transcribed the results of measures on three plates. The lines are difficult to measure and,

[1] O. C. Wendell, *Harvard Annals*, 69, 158, 1914.
[2] H. Shapley, *Astrophysical Journal*, 40, 399, 1914.
[3] H. N. Russell, *Astrophysical Journal*, 36, 74, 1912.
[4] E. B. Frost, *Astrophysical Journal*, 22, 215, 1905; see also 40, 406, 1914.

according to Professor Frost, the velocities may not be nearer than 10 km. of the truth. On plates 558 and 562 the lines of one component are broader and slightly less intense than the companion lines. This is the star eclipsed at primary minimum. On plate 839 the widths and intensities of the two components are said to be equal; on this plate the line of the secondary component at λ4481 was rejected, since it gave a positive velocity more than 100 km. greater than the mean of the two other lines measured.

In the table we take as the primary component the star whose velocity oscillation appears to have the smaller range. This is the star eclipsed at primary minimum and the one whose spectral lines are estimated to have slightly the lesser intensity. Nevertheless we shall designate this as the brighter star and so keep within the rule relative to brightness and mass, and within the apparently less universal rule connecting total brightness and surface intensity. Justification for this course is not lacking. Two stars of the same brightness and spectral class may differ markedly in the conspicuousness of the lines in their spectra; and on superposition of the two spectra a greater difference may be anticipated, if the lines have different degrees of breadth and diffuseness. The

TABLE I

RADIAL VELOCITIES OF RX HERCULIS (YERKES)

Plate No.	Date	Julian Day G. M. T.	Epoch	Phase	Primary Component (brighter star)		Secondary Component (fainter star)	
					Velocity	No. Lines	Velocity	No. Lines
558	1905 July 16	2417043.686	−1473	279°.2	+ 84 km.	2	−130 km.	2
562	July 21	048.709	−1468	215.8	+ 42	3	− 74	4
839	1906 Sept. 8	462.642	−1235	120.0	− 108	3	+ 75	2

brighter component of RX Herculis, having the broader lines, is seen at a disadvantage and appears fainter. The peculiarity of this star appears to be that the two spectra, while they are of nearly the same intensity, are different in character.

Of the three plates measured at the Yerkes Observatory, plate 562 appears to be the least dependable because of the smaller separation of the components. This is probably the second of the two spectrograms first described by Frost (*loc. cit.*). On this plate λ4481 appeared single. The measured relative velocity from the plate is 116 km. Experience in determining the orbits of spectroscopic binaries, when both spectra are visible, has shown that confusion of the components seriously affects the results. Within limits of separation that depend on the character of the lines and the grain of the plate, the measured radial velocities have little meaning. For example, in the case of the star 2 Lacertae the minimum separation for useful measures of plates taken with the Mellon spectrograph was found[1] to be 143 km. with Lumière Sigma plates and 124 km. with Seed

[1] R. H. Baker, *Allegheny Publications*, 1, 93, 1909

23 plates. The Yerkes spectrograms of RX Herculis are on nearly the same scale and are probably Seed 27 plates. If we assume that the lines in the spectrum of this star are not less diffuse than in the case of 2 Lacertæ, there is reason for rejecting plate 562.

The spectroscopic elements given below depend on radial velocities from two Yerkes plates, numbers 558 and 839. Since the light curve gives no evidence of eccentricity, we may put $e=0$ and may readily procure the elements by use of the formulæ:

$$V_b=\gamma-K_b\sin\theta$$
$$V_f=\gamma+K_f\sin\theta$$
$$a\sin i=[4.1383]\cdot K\cdot P$$
$$(m_b+m_f)\sin^3 i=[13.016-20]\cdot(K_b+K_f)^3\cdot P\odot$$
$$m_b K_b=m_f K_f$$

in which P is the period in days, γ the radial velocity of the center of mass of the system, V the observed velocity, K the semi-range of velocity, a the radius of the orbit, i the inclination of the orbit, m the mass. The longitude θ is taken from the epoch of primary minimum. The subscript b denotes the brighter and more massive star that is eclipsed at primary minimum; and f refers to the fainter star. The resulting elements are as follows:

$K_b=104$ km.	Approximate spectroscopic elements of RX Herculis
$K_f=111$ km.	
$\gamma=-19.5$ km.	
$a_b\sin i=2{,}540{,}000$ km.	
$a_f\sin i=2{,}710{,}000$ km.	
$m_b\sin^3 i=0.94\odot$	
$m_f\sin^3 i=0.89\odot$	

PHOTOGRAPHIC OBSERVATIONS

The study of the photographic light variations of RX Herculis is the first to be completed from the program of twelve eclipsing binaries undertaken here. It is based on 46 extrafocal plates obtained with the 5-inch camera. The instruments and methods employed in this work are described in *Laws Observatory Bulletin*, No. 24. Table II contains a list of the plates with data regarding them. The times in columns (2) and (3) and the hour angle in column (8) are given for the middle exposure. The heliocentric phases for the first and the last exposure, column (6), are referred to primary minimum; these and the epoch number in column (4) are based on Shapley's light elements given above.

All the plates were exposed about 1.5 mm. outside of focus and were sensitometrized, developed and measured as usual. They are Seed 27; for plates 183 to 245 the emulsion number is 14447; for 246 to 324, number 14540; for 346 to 456, number 14794; and for

TABLE II

PLATES OF RX HERCULIS

Plate No.	Date, G.M.T.	Julian Day	Epoch	Correction to Sun	Helioc. Phase	Tel.	Hour Angle	No. Exp.	Film	Measured by
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	1914 d. h. m.			days	d. to d.		h.			
183	May 24 16 56	2420277.7056	348	+0.0039	0.1497 0.2052	W	−3.5	9	17	BC
184	24 18 20	277.7639	348	39	0.2149 0.2705	W	−2.1	7	19	BC
191	June 13 16 32	297.6889	359	46	0.5725 0.6305	W	−2.6	12	18	BC
192	13 17 45	297.7396	359	46	0.6375 0.6652	W	−1.4	6	14	BC
197	17 15 41	301.6535	361	47	0.9770 1.0374	W	−3.2	12	26	BC
205	19 15 34	303.6486	362	47	1.2005 1.2463	W	−3.2	12	19	BC
206	19 16 49	303.7007	362	47	1.2526 1.2984	W	−1.9	12	clear	BC
207	19 18 03	303.7521	362	47	1.3039 1.3498	W	−0.7	12	16	BC
208	19 19 12	303.8000	362	47	1.3560 1.3935	W	+0.4	10	14	BC
209	19 20 32	303.8556	362	47	1.4074 1.4536	W	+1.8	12	22	BC
214	23 15 34	307.6486	364	48	1.6435 1.6893	W	−2.9	12	21	BC
215	23 16 48	307.7000	364	48	1.6990 1.7407	W	−1.7	11	clear	BC
216	23 18 03	307.7521	364	48	1.7469 0.0142	W	−0.4	12	clear	BC
217	23 19 18	307.8042	365	48	0.0204 0.0663	W	+0.8	11	clear	BC
218	23 20 33	307.8562	365	48	0.0722 0.1183	W	+2.1	12	14	BC
219	24 15 32	308.6472	365	48	0.8638 0.9097	W	−2.9	12	20	BC
220	24 16 42	308.6958	365	48	0.9159 0.9493	W	−1.7	9	clear	BC
242	July 17 15 30	331.6458	378	45	0.7407 0.7865	W	−1.4	12	clear	BC
243	17 16 45	331.6979	378	45	0.7924 0.8382	W	−0.2	12	clear	BC
244	17 19 12	331.8000	378	45	0.8945 0.9403	W	+2.3	12	clear	BC
245	17 20 10	331.8403	378	45	0.9473 0.9681	W	+3.3	6	clear	BC
246	19 15 30	333.6458	379	45	0.9618 1.0076	W	−1.3	12	18	BC
247	19 16 43	333.6965	379	45	1.0125 1.0583	W	−0.1	12	15	BC
248	19 17 56	333.7472	379	45	1.0632 1.1090	W	+1.2	12	13	BC
249	19 19 09	333.7979	379	45	1.1139 1.1597	W	+2.4	12	18	BC
250	19 20 04	333.8361	379	45	1.1646 1.1854	W	+3.3	6	clear	BC
252	22 15 55	336.6632	381	44	0.4226 0.4705	W	−0.6	9	clear	C
253	22 17 16	336.7194	381	44	0.4767 0.5267	W	+0.7	10	clear	C
254	22 18 29	336.7701	381	44	0.5330 0.5719	W	+1.9	8	clear	C
256	23 17 12	337.7167	381	44	1.4753 1.5212	E	+0.7	12	clear	C
257	23 18 25	337.7674	381	44	1.5260 1.5719	E	+1.9	11	clear	C
258	23 19 38	337.8181	381	44	1.5767 1.6225	E	+3.1	12	clear	C
261	24 16 48	338.7000	382	43	0.6800 0.7216	W	+0.4	7	13	C
263	25 15 32	339.6472	382	43	1.6202 1.6814	W	−0.9	12	18	C
264	25 16 45	339.6979	382	43	1.6876 1.7105	W	+0.4	5	clear	C
265	26 15 43	340.6549	383	43	0.8407 0.9022	W	−0.6	12	clear	C
297	Sept. 13 14 27	389.6021	411	15	0.0006 0.0465	E	+1.4	12	18	C
298	13 15 46	389.6569	411	15	0.0513 0.0972	E	+2.7	10	18	C
323	20 14 51	396.6188	414	09	1.6810 1.7269	E	+2.2	12	17	C
324	20 16 08	396.6722	414	+0.0009	1.7338 1.7762	E	+3.5	11	19	C
346	Oct. 10 13 40	416.5694	426	−0.0007	0.2872 0.3330	E	+2.3	12	26	C
347	1915 10 14 49	416.6174	426	−0.0007	0.3393 0.3768	E	+3.5	10	24	C
455	May 7 19 52	625.8278	543	+0.0030	1.4561 1.5019	W	−1.7	9	27	C
456	7 20 47	625.8660	543	30	1.5067 1.5234	W	−0.8	5	20	C
504	July 6 15 41	685.6535	577	47	0.8119 0.8536	W	−2.0	11	27	C
505	6 16 48	685.7000	577	47	0.8585 0.9001	W	−0.8	11	17	C

the last two, number 15194. An exposure of six minutes was found appropriate for all phases of the light variation, and twelve exposures are the maximum number allowable without blending of the stars to be measured. The total exposure on a plate is thus limited to 72 minutes and a high percentage of clear or nearly clear films is consequent. The numbers in column (10) are wedge readings on the film. Column (11) indicates by initial the measurer of the plate. As an experiment we both measured the first half of the plates, in order to ascertain whether the gain in accuracy might justify the additional labor of duplicate measurements and reductions. The conclusions regarding this procedure are given later.

The choice of comparison stars is readily made. The requirements are that they shall form a magnitude sequence with the variable star intermediate; that their images on the plate shall be generally within limits appropriate for accurate measurement; that they shall be well distributed around the variable and close to it, in order to minimize differential atmospheric absorption and errors in the reduction to center. Six com-

TABLE III

COMPARISON STARS FOR RX HERCULIS

Star	B.D.		R.A.(1900)	Decl.(1900)	Adopted Photographic Mag.		Harvard	
	No.	Mag.			Baker	Cummings	Visual Mag.	Class
			h m					
v	+12° 3557		18 26.0	+12° 33′				
a	11 3481	8.0	24.9	11 51	8.22	8.26		
b	12 3546	7.7	24.9	12 24	8.00	8.06		
c	13 3677	6.8	27.6	13 40	7.68	7.74		
d	13 3658	6.8	24.9	13 48	7.25	7.28		
e	11 3530	6.3	32.5	11 21	6.35	6.35	6.36	A
f	11 3442	6.0	18.0	11 59	5.90	5.90	5.89	A

parison stars, given in Table III, have been selected. Stars *e* and *f* have visual magnitudes 6.36 and 5.89 respectively in *Revised Harvard Photometry* and both are of class A. Their mean visual magnitude is adopted as the mean of their photographic magnitudes, whence the photographic magnitudes of the comparison stars in columns (6) and (7) of the table are deduced from the observed magnitude differences on the increased scale. (See *Laws Observatory Bulletin*, No. 24, Section VI). These differences are taken for both positions of the telescope, east and west of the pier, to eliminate the collimation error.

The measured densities of the extrafocal star images in terms of the microphometer wedge scale are converted to magnitudes by use of the corrected reduction curve (Table IX of *Bulletin* No. 24) for the emulsion concerned. Each observer has used his own reduction curve derived from sensitometer images on these plates. For plates affected by sky fog the reduction curve for the plate is employed up to the point where the fog curve joins the clear-film curve. After correction for atmospheric extinction and for reduction

to center, using Tables III and IV of *Bulletin* No. 24, the magnitude differences $a-v$, $b-v$, etc. are taken for each exposure; these are multiplied by the factor 1.15 in order to bring our scale into agreement with that of the Harvard Sequences. Employing the adopted photographic magnitudes of the comparison stars we now obtain the magnitude of the variable star for each comparison; and finally the mean of six values v_a, v_b, etc. as the observed magnitude of the variable star for each exposure.

The observed photographic magnitudes of RX Herculis are given in Table IV. For plates 183 to 250 they are the means of Cummings' and Baker's magnitudes; the remainder were measured only by Cummings. No correction is required to bring the results of the two observers into agreement. The coördinates of 480 points are available for the determination of the light curve. They are first assembled into 58 normal magnitudes, of which numbers 9 to 18 and 36 to 50 occur outside of eclipse. For convenience in the solutions the number of observations in each normal place without eclipse has been made nearly the same; so also for normal places within eclipse, the number of observations in the former being about twice that in the latter normals.

SOLUTIONS FOR ELEMENTS OF THE SYSTEM

The discussion of the normal magnitudes in Table V for the determination of the photometric orbital elements of RX Herculis follows the notation and formulæ developed at the Princeton Observatory.[1] As frequent reference will be made to tables and formulæ contained in this series of papers on the theory of eclipsing binaries, we shall distinguish them by italics, thus: *Table I*, *formula* (*c*), etc.

We have first to convert normal magnitudes into intensities by the formula

$$\log l = 0.4\ (m_0 - m)$$

where m_0, the magnitude during the intervals between eclipses, is taken to be $7^m.244$. The observed normal intensities read from *Table A*, but using the final value of m_0, appear in column (6) of Table V and are expressed in terms of the total light of the system. On plotting normal intensities against phase, column (2), we find that the time of primary minimum is in exact agreement with Shapley's elements and that the secondary minimum occurs exactly halfway between principal conjunctions; thus $e \cos \omega = 0$. The curves during minima are not symmetrical; also there are marked irregularities in the light curve between eclipses. Outside eclipse the intensities generally increase toward secondary minimum, showing that the light of the star eclipsed at this time is augmented by radiation of its brighter companion; slight upward convexity of the curve suggests a degree of ellipticity in the stars.

[1] H. N. Russell, *Astrophysical Journal*, **35**, 315, 1912; **36**, 54, 1912; H. N. Russell and H. Shapley, *Astrophysical Journal*, **36**, 239, 385, 1912.

TABLE IV

PHOTOGRAPHIC OBSERVATIONS OF RX HERCULIS

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O−C (darkened)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O−C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag.			2420	days		mag.
183	1	277.6778	0.1497	7.27	−.03	205	9	303.6611	1.2338	7.26	.00
	2	.6847	.1566	7.27	−.03		10	.6653	.2380	7.27	+.01
	3	.6917	.1636	7.34	+.04		11	.6694	.2421	7.28	+.02
	4	.6986	.1705	7.32	+.02		12	.6736	.2463	7.27	+.01
	5	.7056	.1775	7.32	+.03						
	6	.7125	.1844	7.29	.00	206	1	303.6799	1.2526	7.29	+.03
	7	.7194	.1913	7.30	+.01		2	.6840	.2567	7.29	+.03
	8	.7264	.1983	7.32	+.03		3	.6882	.2609	7.26	.00
	9	.7333	.2052	7.28	−.01		4	.6924	.2651	7.32	+.06
							5	.6965	.2692	7.26	.00
184	1	277.7430	0.2149	7.29	.00		6	.7007	.2734	7.29	+.03
	2	.7500	.2219	7.26	−.03		7	.7049	.2776	7.29	+.03
	3	.7569	.2288	7.28	−.01		8	.7090	.2817	7.29	+.03
	4	.7639	.2358	7.26	−.03		9	.7132	.2859	7.25	−.01
	6	.7778	.2497	7.25	−.03		10	.7174	.2901	7.27	+.01
	7	.7847	.2566	7.20	−.08		11	.7215	.2942	7.33	+.07
	9	.7986	.2705	7.22	−.06		12	.7257	.2984	7.25	−.01
191	1	297.6642	0.5725	7.29	+.03	207	1	303.7312	1.3039	7.27	+.01
	2	.6691	.5774	7.27	+.01		2	.7354	.3081	7.29	+.03
	3	.6740	.5823	7.29	+.03		3	.7396	.3123	7.29	+.03
	4	.6788	.5871	7.29	+.03		4	.7438	.3165	7.28	+.02
	5	.6826	.5909	7.29	+.03		5	.7479	.3206	7.32	+.06
	6	.6889	.5972	7.29	+.03		6	.7521	.3248	7.33	+.07
	7	.6944	.6027	7.25	−.01		7	.7562	.3289	7.34	+.08
	8	.7000	.6083	7.27	+.01		8	.7604	.3331	7.32	+.06
	9	.7056	.6139	7.21	−.05		9	.7646	.3373	7.32	+.06
	10	.7111	.6194	7.32	+.06		10	.7688	.3415	7.34	+.08
	11	.7167	.6250	7.25	−.01		11	.7729	.3456	7.30	+.04
	12	.7222	.6305	7.29	+.03		12	.7771	.3498	7.29	+.03
192	1	297.7292	0.6375	7.20	−.06	208	1	303.7833	1.3560	7.24	−.02
	2	.7340	.6423	7.27	+.01		2	.7875	.3602	7.29	+.02
	3	.7396	.6479	7.25	−.01		3	.7917	.3644	7.32	+.05
	4	.7458	.6541	7.25	−.01		4	.7958	.3685	7.29	+.02
	5	.7514	.6597	7.27	.00		5	.8000	.3727	7.28	+.01
	6	.7569	.6652	7.25	−.02		6	.8042	.3769	7.27	.00
							7	.8083	.3810	7.27	.00
197	1	301.6257	0.9770	7.41	+.09		8	.8125	.3852	7.29	+.02
	2	.6312	.9825	7.36	+.05		9	.8167	.3894	7.25	−.02
	3	.6368	.9881	7.37	+.07		10	.8208	.3935	7.26	−.01
	4	.6424	.9937	7.34	+.06						
	5	.6478	.9991	7.32	+.04	209	1	303.8347	1.4074	7.22	−.05
	6	.6535	1.0048	7.34	+.07		2	.8389	.4116	7.22	−.05
	7	.6590	.0103	7.32	+.05		3	.8430	.4157	7.22	−.05
	8	.6653	.0166	7.34	+.07		4	.8472	.4199	7.29	+.02
	9	.6701	.0214	7.34	+.07		5	.8514	.4241	7.27	.00
	10	.6757	.0270	7.34	+.07		6	.8556	.4283	7.25	−.02
	11	.6809	.0322	7.32	+.05		7	.8597	.4324	7.28	+.01
	12	.6861	.0374	7.37	+.10		8	.8639	.4366	7.28	+.01
							9	.8680	.4407	7.27	.00
205	1	303.6278	1.2005	7.29	+.03		10	.8722	.4449	7.29	+.02
	2	.6319	.2046	7.25	−.01		11	.8764	.4491	7.26	−.01
	3	.6361	.2088	7.25	−.01		12	.8809	.4536	7.27	.00
	4	.6403	.2130	7.25	−.01						
	5	.6444	.2171	7.28	+.02	214	1	307.6278	1.6435	7.35	+.05
	6	.6486	.2213	7.30	+.04		2	.6319	.6476	7.34	+.04
	7	.6528	.2255	7.26	.00		3	.6361	.6518	7.34	+.04
	8	.6569	.2296	7.27	+.01		4	.6403	.6560	7.32	+.02

TABLE IV—Continued
PHOTOGRAPHIC OBSERVATIONS OF RX HERCULIS

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O−C (darkened)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O−C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag.			2420	days		mag.
214	5	307.6444	1.6601	7.32	+.02	219	1	308.6267	0.8638	7.65	+.01
	6	.6486	.6643	7.34	+.03		2	.6309	.8680	7.68	+.02
	7	.6528	.6685	7.33	+.02		3	.6351	.8722	7.67	−.01
	8	.6569	.6726	7.36	+.04		4	.6392	.8763	7.68	−.01
	9	.6611	.6768	7.33	.00		5	.6434	.8805	7.72	+.02
	10	.6653	.6810	7.32	−.02		6	.6476	.8847	7.68	−.03
	11	.6694	.6851	7.35	.00		7	.6517	.8888	7.68	−.03
	12	.6736	.6893	7.37	+.01		8	.6559	.8930	7.70	−.01
							9	.6601	.8972	7.64	−.06
215	2	307.6833	1.6990	7.43	+.03		10	.6642	.9013	7.67	−.02
	3	.6875	.7032	7.36	−.06		11	.6684	.9055	7.61	−.06
	4	.6917	.7074	7.50	+.06		12	.6726	.9097	7.61	−.04
	5	.6958	.7115	7.48	+.01						
	6	.7000	.7157	7.50	.00	220	1	308.6788	0.9159	7.61	−.01
	7	.7042	.7199	7.55	+.03		2	.6830	.9201	7.60	.00
	8	.7083	.7240	7.59	+.04		3	.6872	.9243	7.61	+.03
	9	.7125	.7282	7.59	+.02		4	.6913	.9284	7.59	+.03
	10	.7167	.7324	7.59	−.01		5	.6955	.9326	7.51	−.01
	11	.7208	.7365	7.66	+.03		6	.6997	.9368	7.50	.00
	12	.7250	.7407	7.71	+.04		7	.7038	.9409	7.50	+.02
							8	.7080	.9451	7.50	+.04
216	1	307.7312	1.7469	7.72	+.02		9	.7122	.9493	7.48	+.04
	2	.7354	.7511	7.73	.00						
	3	.7396	.7553	7.76	.00	242	1	331.6254	0.7407	7.29	+.02
	4	.7438	.7595	7.80	+.01		2	.6295	.7448	7.27	.00
	5	.7479	.7636	7.83	+.03		3	.6337	.7490	7.29	+.02
	6	.7521	.7678	7.82	+.01		4	.6378	.7531	7.29	+.02
	7	.7562	.7719	7.82	−.01		5	.6420	.7573	7.27	.00
	8	.7604	.7761	7.80	−.03		6	.6462	.7615	7.28	+.01
	9	.7646	0.0017	7.80	−.03		7	.6504	.7657	7.26	−.01
	10	.7688	.0059	7.84	+.01		8	.6545	.7698	7.27	.00
	11	.7729	.0100	7.82	.00		9	.6587	.7740	7.27	.00
	12	.7771	.0142	7.76	−.04		10	.6628	.7781	7.27	−.01
							11	.6670	.7823	7.25	−.03
217	1	307.7833	0.0204	7.87	+.10		12	.6712	.7865	7.25	−.04
	2	.7875	.0246	7.82	+.08						
	3	.7917	.0288	7.75	+.02	243	1	331.6771	0.7924	7.29	−.01
	4	.7958	.0329	7.71	+.01		2	.6812	.7965	7.29	−.02
	6	.8042	.0413	7.64	.00		3	.6854	.8007	7.34	+.01
	7	.8083	.0454	7.60	−.01		4	.6896	.8049	7.36	+.02
	8	.8125	.0496	7.59	+.01		5	.6937	.8090	7.34	−.01
	9	.8167	.0538	7.59	+.04		6	.6979	.8132	7.36	−.01
	10	.8208	.0579	7.48	−.04		7	.7021	.8174	7.36	−.03
	11	.8250	.0621	7.45	−.04		8	.7062	.8215	7.38	−.03
	12	.8292	.0663	7.45	−.02		9	.7104	.8257	7.43	.00
							10	.7146	.8299	7.45	.00
218	1	307.8351	0.0722	7.41	−.03		11	.7188	.8341	7.47	.00
	2	.8396	.0767	7.36	−.05		12	.7229	.8382	7.48	−.01
	3	.8438	.0809	7.35	−.04						
	4	.8478	.0850	7.36	−.02	244	1	331.7792	0.8945	7.71	.00
	5	.8521	.0892	7.32	−.04		2	.7833	.8986	7.67	−.03
	6	.8562	.0933	7.29	−.06		3	.7875	.9028	7.65	−.03
	7	.8604	.0975	7.22	−.11		4	.7917	.9070	7.66	−.01
	8	.8646	.1017	7.22	−.11		5	.7958	.9111	7.65	.00
	9	.8688	.1059	7.27	−.05		6	.8000	.9153	7.59	−.04
	10	.8729	.1100	7.27	−.04		7	.8042	.9195	7.61	+.01
	11	.8771	.1142	7.32	+.01		8	.8083	.9236	7.59	+.01
	12	.8812	.1183	7.26	−.05		9	.8125	.9278	7.55	−.01
							10	.8167	.9320	7.52	−.01

TABLE IV—Continued
PHOTOGRAPHIC OBSERVATIONS OF RX HERCULIS

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O−C (darkened)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O−C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag.			2420	days		mag.
244	11	331.8208	0.9361	7.53	+.02	249	11	333.8188	1.1556	7.27	+.01
	12	8250	9403	7.50	+.02		12	.8229	.1597	7.25	−.01
245	1	331.8320	0.9473	7.48	+.03	250	1	333.8278	1.1646	7.29	+.03
	2	.8361	.9514	7.47	+.04		2	.8320	.1688	7.25	−.01
	3	.8403	.9556	7.44	+.03		3	.8361	.1729	7.18	+.02
	4	.8444	.9597	7.43	+.04		4	.8403	.1771	7.29	+.03
	5	.8486	.9639	7.40	+.02		5	.8444	.1812	7.27	+.01
	6	.8528	.9681	7.34	−.01		6	.8486	.1854	7.25	−.01
246	1	333.6250	0.9618	7.34	−.04	252	1	3 6.6431	0.4226	7.27	.00
	2	.6292	.9660	7.36	.00		2	.6472	.4267	7.28	+.01
	3	.6333	.9701	7.29	−.06		3	.6521	.4316	7.22	−.05
	4	.6375	.9743	7.34	.00		5	.6632	.4427	7.29	+.03
	5	.6417	.9785	7.30	−.02		6	.6688	.4483	7.25	−.01
	6	.6458	.9826	7.28	−.03		7	.6743	.4538	7.25	−.01
	7	.6500	.9868	7.26	−.04		8	.6799	.4594	7.22	−.04
	8	.6542	.9910	7.25	−.04		9	.6854	.4649	7.25	−.01
	9	.6583	.9951	7.26	−.02		10	.6910	.4705	7.24	−.02
	10	.6625	.9993	7.25	−.03						
	11	.6667	1.0035	7.27	.00	253	1	336.6972	0.4767	7.26	.00
	12	.6708	.0076	7.29	+.02		2	.7028	.4823	7.27	+.01
							3	.7083	.4878	7.24	−.02
247	1	333.6757	1.0125	7.28	+.01		4	.7139	.4934	7.26	.00
	2	.6799	.0167	7.24	−.03		5	.7194	.4989	7.22	−.04
	3	.6840	.0208	7.25	−.02		6	.7250	.5045	7.21	−.05
	4	.6882	.0250	7.25	−.02		7	.7306	.5101	7.25	−.01
	5	.6924	.0292	7.27	.00		8	.7361	.5156	7.26	.00
	6	.6965	.0333	7.25	−.02		9	.7417	.5212	7.27	+.01
	7	.7007	.0375	7.25	−.02		10	.7472	.5267	7.24	−.02
	8	.7049	.0417	7.25	−.02						
	9	.7090	.0458	7.25	−.02	254	1	336.7535	0.5330	7.28	+.02
	10	.7132	.0500	7.25	−.02		2	.7590	.5385	7.22	−.04
	11	.7174	.0542	7.26	−.01		3	.7646	.5441	7.22	−.04
	12	.7215	.0583	7.25	−.02		4	.7701	.5496	7.25	−.01
							5	.7757	.5552	7.24	−.02
248	1	333.7264	1.0632	7.25	−.02		6	.7812	.5607	7.25	−.01
	2	.7306	.0674	7.25	−.02		7	.7868	.5663	7.26	.00
	3	.7347	.0715	7.25	−.02		8	.7924	.5719	7.30	+.04
	4	.7389	.0757	7.26	−.01						
	5	.7430	.0798	7.24	−.03	256	1	337.6958	1.4753	7.22	−.06
	6	.7472	.0840	7.24	−.03		2	.7000	.4795	7.30	+.02
	7	.7514	.0882	7.27	.00		3	.7042	.4837	7.27	−.01
	8	.7556	.0924	7.25	−.02		4	.7083	.4878	7.30	+.02
	9	.7597	.0965	7.21	−.05		5	.7125	.4920	7.27	−.01
	10	.7639	.1007	7.24	−.02		6	.7167	.4962	7.29	+.01
	11	.7681	.1049	7.28	+.02		7	.7208	.5003	7.27	−.01
	12	.7722	.1090	7.27	+.01		8	.7250	.5045	7.26	−.02
							9	.7292	.5087	7.26	−.02
249	1	333.7771	1.1139	7.27	+.01		10	.7333	.5128	7.27	−.01
	2	.7812	.1180	7.27	+.01		11	.7375	.5170	7.25	−.03
	3	.7854	.1222	7.27	+.01		12	.7417	.5212	7.27	−.01
	4	.7896	.1264	7.26	.00						
	5	.7938	.1306	7.26	.00	257	1	337.7465	1.5260	7.24	−.04
	6	.7979	.1347	7.24	−.02		2	.7507	.5302	7.24	−.04
	7	.8021	.1389	7.25	−.01		3	.7549	.5344	7.25	−.03
	8	.8062	.1430	7.25	−.01		4	.7590	.5385	7.21	−.07
	9	.8104	.1472	7.22	−.04		5	.7632	.5427	7.21	−.08
	10	.8146	.1514	7.25	−.01		6	.7674	.5469	7.20	−.09

TABLE IV—Continued
PHOTOGRAPHIC OBSERVATIONS OF RX HERCULIS

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O−C (darkened)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O−C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag.			2420	days		mag.
257	7	337.7715	1.5510	7.22	−.07	297	1	389.5812	0.0006	7.84	+.01
	8	.7757	.5552	7.21	−.08		2	.5854	.0048	7.87	+.04
	9	.7799	1.5594	7.20	−.09		3	.5896	.0090	7.90	+.08
	11	.7882	.5677	7.20	−.09		4	.5938	.0132	7.80	.00
	12	.7924	.5719	7.26	−.03		5	.5979	.0173	7.88	+.09
							6	.6021	.0215	7.73	−.03
258	1	337.7972	1.5767	7.40	+.11		7	.6062	.0256	7.72	−.02
	2	.8014	.5809	7.34	+.05		8	.6104	.0298	7.68	−.03
	3	.8056	.5851	7.34	+.05		9	.6146	.0340	7.66	− 03
	4	.8097	.5892	7.34	+.05		10	.6188	.0382	7.59	−.07
	5	.8139	.5934	7.35	+.06		11	.6229	.0423	7.66	+.03
	6	.8181	.5976	7.35	+.06		12	.6271	.0465	7.52	−.09
	7	.8222	.6017	7.37	+.08						
	8	.8264	.6059	7.35	+.06	298	1	389.6319	0.0513	7.60	+.03
	9	.8306	.6101	7.34	+.04		2	.6361	.0555	7.40	−.13
	10	.8347	.6142	7.28	−.02		3	.6403	.0597	7.49	−.02
	11	.8389	.6184	7.35	+.05		4	.6438	.0632	7.55	+.06
	12	.8430	.6225	7.30	.00		5	.6569	.0763	7.43	+.01
							6	.6611	.0805	7.45	+.05
261	1	338.6792	0.6800	7.27	.00		7	.6653	.0847	7.44	+.06
	2	.6861	.6869	7.22	−.05		8	.6694	.0888	7.48	+.11
	3	.6931	.6939	7.25	−.02		9	.6736	.0930	7.34	−.01
	4	.7000	.7008	7.25	−.02		10	.6778	.0972	7.45	+.11
	5	.7070	.7078	7.26	−.01						
	6	.7139	.7147	7.28	+.01	323	1	396.5979	1.6810	7.32	−.02
	7	.7208	.7216	7.21	−.06		2	.6021	.6852	7.32	−.03
							3	.6062	.6893	7.29	−.07
263	1	339.6194	1.6202	7.35	+.05		4	.6104	.6935	7.29	−.09
	2	.6250	.6258	7.32	+.02		5	.6146	.6977	7.36	−.04
	3	.6306	.6314	7.32	+.02		6	.6188	.7019	7.37	−.05
	4	.6361	.6369	7.35	+.05		7	.6229	.7060	7.43	−.01
	5	.6417	.6425	7.30	.00		8	.6271	.7102	7.50	+.04
	6	.6472	.6480	7.29	−.01		9	.6312	.7143	7.48	−.01
	7	.6528	.6536	7.30	.00		10	.6354	.7185	7.43	−.08
	8	.6583	.6591	7.32	+.02		11	.6396	.7227	7.47	−.07
	9	.6639	.6647	7.30	−.01		12	.6438	.7269	7.47	−.10
	10	.6694	.6702	7.27	−.05						
	11	.6750	.6758	7.29	−.04	324	1	396.6507	1.7338	7.64	+.02
	12	.6806	.6814	7.29	−.05		2	.6549	.7380	7.64	.00
							3	.6590	.7421	7.61	−.06
264	1	339.6868	1.6876	7.38	+.02		4	.6636	.7467	7.70	.00
	2	.6924	.6932	7.43	+.05		5	.6681	.7512	7.75	+.01
	3	.6979	.6987	7.44	+.04		6	.6722	.7553	7.73	−.03
	4	.7035	.7043	7.56	+.13		7	.6764	.7595	7.78	.00
	5	.7097	.7105	7.47	+.01		8	.6806	.7637	7.75	−.05
							9	.6847	.7678	7.75	−.06
265	1	340.6184	0.8407	7.52	+.01		10	.6889	.7720	7.78	−.04
	2	.6257	.8480	7.61	+.06		11	.6931	.7762	7.89	+.06
	3	.6299	.8522	7.59	+.01						
	4	.6337	.8560	7.51	−.09	346	1	416.5486	0.2872	7.24	−.04
	5	.6507	.8730	7.64	−.04		2	.5528	.2914	7.22	−.06
	6	.6549	.8772	7.68	−.01		3	.5570	.2956	7.24	−.04
	7	.6590	.8813	7.72	+.02		4	.5611	.2997	7.22	−.06
	8	.6632	.8855	7.71	.00		5	.5653	.3039	7.20	−.08
	9	.6674	.8897	7.67	−.04		6	.5694	.3080	7.27	−.01
	10	.6715	.8938	7.71	.00		7	.5736	.3122	7.28	.00
	11	.6757	.8980	7.67	−.03		8	.5778	.3164	7.24	−.04
	12	.6799	.9022	7.67	−.01		9	.5820	.3206	7.28	.00

TABLE IV—Continued
PHOTOGRAPHIC OBSERVATIONS OF RX HERCULIS

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O−C (darkened)	Plate No.	Epx. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O−C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag.			2420	days		mag.
346	10	416.5861	0.3247	7.21	−.06	456	3	625.8660	1.5151	7.29	+.01
	11	.5903	.3289	7.19	−.08		4	.8701	.5192	7.34	+.06
	12	.5944	.3330	7.21	−.06		5	.8743	.5234	7.29	+.01
347	1	416.6007	0.3393	7.26	−.01	504	1	685.6326	0.8119	7.40	+.03
	2	.6049	.3435	7.29	+.02		2	.6368	.8161	7.42	+.04
	3	.6090	.3476	7.33	+.06		3	.6410	.8203	7.41	+.01
	4	.6132	.3518	7.32	+.05		4	.6451	.8244	7.47	+.05
	5	.6174	.3560	7.25	−.02		5	.6493	.8286	7.47	+.03
	6	.6215	.3601	7.29	+.02		6	.6535	.8328	7.47	.00
	7	.6257	.3643	7.32	+.05		7	.6576	.8369	7.50	+.02
	8	.6299	.3685	7.36	+.09		8	.6618	.8411	7.53	+.02
	9	.6340	.3726	7.36	+.09		9	.6660	.8453	7.55	+.02
	10	.6382	.3768	7.32	+.05		10	.6701	.8494	7.60	+.04
							11	.6743	.8536	7.61	+.02
455	1	625.8070	1.4561	7.32	+.04						
	2	.8111	.4602	7.32	+.04	505	1	685.6792	0.8585	7.63	+.02
	3	.8153	.4644	7.32	+.04		2	.6833	.8626	7.68	+.04
	4	.8194	.4685	7.30	+.02		3	.6875	.8678	7.63	−.03
	5	.8236	.4727	7.32	+.04		4	.6917	.8710	7.70	+.03
	6	.8278	.4769	7.32	+.04		5	.6958	.8751	7.70	+.01
	7	.8320	.4811	7.34	+.06		6	.7000	.8793	7.70	.00
	8	.8361	.4852	7.32	+.04		7	.7042	.8835	7.70	.00
	12	.8528	.5019	7.29	+.01		8	.7083	.8876	7.71	.00
							9	.7125	.8918	7.70	−.01
456	1	625.8576	1.5067	7.29	+.01		10	.7167	.8960	7.71	+.01
	2	.8618	.5109	7.34	+.06		11	.7208	.9001	7.68	−.01

TABLE V
NORMAL MAGNITUDES OF RX HERCULIS

No.	Phase	θ	No. Obs.	Observed Magnitude	Observed Intensity	Rectified Intensity	Computed Magnitude (uniform)	O−C (uniform)	Computed Magnitude (darkened)	O−C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10	(11)
	days									
1	0.0053	1°.07	6	7.847	0.585	0.622	7.836	+.011	7.828	+.019
2	0.0173	3.50	5	7.809	0.605	0.642	7.788	+.021	7.791	+.018
3	0.0293	5.93	6	7.724	0.655	0.692	7.711	+.013	7.720	+.004
4	0.0427	8.64	5	7.603	0.732	0.770	7.618	−.015	7.626	−.023
5	0.0546	11.05	6	7.525	0.786	0.824	7.541	−.016	7.542	−.017
6	0.0695	14.07	6	7.442	0.849	0.897	7.468	−.026	7.465	−.023
7	0.0848	17.16	6	7.400	0.882	0.918	7.381	+.019	7.378	+.022
8	0.0965	19.53	5	7.306	0.962	0.998	7.340	−.034	7.340	−.034
9	0.1493	30.22	11	7.293	0.975	1.007	7.300	−.007		
10	0.2369	47.95	10	7.258	1.006	1.028	7.288	−.030		
11	0.3060	61.94	8	7.244	1.018	1.032	7.279	−.035		
12	0.3513	71.11	13	7.285	0.982	0.990	7.273	+.012		
13	0.4527	91.63	11	7.254	1.010	1.010	7.264	−.010		
14	0.5101	103.25	9	7.247	1.016	1.014	7.362	−.015		
15	0.5641	114.18	11	7.263	1.002	1.000	7.262	+.001		

TABLE V—Continued

NORMAL MAGNITUDES OF RX HERCULIS

No.	Phase	θ	No. Obs.	Observed Magnitude	Observed Intensity	Rectified Intensity	Computed Magnitude (uniform)	O−C (uniform)	Computed Magnitude (darkened)	O−C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	days									
16	0.6196	125.42	11	7.263	1.002	1.000	7.263	.000		
17	0.6848	138.61	9	7.255	1.009	1.011	7.266	−.011		
18	0.7538	152.58	10	7.271	0.994	1.000	7.270	+.001		
19	0.7916	160.23	7	7.293	0.974	0.980	7.300	−.007	7.300	−.007
20	0.8146	164.88	6	7.382	0.897	0.905	7.379	+.003	7.376	−.006
21	0.8260	167.19	5	7.440	0.850	0.856	7.428	+.012	7.427	+.013
22	0.8373	169.48	6	7.494	0.809	0.815	7.486	+.008	7.488	+.006
23	0.8519	172.43	7	7.587	0.743	0.750	7.567	+.020	7.574	+.013
24	0.8676	175.61	6	7.669	0.689	0.695	7.655	+.014	7.658	+.011
25	0.8769	177.49	6	7.686	0.678	0.684	7.695	−.009	7.690	−.004
26	0.8852	179.17	6	7.699	0.670	0.676	7.713	−.014	7.705	−.006
27	0.8926	180.67	5	7.695	0.672	0.678	7.714	−.019	7.705	−.010
28	0.8985	181.86	6	7.674	0.686	0.692	7.701	−.027	7.698	−.024
29	0.9064	183.46	6	7.643	0.705	0.712	7.673	−.030	7.673	−.030
30	0.9198	186.18	6	7.605	0.730	0.736	7.605	.000	7.608	−.003
31	0.9323	188.71	6	7.534	0.780	0.786	7.534	.000	7.539	−.005
32	0.9457	191.42	6	7.486	0.815	0.822	7.463	+.023	7.463	+.023
33	0.9625	194.82	6	7.385	0.895	0.903	7.385	.000	7.384	+.001
34	0.9775	197.85	6	7.331	0.940	0.948	7.329	+.002	7.325	+.006
35	0.9933	201.05	7	7.291	0.975	0.981	7.285	+.006	7.286	+.005
36	1.0163	205.71	12	7.293	0.975	0.980	7.270	+.023		
37	1.0494	212.41	12	7.264	1.000	1.004	7.268	−.004		
38	1.0966	221.96	11	7.252	1.012	1.014	7.266	−.014		
39	1.1453	231.82	12	7.254	1.010	1.009	7.263	−.009		
40	1.2007	243.03	11	7.260	1.003	1.002	7.263	−.003		
41	1.2538	253.78	12	7.279	0.987	0.985	7.262	+.017		
42	1.3054	264.23	12	7.291	0.976	0.975	7.263	+.028		
43	1.3551	274.29	11	7.296	0.971	0.972	7.265	+.031		
44	1.4080	284.99	11	7.257	1.007	1.013	7.270	−.013		
45	1.4582	295.16	12	7.290	0.977	0.988	7.276	+.014		
46	1.4940	302°40	12	7.289	0.978	0.994	7.281	+.008		
47	1.5260	308.88	13	7.260	1.003	1.024	7.287	−.027		
48	1.5775	319.30	12	7.298	0.970	0.998	7.295	+.003		
49	1.6247	328.86	11	7.328	0.942	0.973	7.298	+.030		
50	1.6597	335.94	12	7.318	0.952	0.987	7.303	+.015		
51	1.6832	340.69	10	7.326	0.944	0.980	7.344	−.018	7.342	−.016
52	1.6964	343.37	5	7.392	0.889	0.925	7.393	−.001	7.387	+.005
53	1.7062	345.35	7	7.455	0.839	0.875	7.442	+.013	7.436	+.019
54	1.7181	347.76	7	7.498	0.806	0.844	7.506	−.008	7.507	−.009
55	1.7326	350.69	6	7.597	0.736	0.773	7.597	.000	7.602	−.005
56	1.7464	353.49	6	7.703	0.667	0.705	7.690	+.013	7.698	+.005
57	1.7595	356.14	6	7.776	0.624	0.661	7.776	.000	7.777	−.001
58	1.7720	358.67	6	7.810	0.605	0.642	7.831	−.021	7.825	−.015

The intensity l of the light outside eclipse is represented by the expression:

$$l = a - b\cos\theta - c\cos^2\theta$$

in which a is the intensity after correction for reflection and ellipticity, $2b$ is the difference in intensity between the two sides of the star eclipsed at secondary conjunction, $c = \frac{1}{2}z$ is the ratio by which the major axes exceed the minor axes of the prolate spheroidal stars, and $\theta = \frac{2\pi}{P}\cdot t$ is the longitude from principal conjunction. A preliminary solution of the normal intensities is made to determine approximate values of the constants. Plotting the observed l^2 against $\cos^2\theta$, we find the points represented by a straight line whose slope, z, is 0.069; thus $c = 0.0345$. The constant b is determined by comparing pairs of normal intensities outside eclipse, for which the values of $\cos\theta$ are near unity and about the same numerically, but opposite in sign. One-half the mean of these differences, each divided by $\cos\theta$, gives $b = 0.0172$. By substitution in the equation given above we have, in the mean, $a = 0.985$. With the preliminary constants the intensities of the normal places are computed by the formula.

We have for each normal intensity an equation of condition of the form:

$$\delta a - \cos\theta\cdot\delta b - \cos^2\theta\cdot\delta c = l_o - l_c$$

All the normals outside eclipse receive weight unity. The resulting normal equations are:

$$\begin{aligned} +25.000\,\delta a + 0.037\,\delta b - 9.255\,\delta c &= +0.0126 \\ +9.255\,\delta b - 0.078\,\delta c &= -0.0106 \\ +5.400\,\delta c &= -0.0231 \end{aligned}$$

whence we find:

	Preliminary	Correction	Final	Probable Error	Weights
a	$=0.985$	$-0.0030=$	0.982	± 0.0038	9.137
b	$=0.0172$	$-0.0012=$	0.016	± 0.0037	9.253
c	$=0.0345$	$-0.0094=$	0.025	± 0.0081	1.974
m_0	$=7^m.244$	$+0^m.020=$	$7^m.264$	$\pm 0^m.0041$	

The correction to m_0 is the magnitude difference corresponding to the intensity difference between the final value of a and unity. The final m_0 is the magnitude of the system between eclipses, after the effects of radiation and ellipticity have been eliminated. The probable errors of the constants are found from the weights in the solution. The probable error of a normal magnitude, weight 1, is $\pm\ 0^m.012$, and of a normal intensity is ± 0.011. It is unsafe to compute the probable error of a single observation from that of a normal place, since there is conspicuous system in their arrangement. We have now established the existence and amounts of the ellipticity of the stars and of the radiation effect; and have completed the solution of the light curve independent of the eclipses.

The effect of ellipticity and the radiation effect are removed from the normal intensities during eclipse by use of the formula:

$$R=\frac{l+b(1+\cos\theta)+c\cdot l\cdot\cos^2\theta}{a+b}$$

where R is the rectified intensity. These values appear in the seventh column of Table V. It remains to determine the orbital elements of the system in accordance with the theory.

We assume at first that the disks of the two stars are uniformly luminous. The losses of light at primary and secondary minimum, in terms of the total light of the system, are taken respectively to be $1-\lambda_b=0.375$ and $1-\lambda_f=0.335$. The two minima have so nearly the same depth that their normal places may enter the solution with equal weight. Accordingly we increase the scale of the intensities on the secondary minimum, multiplying them by 1.119. A symmetrical curve is drawn to represent the plotted points so combined.

We now read from the curve the phase, t', for twelve fractions, n, of the greatest loss of light and compute $\sin^2\theta(n)$ for each one. By use of *formula* (g'):

$$\frac{\sin^2\theta(n)}{1-z\cos^2\theta(n)}=C\omega_2(n)+D\omega_1(n),\quad \frac{C}{D}=\chi(k,\alpha_0,\tfrac{1}{4})$$

where k is the ratio of the radii of the two stars, α_0 is the greatest loss of light at minimum, and the functions $\omega_1(n)$ and $\omega_2(n)$ are taken from *Table IIIa*, we find:

$$C=0.0764,\qquad D=0.0390,\qquad \chi(k,\ \alpha_0,\ \tfrac{1}{4})=1.96.$$

On the whole this appears to be the most satisfactory result, but without altering the curve enough to seriously misrepresent the normal places, the value of $\chi(k,\alpha_0,\frac{1}{4})$ may be varied by several hundredths. The degree of indetermination in the solution thus depends on the rapidity with which the χ function changes as k is varied.

In Table VI we have given the value of $\chi(k,\ \alpha_0,\ \frac{1}{4})$, the corresponding intensity of the star eclipsed at primary minimum in terms of the total, and the difference in magnitude between the two stars, for possible combinations of k and α_0 between the limits of grazing total and grazing annular at primary minimum. The table is derived from *Table III* and *formulæ* (j) and (k):

$$\alpha_0=1-\lambda_1+\frac{1-\lambda_2}{k^2};\quad 1-\lambda_1=\alpha_0 L_2,\quad L_2=1-L_1$$

Where L_1 is the light of the larger star, L_2 the light of the smaller star. The χ function thus changes slowly and the solution would be highly indeterminate if it were not possible to make certain reasonable assumptions. The description of the spectrum indicates that the star eclipsed at principal conjunction is the fainter. But this star has the greater

surface brightness and probably the greater mass. In all other cases of binary stars where this information is available, the brighter star is the more massive and usually has the greater surface intensity. Moreover, we have found clear evidence of the greater radiation effect of this star on its companion. It has been pointed out above that the testimony of the spectrograms may be misleading in this case wherein the lines of the two components have different degrees of diffuseness.

On the assumption that the star eclipsed at primary minimum is the brighter, we see from the fifth column of Table VI that k must be between 1.00 and 0.945, if this star is the smaller; and from the sixth column, that k in all probability is between 1.00 and 0.95, if this star is the larger. For, in the latter event, if k is less than 0.95 the difference in magnitude between the two stars exceeds $0^m.24$. This would be in serious conflict with the evidence of the spectrograms, on which the components appear of nearly equal intensity. The approximate value of k thus lies between 0.945 on one side of unity and

TABLE VI

POSSIBLE SOLUTIONS (UNIFORM)

Star Eclipsed at Primary Minimum	k	α_0	$\chi(k, \alpha_0, \frac{1}{4})$	Light of Star Eclipsed at Primary Minimum	Magnitude Difference
Smaller star	0.732	1.00	1.883	0.375	0.55
	0.80	0.898	1.949	0.418	0.26
	0.90	0.789	1.960	0.475	0.11
	0.945	0.750	1.964	0.500	0.00
	1.00	0.710	1.966	0.528	0.12
Larger star	0.95	0.751	1.969	0.554	0.24
	0.90	0.798	1.967	0.580	0.35
	0.80	0.921	1.965	0.636	0.61
	0.751	1.00	1.890	0.665	0.74

0.95 on the other. Having no preference within these limits, we take $k=1.00$; whence $\alpha_0=0.710$ and $\chi\ (k, \alpha_0, \frac{1}{4})=1.966$.

The solution for stars completely darkened at the limb is readily made in this case. We take $k=1.00$, as before, and decrease the depths of the minima a little, so the $1-\lambda_b=0.370$ and $1-\lambda_f=0.330$; then $\alpha_0=0.700$. From *Table IIIx* we find:

$$\chi(k, \alpha_0, 0)=4.43, \quad \chi(k, \alpha_0, \tfrac{1}{4})=1.884, \quad \chi(k, \alpha_0, \tfrac{3}{4})=0.413$$

and from the light curve $\dfrac{\sin^2\theta(\frac{1}{2})}{1-z\cos^2\theta}=0.0400$. In addition, of course, $\sin^2\theta(1)=0$.

The phase in each case is obtained by *formula* (g'). The theoretical curve for darkened stars may now be drawn from the five points thus derived; it differs slightly from the curve for the uniform solution.

The remaining elements for both uniform and darkened solutions are now computed. The orbital inclination, i, and the semi-major axis of either star, $a_b = a_f$ are deduced by *formula* (h'):

$$\cos^2 i \cos^2\theta' + \sin^2\theta' = a_b^2(1 - z\cos^2\theta')(1+k)^2$$
$$\cos^2 i = a_b^2(1-z)[1+kp(k, \alpha_0)]^2$$

where $\theta' = \theta(0)$, $z = 2c$ and $\frac{5}{4}c$ for uniform and darkened solutions respectively, and the p functions are taken from *Tables I* and *Ix*. The ratio $\frac{J_1}{J_2}$ of the surface intensities of the two stars, and the densities ρ_b and ρ_f referred to the sun's density are found by the aid of *formulæ* (s) and (t):

$$\frac{J_b}{J_f} = \frac{K^2 L_1}{L_2}$$

$$\rho_b = 0.01344 \cdot \frac{y}{P^2 a_b b_b c_b}, \quad \rho_f = 0.01344 \cdot \frac{1-y}{P^2 a_f b_f c_f}$$

where y is the mass of the brighter star in terms of the total mass of the system. For RX Herculis we have from the radial velocities $y = 0.516$. From the photometric elements alone we must take $y = 0.5$. The semi-minor axis $b_b = b_f$ is found from the semi-major axis, when their ratio $1-c$ is known, and the semi-polar axis $c_b = c_f$ is derived by Russell's *formula* (38).

The elements of the system appear in Table VII. The photometric elements are given for both uniform and darkened solutions. The unit of length is the radius of the relative orbit, $a_b + a_f$; the unit of light is the total light of the system, $L_b + L_f$; and the sun's density is taken as the unit of density. Comparing these elements with Shapley's (*loc. cit.*), derived from visual observations, we find that there are differences due mostly to the different asumptions as to the relative brightness of the two stars, and to the allowance we have made for ellipticity of the stars and the radiation effect. Comparing especially the darkened solutions, we find the semi-duration of eclipse longer by 13 minutes than that derived by Shapley. The sizes of the two stars and their distance apart are greater in the present discussion, the masses are a little greater and are unequal, and the densities are somewhat smaller. The inclination of the orbit is less by nearly 2°.

The photographic magnitude of the system is $7^m.264$, as compared with $7^m.07$ derived from Wendell's visual observations. As both determinations are on the Harvard system, the color index of RX Herculis, $+0^m.19$, has a definitive meaning. The photographic ranges of variation, from the darkened solution, for primary and secondary minima are $0^m.573$ and $0^m.447$ respectively; the corresponding visual ranges according to Shapley are $0^m.55$ and $0^m.49$. This comparison is interesting. In photographic light the primary minimum is deeper than it is in visual light, and the secondary mini-

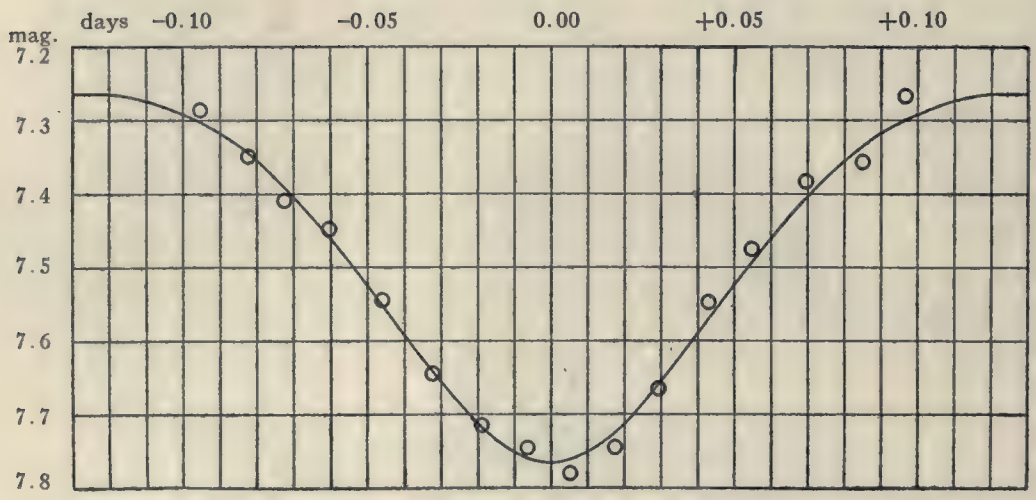

FIGURE 1. RECTIFIED NORMAL MAGNITUDES AND LIGHT CURVE OF RX HERCULIS DURING PRIMARY MINIMUM (DARKENED SOLUTION)

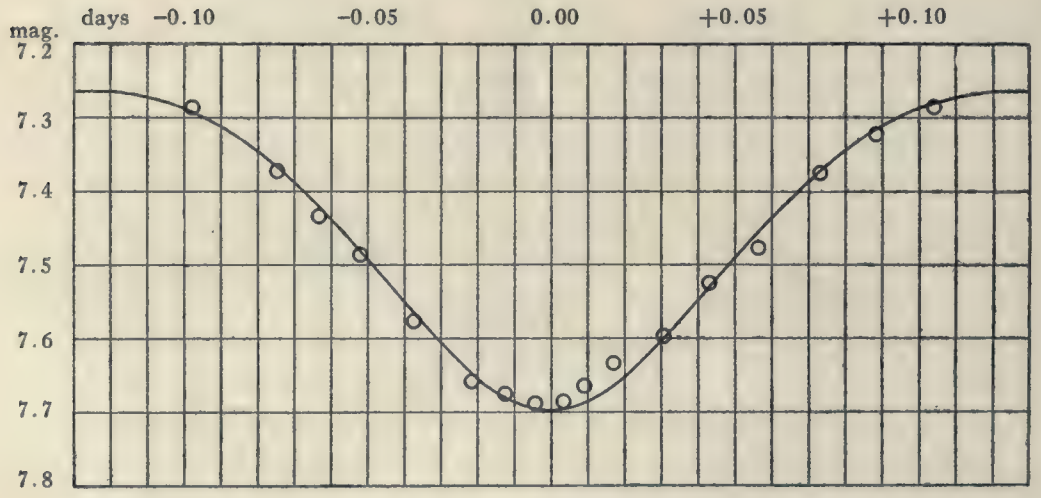

FIGURE 2. RECTIFIED NORMAL MAGNITUDES AND LIGHT CURVE OF RX HERCULIS DURING SECONDARY MINIMUM (DARKENED SOLUTION)

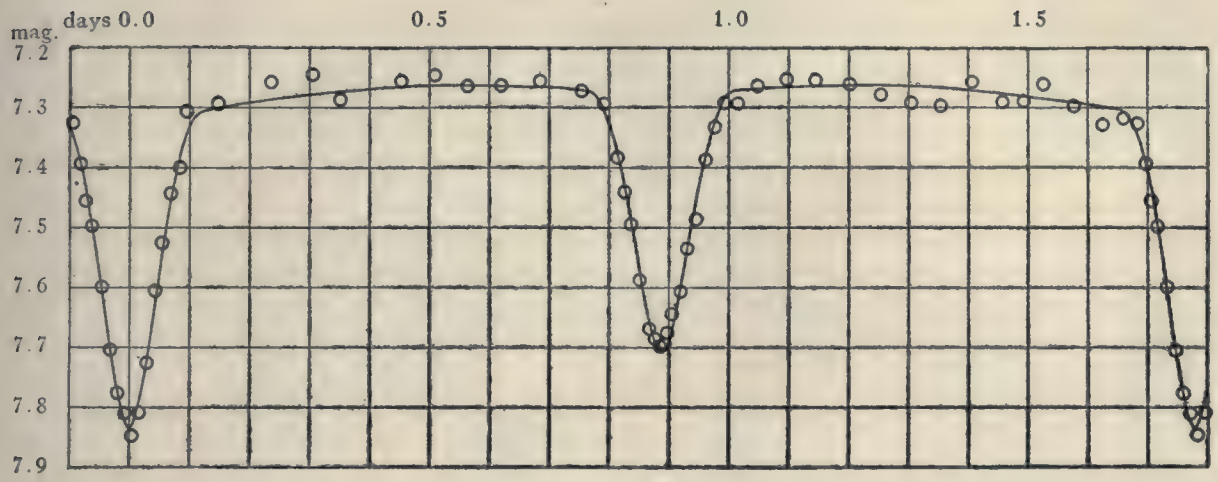

FIGURE 3. NORMAL MAGNITUDES AND THEORETICAL LIGHT CURVE OF RX HERCULIS

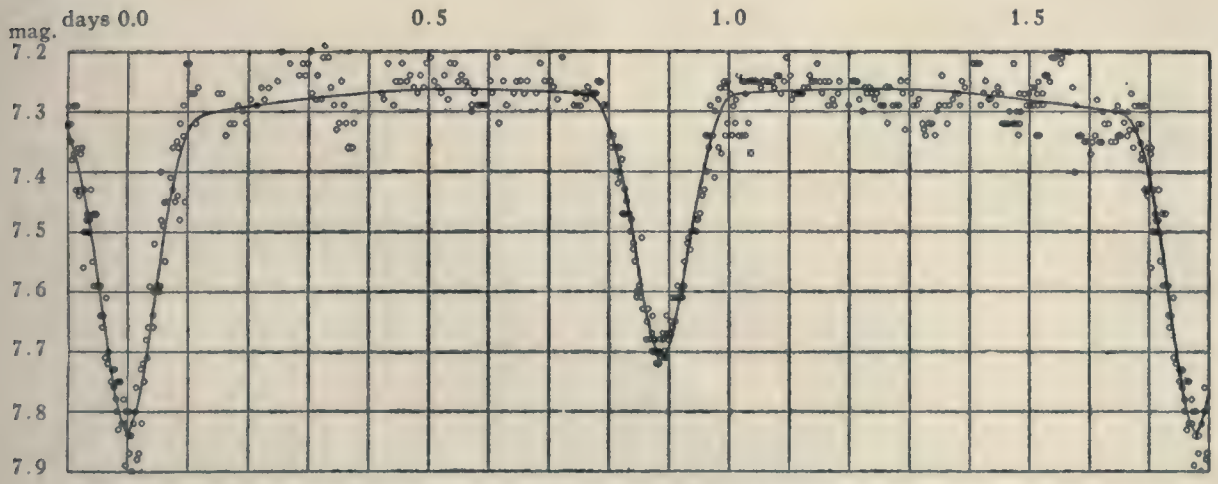

FIGURE 4. OBSERVED MAGNITUDES AND THEORETICAL LIGHT CURVE OF RX HERCULIS

mum is shallower. It follows that the two stars are of slightly different color index. The brighter star that is eclipsed at principal conjunction, having a smaller color index, is presumably of earlier spectral class.

On combining the photometric and spectroscopic elements, we have at once the absolute value of the linear unit a_b+a_f and so of the elements expressed in terms of it. The absolute masses and densities of the two stars are now available. It is to be noted, however, that the spectroscopic elements are based on radial velocities from only two

TABLE VII

ELEMENTS OF THE SYSTEM OF RX HERCULIS

		Uniform	Darkened	Absolute (darkened)
Period of revolution	P	1.7785740 days		
Daily motion in longitude	μ	202°.409		
Epoch of primary minimum	T	J.D. 2419658.5882 G.M.T.		
Semi-duration of eclipse	.....	0.1136 days	0.1202 days	
Eccentricity of orbit	e	0	0	
Inclination of orbit	i	84°.8	83°.9	
Least apparent distance of centers	$\cos i$	0.091	0.107	0.81 ⊙
Radius of relative orbit	a_b+a_f	1	1	7.59 ⊙
Ratio of radii	k	1.00	1.00	
Ratio of major and minor axes	$1+\frac{1}{2}\epsilon$	1.025	1.016	
Semi-major axis	$a_b=a_f$	0.204	0.216	1.64 ⊙
Semi-minor axis	$b_b=b_f$	0.199	0.212	1.61 ⊙
Semi-polar axis	$c_b=c_f$	0.194	0.203	1.54 ⊙
Mass of brighter star	m_b	.....	.....	0.96 ⊙
Mass of fainter star	m_f	.....	.....	0.91 ⊙
Density of brighter star	ρ_b	0.27	0.23	0.24 ⊙
Density of fainter star	ρ_f	0.27	0.23	0.22 ⊙
Maximum obscuration during eclipse	α_0	0.71	0.70	
"Radiation effect" constant	b	0.016	0.016	
Light of brighter star	L_b	0.528	0.529	
Light of fainter star, brighter side	L_f	0.472	0.471	
Light of fainter star, fainter side	L_f-2b	0.440	0.439	
Ratio of surface intensities	$\frac{J_b}{J_f}$	1.12	1.12	
Magnitude of system	.....	7.264	7.264	
Color index of system	.....	+0m.19	+0m.19	
Magnitude of brighter star	.....	7.96	7.96	
Magnitude of fainter star	.....	8.08	8.08	
Magnitude at primary minimum	.....	7.843	7.835	
Magnitude at secondary mimimum	.....	7.717	7.709	
Range of magnitude, primary	.....	0.581	0.573	
Range of magnitude, secondary	.....	0.455	0.447	

spectrograms, and that the lines on these plates are few and not well defined. The absolute elements for the darkened solution are given in the last column of Table VII in terms of the sun's radius (695,553 km.), mass and density.

Figures 1 and 2 show the arrangement of the rectified normal magnitudes at the two minima relative to the theoretical curve from the darkened solution. Asymmetry in the arrangement of the points is to be noted. At primary minimum the points on the

ascending branch indicate a steeper slope than those on the descending branch of the curve; at secondary minimum the descending branch is the steeper. This fact is substantiated by the behavior of the visual observations, plotted on the uniform curve in *Astrophysical Journal*, **40**, 404, 1914. The sum of the squares of the residuals for the normal places during eclipse is 0.008271 for the uniform solution and 0.007520 for the darkened solution; the latter appears to be the more satisfactory and is taken as the definitive solution.

The theoretical light curve is shown in Figure 3, with the observed normal magnitudes. Attention is called to the systematic trend of the residuals. This is more conspicuous for the separate observations, and for this reason the light curve is repeated in Figure 4, with the observed magnitudes for the individual exposures. The arrangement of the points is roughly symmetrical. On either side of secondary minimum they keep to the curve with reasonable fidelity, until points are reached about midway between secondary minimum and primary minimum, to the right and left, where they break into

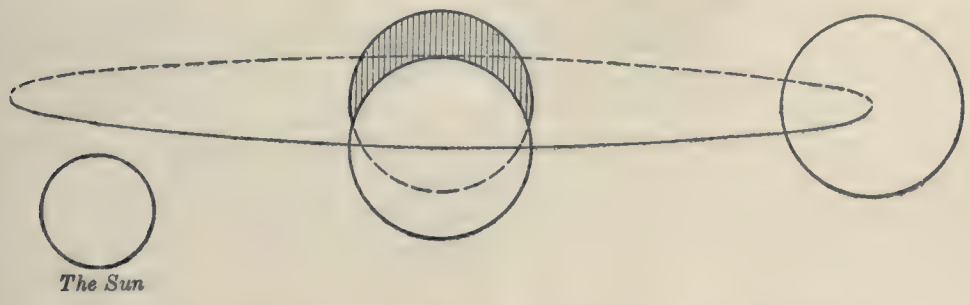

FIGURE 5. THE SYSTEM OF RX HERCULIS COMPARED WITH THE SUN, AT SECONDARY MINIMUM AND AT GREATEST ELONGATION

waves. The status of the irregularities would be even more convincing, if it were expedient to distinguish in the figure between points from different plates. For example, the wave between phases 1.33 and 1.48 days contains points from five plates. The inference is that the light of the system outside eclipse is variable, and that the fluctuations recur with considerable regularity at intervals of the eclipse period. We suspect similar fluctuations during eclipse. Our experience with this star and with others on the program is not in accord with the conclusion reached elsewhere that good photometric curves are uniformly smooth. The irregularities in the light variation of RX Herculis are typical of those found in other stars under investigation at the Laws Observatory. In respect to the periodic recurrence of the irregularities, one is reminded of the secondary oscillations remanent when the radial velocities of certain spectroscopic binaries are represented on the basis of simple elliptic motion.

The residuals for the observed magnitudes, scaled from the final theoretical light curve, appear in the sixth column of Table IV. The residuals for the normal magnitudes from the uniform and the darkened curve, are given in Table V, columns (9) and (11). For

that half of the plates measured by us both the probable error of a single observed magnitude of the variable is $\pm 0^m.027$ for Baker, $\pm 0^m.027$ for Cummings, and $\pm 0^m.023$ for a single combination. The probable error of a single magnitude for all the observations is $\pm 0^m.026$. The small amount of the probable error shows that the extrafocal method is highly satisfactory for investigations of this character. It is understood, of course, that the probable error would be greatly diminished, if a light curve were drawn to faithfully represent the observations. The probable error of a normal magnitude, computed from the residuals for the normals, is $\pm 0^m.012$ for the points without eclipse, $\pm 0^m.011$ within eclipse on the uniform solution, and $\pm 0^m.010$ within eclipse on the darkened solution.

Our study of the photographic light curve of RX Herculis derived from extrafocal plates made at the Laws Observatory, in combination with the spectroscopic evidence, leads to the following conclusions:

The system is composed of two slightly ellipsoidal stars of the same size, each having a radius about one and one-half times the sun's radius. The mass of each star is a little less than that of the sun, and the density is one-fourth the sun's density.

The star eclipsed at principal conjunction is of magnitude 7.96 and is brighter by $0^m.12$ than its companion. It contributes 52.8 per cent to the light of the system. Its surface brightness, mass and density are greater than those of the fainter star by 12, 6 and 9 per cent respectively; and it is of slightly earlier spectral class. A radiation effect of the brighter star on its companion is established by the least-squares solution.

The orbits of the two stars are circular and their inclination is $83^\circ.9$. The distance between the centers of the two stars is about five times the radius of either star, or 5,280,000 km. At conjunction 70 per cent of the disk of one star is eclipsed by the other. The semi-duration of eclipse is $2^h 53^m$.

Of the two solutions, the one assuming the stars to be completely darkened at the limb is a little more satisfactory. The theoretical light curve, however, does not closely represent the observed magnitudes. During the minima the points require an asymmetrical curve, and outside eclipse they fall into waves, especially near the primary minimum. The probable error of a single observed magnitude is $\pm 0^m.026$; this is derived from the residuals relative to the theoretical curve for the entire set of 480 observations.

1916 March 31

THE UNIVERSITY OF MISSOURI BULLETIN

ASTRONOMICAL SERIES

LAWS OBSERVATORY BULLETIN NO. 26

THE ECLIPSING BINARY Z VULPECULAE

By Robert H. Baker

The variability in the light of Z Vulpeculae (R.A. = $19^h17^m.5$, Decl. = $+25°\ 23'$) was suspected by Flint[1] in 1900. It was independently detected in 1908 by Astbury[2] whose observations identified it with the Algol type and who later[3] found a period of 2.445 days. A series of 68 observations of this star, made by the writer[4] with the polarizing photometer, showed that the light continues to vary outside eclipse. A well-defined secondary minimum appeared at phase $1^d.20$, slightly earlier than the time midway between principal minima. The light of the system, referred to the star B.D + 25°3802, magnitude 7.26, was found to vary from $7^m.06$ to $8^m.79$ at primary minimum and to $7^m.44$ at secondary minimum. By comparison with previous observations the following elements were derived:

$$\text{Primary Minimum} = \text{J.D. } 2419679.675 + 2^d.45492 \cdot E, \text{ G.M.T. (Baker)}$$

Nijland's visual observations[5] show the same range at primary minimum; they do not include the secondary minimum.

Orbital elements have been computed by Shapley[6] from the Laws Observatory results. The brighter star which he finds slightly the smaller, contributes three-fourths of the light of the system. The eclipse at principal conjunction is grazing total. The solution that supposes uniformly luminous disks is considered unique; but when the stars are assumed to be completely darkened at the limb, the solution is somewhat indeterminate, so that the eclipse might be grazing annular without sensibly affecting the light curve. From the displacement of the secondary minimum he finds $e = \cos\omega = -0.013$. A difference in the widths of the two minima is noted, but the observations are not sufficient to derive the other component of the eccentricity.

This star is one of the twelve eclipsing binaries whose variations in photographic

[1] A. S. Flint, *Astronomical Journal*, 21, 74, 1901.
[2] H. H. Turner, *Astronomische Nachrichten*, 179, 225, 1908.
[3] T. H. Astbury, *Astronomische Nachrichten*, 182, 389, 1909.
[4] R. H. Baker, *Laws Observatory Bulletin* No. 20, 58, 1913.
[5] A. A. Nijland, *Hemel en Dampkring*, April, 1913.
[6] H. Shapley, *Princeton Contributions*, No. 3, 86, 1915.

TABLE I

PLATES OF Z VULPECULAE

Plate No.	Date, G.M.T.	Julian Day	Epoch	Correction to Sun	Helioc. Phase	Tel.	Hour Angle	No. Exp.	Film
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	1914 d. h. m.			days	d. to d.		h.		
225	June 26 15 48	2420310.6583	257	+0.0037	0.0469 0.1024	W	−3.4	9	21
226	26 17 18	310.7208	257	37	0.1094 0.1701	W	−1.8	11	16
227	26 18 45	310.7812	257	37	0.1767 0.2184	W	−0.4	11	13
228	27 16 48	311.7000	257	37	1.0871 1.1455	W	−2.3	14	16
229	27 18 19	311.7632	257	37	1.1503 1.2087	W	−0.8	15	15
230	27 19 45	311.8229	257	37	1.2142 1.2649	W	+0.7	13	clear
231	27 20 53	311.8701	257	37	1.2739 1.2989	E	+1.8	7	clear
232	28 16 59	312.7076	257	38	2.0949 2.1532	W	−2.0	15	24
233	28 18 12	312.7583	257	38	2.1581 2.1949	W	−0.8	10	18
234	July 11 15 51	325.6604	263	39	0.3265 0.3681	W	−2.3	11	14
235	13 15 45	327.6562	263	39	2.3154 2.3709	W	−2.3	9	19
236	13 16 55	327.7049	263	39	2.3782 2.4043	W	−1.1	5	13
237	14 15 40	328.6528	264	40	0.8557 0.9141	W	−2.3	15	19
238	14 17 11	328.7160	264	40	0.9189 0.9772	W	−0.8	15	19
241	15 17 00	329.7083	264	40	1.9279 1.9536	W	−0.9	7	clear
255	23 15 37	337.6507	268	40	0.0353 0.0978	W	−1.8	10	17
260	24 15 25	338.6424	268	40	1.0325 1.0770	W	−1.9	9	15
267	27 17 17	341.7201	269	39	1.6567 1.7025	E	+0.2	12	17
268	27 18 24	341.7667	269	39	1.7116 1.7366	E	+1.3	7	clear
272	28 18 00	342.7500	270	39	0.2317 0.2775	E	+1.0	12	16
273	28 19 13	342.8007	270	39	0.2824 0.3240	E	+2.2	11	15
276	29 19 30	343.8125	270	39	1.2942 1.3358	E	+2.5	11	20
277	29 20 27	343.8521	270	39	1.3463 1.3608	E	+3.5	5	18
278	Aug. 9 15 29	354.6451	274	38	2.3167 2.3389	W	−0.8	5	37
279	11 15 00	356.6250	275	37	1.8322 1.8781	W	−1.1	12	17
280	11 15 58	356.6652	275	37	1.8843 1.9010	W	−0.2	5	clear
287	17 15 20	362.6389	278	35	0.4731 0.5308	W	−0.4	15	21
288	17 16 35	362.6910	278	35	0.5370 0.5700	W	+0.8	9	16
289	17 18 13	362.7590	278	35	0.5919 0.6509	E	+2.5	15	22
290	17 19 39	362.8188	278	35	0.6606 0.7106	E	+3.9	12	21
299	Sept. 15 14 23	391.5993	289	22	2.4281 0.0371	E	+0.5	11	19
301	15 16 47	391.6993	290	22	0.0850 0.1239	E	+2.9	8	17
302	15 17 54	391.7458	290	22	0.1316 0.1705	E	+4.0	8	18
303	17 14 15	393.5938	290	21	1.9669 2.0252	E	+0.5	15	16
304	17 15 53	393.6618	290	21	2.0391 2.0849	E	+2.1	12	18
318	19 16 13	395.6757	291	20	1.6022 1.6480	E	+2.6	12	17
348	Oct. 11 13 33	417.5646	300	06	1.3954 1.4412	E	+1.4	12	21
	1915								
493	July 4 15 41	683.6535	408	38	2.3560 2.3977	W	−3.0	7	22
494	4 16 52	683.7028	408	38	2.4054 2.4484	W	−1.8	7	17
495	4 17 55	683.7465	409	38	0.0011 0.0289	W	−0.7	5	14
502	5 17 55	684.7465	409	39	0.9894 1.0401	W	−0.6	13	20
518	17 18 29	696.7701	414	40	0.7469 0.7851	E	+0.7	10	13
519	17 19 31	696.8132	414	40	0.7899 0.8240	E	+1.7	9	12
533	Sept. 5 14 49	746.6174	434	27	1.4951 1.5340	E	+0.3	8	14
534	5 15 54	746.6625	434	27	1.5402 1.5791	E	+1.4	8	15
544	10 14 03	751.5854	436	24	1.5572 1.5864	W	−0.1	8	13
562	13 16 37	754.6924	437	23	2.1960 2.2522	E	+2.6	14	22
563	13 17 58	754.7486	437	23	2.2571 2.2987	E	+4.0	11	13

light we have undertaken to investigate by the extrafocal method. The study of Z Vulpeculae is based on 48 plates containing 495 exposures, obtained by Miss Cummings and the writer with the 5-inch camera. The instruments and methods employed in this work are described in *Laws Observatory Bulletin*, No. 24. Table I gives a list of the plates with data regarding them. The times in columns (2) and (3) and the hour angle in column (8) are given for the middle exposure. The heliocentric phases for the first and the last exposure, column (6), are referred to primary minimum; these and the epoch number in column (4) are based on the light elements given above, with the addition of the correction Δ Epoch $= -0.002$ days, that is required by the present series of observations. The corrected elements are accordingly:

Primary Minimum = J.D. 2419679.673 + $2^d.45492 \cdot E$, G.M.T. (Final Elements)

The plates were exposed about 1.5 mm. outside focus and were sensitometrized, developed and measured as usual; all measurements were made by the writer. They are

TABLE II

COMPARISON STARS FOR Z VULPECULAE

Star	B.D.		R.A.(1900)	Decl.(1900)	Adopted Photg. Mag.	Harvard	
	No.	Mag.				Visual Mag.	Class
			h m				
v	+25°3803	7.5	19 17.5	+25°23′			
a	25 3802	7.0	19 17.3	25 25	7.45		
b	26 3533	7.7	19 17.6	26 27	8.16		
c	24 3706	7.8	19 13.3	25 04	7.75		
d	24 3737	6.1	19 21.3	24 44	6.75	6.17	K
h	27 3379	7.0	19 20.4	27 53	6.32	6.36	A
i	27 3313	7.0	19 11.9	27 17	6.31	6.26	A

Seed 27 plates whose emulsion number is 14540 for the 1914 series, and 15194 for the 1915 series, with the exception of plate 348 whose number is 14794. The separate exposures vary from six to ten minutes; the total exposure time on a plate does not exceed 100 minutes. Column (10) of the table gives the wedge readings on the film. The excessive fogging of plate 278 is due to the moon.

The six stars measured in connection with the variable are listed in Table II. Stars *a*, *b*, *c* and *d* were used regularly as comparison stars. They are all within 16.4 mm. of the center of the plate. Stars *h* and *i* serve as standards; their spectra are of class A and their visual magnitudes are given in *Revised Harvard Photometry* as $6^m.36$ and $6^m.26$ respectively. Measurements of the images of these two stars were made for three exposures on each of twelve plates and are used to establish the magnitude of comparison star *a*. The mean magnitude differences are: $a - h = 1^m.13$, $a - i = 1^m.14$. The adopted photographic magnitudes are accordingly: $a = 7^m.45$, $h = 6^m.32$, $i = 6^m.31$, and the corresponding values for the other stars are derived from the observed differences $b - a$, $c - a$, $d - a$.

TABLE III

PHOTOGRAPHIC OBSERVATIONS OF Z VULPECULAE

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O–C (uniform)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O–C (uniform)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag.			2420	days		mag.
225	1	310.6306	0.0469	8.58	+.01	229	11	311.7757	1.1920	7.69	+.02
	2	.6375	.0538	8.46	−.01		12	.7799	.1962	7.72	+.04
	3	.6444	.0607	8.42	+.02		13	.7840	.2003	7.68	.00
	4	.6514	.0677	8.35	+.01		14	.7882	.2045	7.66	−.03
	5	.6583	.0746	8.25	−.02		15	.7924	.2087	7.72	+.03
	6	.6653	.0816	8.21	−.01						
	7	.6722	.0885	8.13	−.04	230	1	311.7979	1.2142	7.70	+.01
	8	.6792	.0955	8.10	−.01		2	.8021	.2184	7.68	−.02
	9	.6861	.1024	8.02	−.05		3	.8062	.2225	7.72	+.02
							4	.8104	.2267	7.66	−.04
226	1	310.6931	0.1094	8.02	.00		5	.8146	.2309	7.66	−.04
	2	.6986	.1149	8.00	+.02		6	.8188	.2351	7.76	+.05
	3	.7042	.1205	8.02	+.06		7	.8229	.2392	7.71	+.01
	4	.7097	.1260	7.99	+.07		8	.8271	.2434	7.71	+.01
	5	.7153	.1316	7.89	.00		9	.8312	.2475	7.68	−.01
	6	.7208	.1371	7.87	.00		11	.8361	.2524	7.72	+.03
	7	.7264	.1427	7.84	.00		12	.8403	.2566	7.66	−.03
	8	.7319	.1482	7.82	.00		13	.8444	.2607	7.66	−.02
	9	.7375	.1538	7.82	+.03		14	.8486	.2649	7.72	+.04
	10	.7430	.1593	7.78	+.01						
	12	.7538	.1701	7.68	−.04	231	1	311.8576	1.2739	7.64	−.03
							2	.8618	.2781	7.70	+.04
227	1	310.7604	0.1767	7.72	+.02		3	.8660	.2823	7.70	+.05
	2	.7646	.1809	7.70	+.02		4	.8701	.2864	7.70	+.05
	3	.7688	.1851	7.67	.00		5	.8743	.2906	7.64	.00
	4	.7729	.1892	7.70	+.04		6	.8785	.2948	7.63	−.01
	5	.7771	.1934	7.61	−.04		7	.8826	.2989	7.68	+.04
	6	.7812	.1975	7.63	.00						
	7	.7854	.2017	7.55	−.07	232	1	312.6785	2.0949	7.53	+.03
	8	.7896	.2059	7.60	−.01		2	.6826	.0990	7.60	+.09
	9	.7938	.2101	7.59	−.01		3	.6868	.1032	7.53	+.02
	10	.7979	.2142	7.56	−.03		4	.6910	.1074	7.53	+.02
	11	.8021	.2184	7.59	.00		5	.6951	.1115	7.56	+.05
							6	.6993	.1157	7.60	+.09
228	1	311.6708	1.0871	7.55	−.01		7	.7035	.1199	7.56	+.05
	2	.6750	.0913	7.61	+.05		8	.7076	.1240	7.52	+.01
	3	.6792	.0955	7.56	−.01		9	.7118	.1282	7.56	+.05
	4	.6833	.0996	7.55	−.02		10	.7160	.1324	7.55	+.04
	5	.6875	.1038	7.60	+.03		11	.7201	.1365	7.59	+.08
	6	.6917	.1080	7.63	+.05		12	.7243	.1407	7.59	+.07
	7	.6958	.1121	7.56	−.02		13	.7285	.1449	7.54	+.02
	8	.7000	.1163	7.54	−.05		14	.7326	.1490	7.53	+.01
	9	.7042	.1205	7.58	−.01		15	.7368	.1532	7.48	−.04
	10	.7083	.1246	7.60	+.01						
	11	.7125	.1288	7.55	−.05	233	1	312.7417	2.1581	7.54	+.02
	12	.7167	.1330	7.61	+.01		2	.7458	.1622	7.55	+.03
	13	.7208	.1371	7.62	+.01		3	.7500	.1664	7.53	.00
	15	.7292	.1455	7.60	−.01		4	.7542	.1706	7.58	+.05
							5	.7583	.1747	7.62	+.09
229	1	311.7340	1.1503	7.64	+.02		6	.7625	.1789	7.54	+.01
	2	.7382	.1545	7.63	+.01		7	.7667	.1831	7.52	−.01
	3	.7424	.1587	7.64	+.01		8	.7708	.1872	7.53	.00
	4	.7465	.1628	7.63	.00		9	.7747	.1911	7.52	−.02
	5	.7507	.1670	7.61	−.03		10	.7785	.1949	7.51	−.03
	6	.7549	.1712	7.62	−.03						
	7	.7590	.1753	7.68	+.03	234	1	325.6396	0.3265	7.48	−.04
	8	.7632	.1795	7.66	.00		2	.6438	.3307	7.55	+.04
	9	.7674	.1837	7.66	.00		3	.6479	.3348	7.56	+.05
	10	.7715	.1878	7.66	−.01		4	.6521	.3390	7.48	−.03

TABLE III—Continued
PHOTOGRAPHIC OBSERVATIONS OF Z VULPECULAE

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O—C (uniform)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O—C (uniform)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag.			2420	days		mag.
234	5	325.6562	0.3431	7.53	+.02	241	4	329.7083	1.9404	7.44	−.02
	6	.6604	.3473	7.55	+.04		5	.7125	.9446	7.43	−.03
	7	.6646	.3515	7.54	+.03		6	.7170	.9491	7.50	+.04
	8	.6688	.3557	7.53	+.02		7	.7215	.9536	7.48	+.02
	9	.6729	.3598	7.51	.00						
	10	.6771	.3640	7.50	.00	255	1	337.6229	0.0353	8.67	−.05
	11	.6812	.3681	7.48	−.02		2	.6299	.0423	8.55	−.06
							3	.6368	.0492	8.56	+.01
235	1	327.6285	2.3154	7.82	−.03		4	.6438	.0562	8.50	+.04
	2	.6354	.3223	7.90	+.01		5	.6507	.0631	8.44	+.05
	3	.6424	.3293	7.96	+.04		6	.6576	.0700	8.35	+.03
	4	.6493	.3362	7.98	+.02		7	.6646	.0770	8.30	+.04
	5	.6562	.3431	8.01	.00		8	.6715	.0839	8.23	+.02
	6	.6632	.3501	8.05	.00		9	.6785	.0909	8.17	+.02
	7	.6701	.3570	8.05	−.05		10	.6854	.0978	8.12	+.02
	8	.6771	.3640	8.15	.00						
	9	.6840	.3709	8.16	−.04	260	1	338.6201	1.0325	7.48	−.04
							2	.6257	.0381	7.50	−.02
236	1	327.6913	2.3782	8.30	+.04		3	.6312	.0436	7.50	−.03
	2	.6979	.3848	8.43	+.11		4	.6368	.0492	7.53	.00
	3	.7049	.3918	8.43	+.05		5	.6424	.0548	7.56	+.03
	4	.7118	.3987	8.58	+.13		6	.6479	.0603	7.53	−.01
	5	.7174	.4043	8.53	+.01		7	.6535	.0659	7.58	+.04
							8	.6590	.0714	7.53	−.02
237	1	328.6236	0.8557	7.52	+.04		9	.6646	.0770	7.55	.00
	2	.6278	.8599	7.50	+.02						
	3	.6319	.8640	7.51	+.03	267	1	341.6993	1.6567	7.45	−.01
	4	.6361	.8682	7.47	−.01		2	.7035	.6609	7.44	−.02
	5	.6403	.8724	7.50	+.02		3	.7076	.6650	7.39	−.07
	6	.6444	.8765	7.48	.00		4	.7118	.6692	7.43	−.03
	7	.6486	.8807	7.46	−.02		5	.7160	.6734	7.47	+.01
	8	.6528	.8849	7.48	.00		6	.7201	.6775	7.51	+.06
	9	.6569	0.8890	7.47	−.02		7	.7243	.6817	7.41	−.04
	10	.6611	.8932	7.47	−.02		8	.7285	.6859	7.45	.00
	11	.6653	.8974	7.47	−.02		9	.7326	.6900	7.45	.00
	12	.6694	.9015	7.48	−.01		10	.7368	.6942	7.50	+.05
	13	.6736	.9057	7.52	+.03		11	.7410	.6984	7.44	−.01
	14	.6778	.9099	7.50	+.01		12	.7451	.7025	7.47	+.02
	15	.6820	.9141	7.46	−.03						
						268	2	341.7542	1.7116	7.47	+.02
238	1	328.6868	0.9189	7.53	+.04		3	.7583	.7157	7.41	−.04
	2	.6910	.9231	7.51	+.02		4	.7625	.7199	7.50	+.05
	3	.6951	.9272	7.56	+.07		5	.7667	.7241	7.54	+.09
	4	.6993	.9314	7.51	+.02		6	.7708	.7282	7.48	+.03
	5	.7035	.9356	7.48	−.01		7	.7750	.7324	7.46	+.01
	6	.7076	.9397	7.53	+.04		8	.7792	.7366	7.43	−.02
	7	.7118	.9439	7.53	+.03						
	8	.7160	.9481	7.52	+.02	272	1	342.7292	0.2317	7.46	−.10
	9	.7201	.9522	7.52	+.02		2	.7333	.2358	7.56	.00
	10	.7243	.9564	7.51	+.01		3	.7375	.2400	7.53	−.03
	11	.7285	.9606	7.54	+.04		4	.7417	.2442	7.52	−.03
	12	.7326	.9647	7.53	+.03		5	.7458	.2483	7.53	+.02
	13	.7368	.9689	7.51	+.01		6	.7500	.2525	7.46	−.08
	14	.7410	.9731	7.52	+.02		7	.7542	.2567	7.53	−.01
	15	.7451	.9772	7.50	−.01		8	.7583	.2608	7.51	−.03
							9	.7625	.2650	7.55	+.01
241	1	329.6958	1.9279	7.36	−.09		10	.7667	.2692	7.55	+.02
	2	.7000	.9321	7.40	−.05		11	.7708	.2733	7.54	+.01
	3	.7042	.9363	7.48	+.02		12	.7750	.2775	7.50	−.03

TABLE III—Continued

PHOTOGRAPHIC OBSERVATIONS OF Z VULPECULAE *

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O–C (uniform)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O–C (uniform)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag.			2420	days		mag.
273	1	342.7799	0.2824	7.56	+.03	287	5	362.6264	0.4891	7.43	−.04
	2	.7840	.2865	7.53	+.01		6	.6306	.4933	7.46	−.01
	3	.7882	.2907	7.55	+.02		7	.6347	.4974	7.48	+.01
	4	.7924	.2949	7.56	+.04		8	.6389	.5016	7.45	−.02
	5	.7965	.2990	7.48	−.04		9	.6431	.5058	7.45	−.02
	6	.8007	.3032	7.47	−.05		10	.6472	.5099	7.45	−.02
	7	.8049	.3074	7.52	.00		11	.6514	.5141	7.45	−.02
	8	.8090	.3115	7.48	−.04		12	.6556	.5183	7.53	+.06
	9	.8132	.3157	7.51	−.01		13	.6597	.5224	7.47	.00
	10	.8174	.3199	7.48	−.04		14	.6639	.5266	7.47	.00
	11	.8215	.3240	7.52	.00		15	.6681	.5308	7.45	−.01
276	1	343.7917	1.2942	7.61	−.03	288	1	362.6743	0.5370	7.40	−.06
	2	.7958	.2983	7.69	+.05		2	.6785	.5412	7.44	−.02
	3	.8000	.3025	7.64	+.01		3	.6826	.5453	7.46	.00
	4	.8042	.3067	7.58	−.05		4	.6868	.5495	7.51	+.05
	5	.8083	.3108	7.68	+.06		5	.6910	.5537	7.51	+.05
	6	.8125	.3150	7.55	−.06		6	.6951	.5578	7.47	+.01
	7	.8167	.3192	7.63	+.02		7	.6993	.5620	7.50	+.04
	8	.8208	.3233	7.54	−.07		8	.7035	.5662	7.46	.00
	9	.8250	.3275	7.54	−.06		9	.7073	.5700	7.43	−.03
	10	.8292	.3317	7.60	.00						
	11	.8333	.3358	7.56	−.03	289	1	362.7292	0.5919	7.47	+.01
							2	.7333	.5960	7.41	−.05
277	1	343.8438	1.3463	7.58	.00		3	.7375	.6002	7.45	−.01
	2	.8479	.3504	7.62	+.04		4	.7417	.6044	7.45	−.01
	3	.8521	.3546	7.62	+.05		5	.7462	.6089	7.43	−.03
	4	.8562	.3587	7.63	+.06		6	.7507	.6134	7.43	−.02
	5	.8583	.3608	7.63	+.06		7	.7549	.6176	7.46	+.01
							8	.7590	.6217	7.47	+.02
278	1	354.6340	2.3167	7.90	+.04		9	.7632	.6259	7.45	.00
	2	.6396	.3223	7.85	−.04		10	.7674	.6301	7.45	.00
	3	.6451	.3278	7.89	−.02		11	.7715	.6342	7.52	+.07
	4	.6507	.3334	7.93	−.01		12	.7757	.6384	7.46	+.01
	5	.6562	.3389	7.93	−.04		13	.7799	.6426	7.47	+.02
							14	.7840	.6467	7.44	−.01
279	1	356.6045	1.8322	7.41	−.04		15	.7882	.6509	7.50	+.05
	2	.6087	.8364	7.36	−.09						
	3	.6128	.8405	7.38	−.06	290	1	362.7979	0.6606	7.39	−.06
	4	.6170	.8447	7.41	−.04		2	.8021	.6648	7.43	−.02
	5	.6212	.8489	7.40	−.05		3	.8062	.6689	7.40	−.05
	6	.6254	.8531	7.37	−.08		4	.8104	.6731	7.47	+.02
	7	.6295	.8572	7.38	−.07		5	.8146	.6773	7.45	.00
	8	.6337	.8614	7.43	−.02		6	.8188	.6815	7.43	−.02
	9	.6378	.8655	7.52	+.07		7	.8229	.6856	7.48	+.03
	10	.6420	.8697	7.45	.00		8	.8271	.6898	7.44	−.01
	11	.6462	.8739	7.45	.00		9	.8312	.6939	7.50	+.05
	12	.6504	.8781	7.38	−.07		10	.8354	.6981	7.45	.00
							11	.8410	.7037	7.48	+.03
280	1	356.6566	1.8843	7.48	+.03		12	.8479	.7106	7.48	+.03
	2	.6608	.8885	7.47	+.02						
	3	.6649	.8926	7.46	+.01	299	1	391.5708	2.4281	8.86	+.03
	4	.6691	.8968	7.52	+.07		2	.5764	.4327	8.79	−.10
	5	.6733	.9010	7.46	+.01		3	.5820	.4393	8.96	−.02
							4	.5875	.4448	9.06	.00
287	1	362.6104	0.4731	7.46	−.02		5	.5930	.4503	9.07	−.03
	2	.6146	.4773	7.53	+.05		6	.5993	0.0017	8.98	−.14
	3	.6188	.4815	7.43	−.04		7	.6066	.0090	9.09	+.01
	4	.6226	.4853	7.45	−.02		8	.6139	.0163	8.96	−.03

TABLE III—Continued

PHOTOGRAPHIC OBSERVATIONS OF Z VULPECULAE

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O–C (uniform)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O–C (uniform)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag.			2420	days		mag.
299	9	391.6208	0.0232	8.85	−.04	318	9	395.6882	1.6355	7.47	+.01
	10	.6278	.0302	8.83	+.03		10	.6924	.6397	7.45	−.01
	11	.6347	.0371	8.73	+.03		11	.6965	.6438	7.45	−.01
							12	.7007	.6480	7.48	+.02
301	1	391.6826	0.0850	8.17	−.03						
	2	.6882	.0906	8.17	+.02	348	1	417.5438	1.3954	7.54	.00
	3	.6938	.0962	8.08	−.03		2	.5479	.3995	7.53	−.01
	4	.6993	.1017	8.02	−.05		3	.5521	.4037	7.52	−.01
	5	.7049	.1073	8.05	+.01		4	.5562	.4078	7.50	−.03
	6	.7104	.1128	7.94	−.06		5	.5604	.4120	7.52	−.01
	7	.7160	.1184	7.98	+.01		6	.5646	.4162	7.52	−.01
	8	.7215	.1239	7.91	−.03		7	.5688	.4204	7.54	+.02
							8	.5729	.4245	7.55	+.03
302	1	391.7292	0.1316	7.91	+.01		9	.5771	.4287	7.50	−.02
	2	.7347	.1371	7.97	+.10		10	.5812	.4328	7.47	−.04
	3	.7403	.1427	7.81	−.03		11	.5854	.4370	7.47	−.04
	4	.7458	.1482	7.84	+.02		12	.5896	.4412	7.43	−.08
	5	.7514	.1538	7.75	−.04						
	6	.7569	.1593	7.76	−.01	493	1	683.6326	2.3560	8.07	−.04
	7	.7625	.1649	7.85	+.11		2	.6396	.3630	8.16	+.02
	8	.7681	.1705	7.83	+.11		3	.6465	.3699	8.25	+.05
							4	.6535	.3769	8.28	+.03
303	1	393.5646	1.9669	7.39	−.07		5	.6604	.3838	8.32	+.02
	2	.5688	.9711	7.35	−.11		6	.6674	.3908	8.39	+.01
	3	.5729	.9752	7.48	+.02		7	.6743	.3977	8.46	+.02
	4	.5771	.9794	7.39	−.07						
	5	.5812	.9835	7.47	.00	494	1	683.6820	2.4054	8.58	+.05
	6	.5854	.9877	7.50	+.03		2	.6889	.4123	8.67	+.04
	7	.5896	.9919	7.43	−.04		3	.6958	.4192	8.76	+.06
	8	.5938	.9961	7.46	−.01		4	.7028	.4262	8.86	+.05
	9	.5979	2.0002	7.53	+.06		5	.7104	.4338	8.94	+.03
	10	.6021	.0044	7.41	−.06		6	.7180	.4414	9.05	+.02
	11	.6062	.0085	7.48	+.01		7	.7250	.4484	9.15	+.06
	12	.6104	.0127	7.53	+.05						
	13	.6146	.0169	7.50	+.02	495	1	683.7326	0.0011	9.21	+.09
	14	.6188	.0211	7.56	+.08		2	.7396	.0081	9.17	+.08
	15	.6229	.0252	7.58	+.10		3	.7465	.0150	9.06	+.04
							4	.7535	.0220	9.01	+.09
304	2	393.6368	2.0391	7.41	−.07		5	.7604	.0289	8.90	+.08
	3	.6410	.0433	7.47	−.01						
	4	.6451	.0474	7.45	−.04	502	1	684.7208	0.9894	7.51	.00
	5	.6493	.0516	7.52	+.03		2	.7257	.9943	7.50	−.01
	6	.6535	.0558	7.46	−.03		3	.7299	.9985	7.51	.00
	7	.6576	.0599	7.47	−.02		4	.7340	1.0026	7.45	−.06
	8	.6618	.0641	7.44	−.05		5	.7382	.0068	7.52	+.01
	9	.6660	.0683	7.51	+.02		6	.7424	.0110	7.53	+.02
	10	.6701	.0724	7.51	+.02		7	.7465	.0151	7.56	+.05
	11	.6743	.0766	7.54	+.05		8	.7507	.0193	7.53	+.02
	12	.6785	.0808	7.45	−.05		9	.7549	.0235	7.48	−.04
	13	.6826	.0849	7.48	−.02		10	.7590	.0276	7.50	−.02
							11	.7632	.0318	7.60	+.08
318	1	395.6549	1.6022	7.48	+.01		12	.7674	.0360	7.53	+.01
	2	.6590	.6063	7.52	+.05		13	.7715	.0401	7.54	+.01
	3	.6632	.6105	7.48	+.01						
	4	.6674	.6147	7.50	+.03	518	1	696.7528	0.7469	7.53	+.07
	5	.6715	.6188	7.50	+.03		2	.7576	.7517	7.48	+.02
	6	.6757	.6230	7.52	+.06		3	.7618	.7559	7.47	+.01
	7	.6799	.6272	7.40	−.06		4	.7660	.7601	7.47	+.01
	8	.6840	.6313	7.45	−.01		5	.7701	.7642	7.43	−.03

TABLE III—Continued
PHOTOGRAPHIC OBSERVATIONS OF Z VULPECULAE

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O–C (uniform)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O–C (uniform)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag.			2420	days		mag.
518	6	696.7743	0.7684	7.47	+.01	544	2	751.5771	1.5614	7.51	+.03
	7	.7785	.7726	7.51	+.05		3	.5812	.5655	7.47	.00
	8	.7826	.7767	7.47	+.01		4	.5854	.5697	7.52	+.05
	9	.7868	.7809	7.52	+.06		5	.5896	.5739	7.52	+.05
	10	.7910	.7851	7.46	.00		6	.5938	.5781	7.52	+.05
							7	.5979	.5822	7.50	+.03
519	1	696.7958	0.7899	7.47	+.01		8	.6021	.5864	7.45	−.02
	2	.8000	.7941	7.45	−.02						
	3	.8049	.7990	7.45	−.02	562	1	754.6667	2.1960	7.58	+.04
	4	.8090	.8031	7.46	−.01		2	.6708	.2001	7.61	+.07
	5	.8132	.8073	7.47	.00		3	.6750	.2043	7.60	+.05
	6	.8174	.8115	7.47	.00		4	.6792	.2085	7.61	+.06
	7	.8215	.8156	7.44	−.03		5	.6833	.2126	7.63	+.08
	8	.8257	.8198	7.45	−.02		6	.6882	.2185	7.63	+.07
	9	.8299	.8240	7.48	+.01		7	.6924	.2217	7.61	+.05
							8	.6965	.2258	7.62	+.05
533	1	746.6007	1.4951	7.50	+.01		9	.7007	.2300	7.61	+.04
	2	.6062	.5006	7.48	−.01		10	.7049	.2342	7.60	+.02
	3	.6118	.5062	7.47	−.02		11	.7097	.2390	7.62	+.03
	4	.6174	.5118	7.51	+.02		12	.7146	.2439	7.63	+.03
	5	.6229	.5173	7.47	−.02		13	.7188	.2481	7.60	.00
	6	.6285	.5229	7.45	−.03		14	.7229	.2522	7.61	.00
	7	.6340	.5284	7.40	−.08						
	8	.6396	.5340	7.46	−.02	563	1	754.7278	2.2571	7.59	−.04
							2	.7319	.2612	7.62	−.02
534	1	746.6458	1.5402	7.43	−.05		3	.7361	.2654	7.59	−.06
	2	.6514	.5458	7.47	−.01		4	.7403	.2696	7.63	−.04
	3	.6569	.5513	7.47	−.01		5	.7444	.2737	7.67	−.01
	4	.6625	.5569	7.45	−.03		6	.7486	.2779	7.64	−.06
	5	.6681	.5625	7.44	−.04		7	.7528	.2821	7.68	−.03
	6	.6736	.5680	7.47	.00		8	.7569	.2862	7.75	+.02
	7	.6792	.5736	7.51	+.04		9	.7611	.2904	7.72	−.02
	8	.6847	.5791	7.45	−.02		10	.7653	.2946	7.74	−.02
							11	.7694	.2987	7.79	+.02
544	1	751.5729	1.5572	7.51	+.03						

Plate V of *Bulletin* No. 24 shows a sample plate of Z Vulpeculae on which the comparison stars are marked.

The measured densities of the extrafocal star images in terms of the microphotometer wedge scale are converted to Δmagnitudes by use of the corrected reduction curve (Table IX of *Bulletin* No. 24) for the emulsion concerned. For plates affected by sky fog the reduction curve for the plate is employed up to the point where the fog curve joins the clear-film curve. After correction for atmospheric extinction and for reduction to center, using Tables III and IV of *Bulletin* No. 24, the magnitude differences $a - v$, $b - v$, etc. are taken for each exposure; these are multiplied by the factor 1.15 in order to bring the scale into agreement with that of the Harvard Sequences. Employing the adopted photographic magnitudes of the comparison stars we now obtain the magnitude of the variable star for each comparison, and finally the mean of four values v_a, v_b, etc. as the observed magnitude of the variable star for each exposure.

The observed photographic magnitudes appear in Table III. The coordinates of 495 points are now available for the determination of the light curve. They are first assembled into 64 normal magnitudes, of which numbers 15 to 26 and 42 to 52 occur outside eclipse. For convenience in the solutions the number of observations in each normal place outside eclipse has been made nearly the same; so also for the normal places within eclipse, the number of observations in the former being about twice that in the latter normals.

The discussion of the normal magnitudes in Table IV for the determination of the orbital elements follows the notation and formulae developed at the Princeton Observatory.[7] The normal magnitudes are first converted into intensities by the formula:

$$\log l = 0.4\,(m_0 - m),$$

where m_0, the magnitude during the intervals between eclipses, is taken to be $7^m.421$. The observed normal intensities in the sixth column of Table IV are computed with the final value, $m_0 = 7^m.451$, and are expressed in terms of the total light of the system. On plotting normal intensities against phases, column (2), we find the time of primary minimum in agreement with the corrected elements given above. The secondary minimum occurs $0^d.005$ after the time midway between primary minima and is of less duration. Outside eclipse the light is not constant. The effect of ellipticity in the stars is marked, and a "reflection effect" is evident from the slight rise of the curve toward secondary minimum; in addition, the curve is found to be somewhat higher after secondary minimum than before, indicating brightening of the stars at periastron.

The intensity l of the light outside eclipse is represented by the expression:

$$l = a - b\cos\theta - c\cos^2\theta - f\sin\theta,$$

in which a is the rectified intensity which is constant between eclipses, $2b$ is the difference in intensity between the two sides of the star eclipsed at secondary minimum, $c = \frac{1}{2}z$ is the ellipticity constant that determines the ratio between the major and the minor axes of the stars, $2f$ is that part of the difference in intensity between periastron and apastron which is not taken up in the reflection constant[8] and θ is the longitude from principal conjunction.

We have for each normal intensity outside eclipse an equation of condition of the form:

$$\delta a - \cos\theta\cdot\delta b - \cos^2\theta\cdot\delta c - \sin\theta\cdot\delta f = l_o - l_c,$$

where l_c is computed from preliminary values of the constants derived by a graphical solution. Weight unity is assigned to each normal intensity. The following normal

[7] H. N. Russell, *Astrophysical Journal*, 35, 315, 1912; 36, 54, 1912; H. N. Russell and H. Shapley, *Astrophysical Journal*, 36, 239, 385, 1912.

[8] H. N. Russell, *Astrophysical Journal*, 36, 69, 1912.

TABLE IV

NORMAL MAGNITUDES OF Z VULPECULAE

No.	Phase	θ	No. Obs.	Observed Magnitude	Intensity Observed	Rectified Intensity	Computed Magnitude (uniform)	O—C (uniform)
	days	°						
1	0.0121	1.77	4	9.070	0.225	0.273	9.050	+0.020
2	0.0261	3.83	4	8.898	0.264	0.315	8.855	+0.043
3	0.0422	6.19	5	8.617	0.342	0.401	8.622	−0.005
4	0.0584	8.56	4	8.452	0.398	0.460	8.446	+0.006
5	0.0742	10.88	5	8.291	0.461	0.527	8.281	+0.010
6	0.0878	12.88	5	8.176	0.513	0.583	8.176	0.000
7	0.0987	14.47	5	8.070	0.565	0.638	8.097	−0.027
8	0.1139	16.70	6	8.003	0.601	0.675	8.003	0.000
9	0.1312	19.24	6	7.923	0.647	0.723	7.901	+0.022
10	0.1482	21.73	6	7.811	0.718	0.797	7.821	−0.010
11	0.1668	24.46	6	7.771	0.745	0.824	7.743	+0.028
12	0.1913	28.05	6	7.644	0.837	0.920	7.654	−0.010
13	0.2161	31.69	5	7.559	0.905	0.985	7.589	−0.030
14	0.2525	37.03	9	7.527	0.932	1.016	7.540	−0.013
15	0.2968	43.52	12	7.516	0.942	1.010	7.527	−0.011
16	0.3454	50.65	12	7.520	0.938	0.997	7.517	+0.003
17	0.4956	72.68	12	7.463	0.989	1.009	7.473	−0.010
18	0.5469	80.20	12	7.463	0.989	1.001	7.464	−0.001
19	0.6089	89.29	9	7.447	1.004	1.008	7.456	−0.009
20	0.6486	95.11	9	7.450	1.001	1.005	7.455	−0.005
21	0.6904	101.24	9	7.464	0.988	0.991	7.454	+0.010
22	0.7621	111.76	8	7.479	0.974	0.984	7.462	+0.017
23	0.8028	117.73	11	7.466	0.986	1.000	7.466	0.000
24	0.8786	128.84	12	7.485	0.969	0.999	7.484	+0.001
25	0.9292	136.26	12	7.513	0.944	0.984	7.496	+0.017
26	0.9836	144.24	12	7.509	0.948	0.999	7.508	+0.001
27	1.0265	150.53	7	7.526	0.933	0.991	7.520	+0.006
28	1.0503	154.02	7	7.533	0.927	0.990	7.536	−0.003
29	1.0870	159.40	6	7.561	0.904	0.970	7.561	0.000
30	1.1142	163.39	6	7.585	0.884	0.952	7.585	0.000
31	1.1415	167.39	6	7.610	0.864	0.932	7.612	−0.002
32	1.1691	171.44	6	7.641	0.840	0.907	7.643	−0.002
33	1.1941	175.11	6	7.677	0.812	0.878	7.673	+0.004
34	1.2224	179.26	7	7.701	0.794	0.859	7.702	−0.001
35	1.2521	183.61	7	7.695	0.799	0.864	7.695	0.000
36	1.2823	188.04	5	7.679	0.811	0.874	7.658	+0.021
37	1.2992	190.52	6	7.639	0.841	0.905	7.638	+0.001
38	1.3171	193.15	4	7.601	0.871	0.937	7.614	−0.013
39	1.3457	197.34	8	7.598	0.873	0.937	7.586	+0.012
40	1.4058	206.15	6	7.520	0.938	0.997	7.535	−0.015
41	1.4441	211.77	6	7.494	0.961	1.013	7.513	−0.019
42	1.5231	223.35	11	7.465	0.987	1.022	7.489	−0.024
43	1.5690	230.08	12	7.487	0.967	0.991	7.477	+0.010
44	1.6134	236.60	9	7.477	0.976	0.992	7.468	+0.009
45	1.6547	242.65	11	7.450	1.001	1.007	7.458	−0.008
46	1.7116	251.00	12	7.466	0.986	0.985	7.450	+0.016
47	1.8596	272.70	14	7.421	1.027	1.023	7.446	−0.025
48	1.9310	283.17	11	7.447	1.004	1.009	7.457	−0.010
49	1.9940	292.41	12	7.460	0.992	1.008	7.469	−0.009
50	2.0521	300.93	12	7.493	0.962	0.997	7.490	+0.003

TABLE IV—Continued
NORMAL MAGNITUDES OF Z VULPECULAE

No.	Phase	θ	No. Obs.	Observed Magnitude	Observed Intensity	Rectified Intensity	Magnitude Computed (uniform)	O–C (uniform)
	days							
51	2.1085	309°.20	12	7.540	0.921	0.970	7.508	+0.032
52	2.1699	318.20	12	7.550	0.913	0.983	7.532	+0.018
53	2.2096	324.03	13	7.589	0.881	0.946	7.544	+0.045
54	2.2567	330.93	9	7.618	0.857	0.933	7.629	−0.011
55	2.2883	335.57	6	7.722	0.779	0.856	7.733	−0.011
56	2.3223	340.55	6	7.884	0.671	0.746	7.886	−0.002
57	2.3403	343.19	5	7.980	0.614	0.687	7.984	−0.004
58	2.3635	346.59	6	8.141	0.530	0.599	8.143	−0.002
59	2.3809	349.15	4	8.332	0.444	0.507	8.292	+0.040
60	2.3948	351.18	4	8.463	0.394	0.454	8.426	+0.037
61	2.4103	353.46	4	8.634	0.336	0.392	8.596	+0.038
62	2.4304	356.40	4	8.866	0.272	0.313	8.871	−0.005
63	2.4435	358.33	4	9.053	0.229	0.277	9.053	0.000
64	2.4543	359.91	3	9.085	0.222	0.270	9.122	−0.037

equations result:

$$\begin{aligned} +25.000\,\delta a + 1.645\,\delta b - 7.360\,\delta c - 1.004\,\delta f &= -0.0069 \\ +7.360\,\delta b - 0.829\,\delta c + 0.004\,\delta f &= +0.0242 \\ +3.464\,\delta c - 0.101\,\delta f &= +0.0161 \\ +26.642\,\delta f &= -0.0288 \end{aligned}$$

The solution of these equations gives:

	Preliminary	Correction		Final	Probable Error	Weight
a =	0.970	+0.0029	=	0.973	±0.0029	9.305
b =	0.0115	+0.0039	=	0.015	±0.0032	7.162
c =	0.0862	+0.0116	=	0.098	±0.0077	1.275
f =	+0.0058	−0.0009	=	+0.005	±0.0017	26.480
m_0 =	$7^m.421$	$+0^m.030$	=	$7^m.451$	$\pm 0^m.0031$	

The correction to m_0 is the magnitude difference corresponding to the intensity difference between the final value of a and unity. The final m_0 is the rectified magnitude of the system between eclipses, i. e., after the effects of ellipticity, radiation and periastron brightening are removed. The probable error of a normal intensity, weight unity, is ± 0.0083. This completes the solution of the light curve independent of the eclipses.

The effects of ellipticity, radiation, and brightening at periastron are removed from the normal intensities during eclipse by use of the formula:

$$R = \frac{l + b(1 + \cos\theta) + cl\cos^2\theta + f\sin\theta}{a + b},$$

where R is the rectified intensity. These values, that appear in the seventh column of Table IV, are now plotted against the corresponding phases in the second column of the table and symmetrical curves are drawn to represent them. The losses of light at primary and secondary minima, in terms of the total light of the system, are taken respectively to be $1-\lambda_p=0.740$ and $1-\lambda_s=0.145$. Trial solutions of the light curve during primary minimum indicate that the eclipse is not partial; this must also obtain during secondary minimum, since the shorter duration of the latter shows that the stars are nearer periastron at this time. Inasmuch as neither minimum has a constant phase, we assume the eclipses grazing. The fainter star is, of course, in front at primary minimum, since the loss of light here exceeds one-half of the total. We readily decide by computing k, the ratio of the radii of the two stars, whether this eclipse is total or annular:

$$\text{If primary is total,} \quad k^2=\frac{1-\lambda_s}{\lambda_p}=0.5577,\ k=0.747$$

$$\text{If primary is annular,} \quad k^2=\frac{1-\lambda_p}{\lambda_s}=0.8656,\ k=0.930$$

The latter value of k is required by the slope of the light curve. Hence the eclipse is grazing annular.

The theoretical light curve that best represents the observed normal intensities at primary minimum is now computed for suitable fractions, a_1, of the greatest loss of light, by use of the formula:

$$\sin^2\theta-A=B(1-z\cos^2\theta)\psi(k,a_1)$$

where $A=\sin^2\theta_2$, $B=\sin^2\theta_2-\sin^2\theta_3$; θ_2, θ_3 are values of θ corresponding respectively to $a_1=0.6$ and 0.9; $z=2c=0.196$, $k=0.930$, and the function $\psi(k, a_1)$ is taken from Russell's *Table II*. The required values of A and B are found respectively to be 0.0424 and 0.0467. At the beginning and end of eclipse $\sin^2\theta'=0.3711$.

Although the secondary minimum is shallow, the observations at this phase are sufficiently numerous and accurate to warrant an independent determination of the light curve. By a similar procedure we derive for this minimum $A=0.0340$, $B=0.0368$ and $\sin^2\theta'=0.289$.

On the assumption that the orbits of the two stars are circles, the orbital inclination, i, and the semi-major axis of the larger star, a_b, are found by the formulae:

$$\cos^2 i\cos^2\theta'+\sin^2\theta'=a_b^2\,(1-z\cos^2\theta')(1+k)^2$$
$$\cos^2 i=a_b^2\,(1-z)[1+kp(k,a_0)]^2$$

These circular elements are:

At primary minimum, $a'=0.337$, $i'=88°48'$
At secondary minimum, $a''=0.300$, $i''=88°56'$

The lack of agreement in these values is ascribed to the eccentricity of the orbit. It remains to derive the elliptic elements.

The eccentricity of the orbit, e, and the longitude of periastron, ω, are found from the formulae:

$$e\cos\omega = \frac{\pi(t_2 - t_1 - \frac{P}{2})}{P(1+\operatorname{cosec}^2 i)}, \quad e\sin\omega = \frac{a_b'' - a_b'}{\eta(a_b'' + a_b')} = \frac{\cot i'' - \cot i'}{2\eta(\cot i'' + \cot i')},$$

in which $t_2 - t_1 - \frac{P}{2}$, the displacement of the secondary minimum, is $+0.005$ days and $\eta = \frac{1}{3}(2+\cos\theta') = 0.931$.
Whence we derive:

$$e = 0.047, \quad \omega = 274°$$

By the formulae:

$$a_b' = a_b(1 - \eta\cdot e\sin\omega), \quad \cot i' = \cot i\,(1 - 2\eta\cdot e\sin\omega)$$

we have:

$$a_b = 0.323, \quad i = 88°54'.$$

The semi-minor axis of the brighter star is found by the formulae:

$$\epsilon^2 = z\operatorname{cosec}^2 i, \quad b_b^2 = a_b^2\,(1-\epsilon^2),$$

and the semi-polar axis by Russell's *formula* (*38*).

The ratio $\frac{J_1}{J_2}$ of the surface intensities of the two stars, and the densities, ρ_b and ρ_f, referred to the sun's density with the assumption that the two stars are equally massive, are found by aid of *formulae* (*s*) and (*t*):

$$\frac{J_b}{J_f} = \frac{k^2 L_1}{L_2}, \quad \rho = \frac{0.00672}{P^2\cdot a\cdot b\cdot c}$$

The elements of the system, given in Table V, are derived on the supposition that the disks of the two stars are uniformly luminous. A number of trials indicated that the light variations at the two conjunctions can not be so well represented by curves computed for stars completely darkened at the limb. The unit of length is the mean distance between the centers of the two stars and the unit of light is the total light of the system. In the absence of spectroscopic data, the absolute linear dimensions, masses and densities are unknown.

It is interesting to compare these results with the elements derived by Shapley (*loc. cit.*) from the writer's visual observations of this star. The photographic range of magnitude is $0^m.03$ greater at primary minimum and $0^m.08$ less at secondary minimum than the visual range, which indicates that the fainter star is of slightly later spectral class

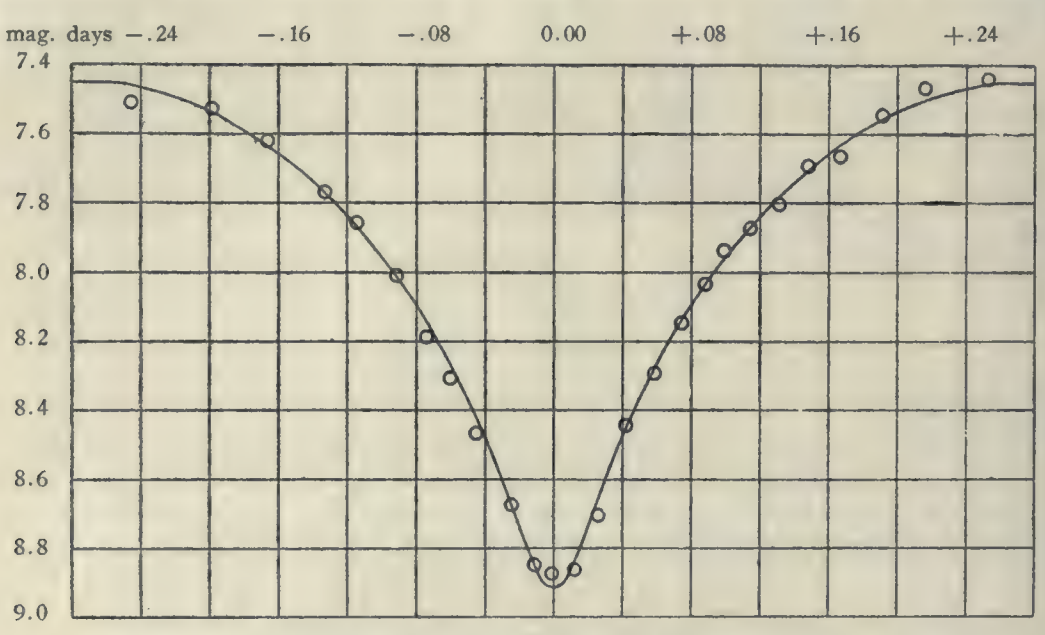

FIGURE 1. RECTIFIED NORMAL MAGNITUDES AND LIGHT CURVE OF Z VULPECULAE DURING PRIMARY MINIMUM (UNIFORM SOLUTION)

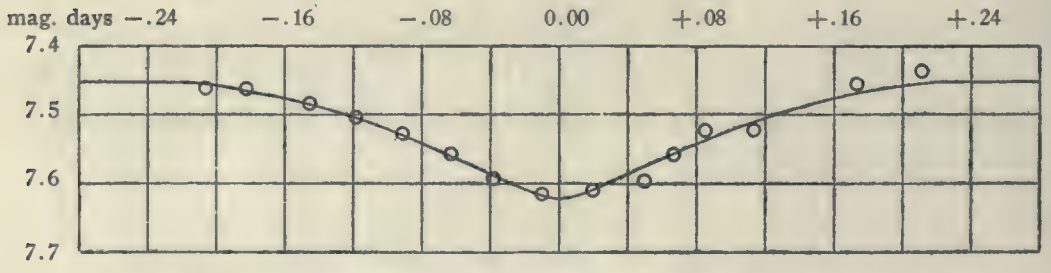

FIGURE 2. RECTIFIED NORMAL MAGNITUDES AND LIGHT CURVE OF Z VULPECULAE DURING SECONDARY MINIMUM (UNIFORM SOLUTION)

than its brighter companion. As in the case of RX Herculis[9] the duration of minimum is longer from the photographic observations. At primary minimum the eclipse is here found grazing annular, while from the visual observations it appears to be grazing total.

[9]R. H. Baker and E. E. Cummings, *Laws Observatory Bulletin*, No. 25.

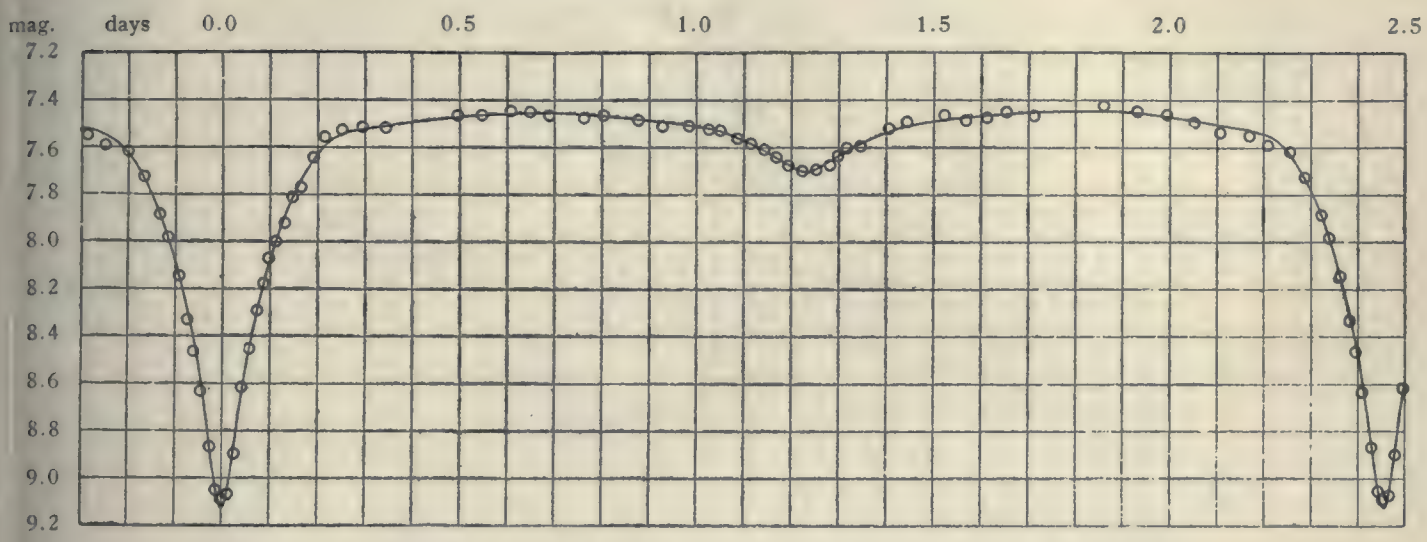

FIGURE 3. NORMAL MAGNITUDES AND THEORETICAL LIGHT CURVE OF Z VULPECULAE

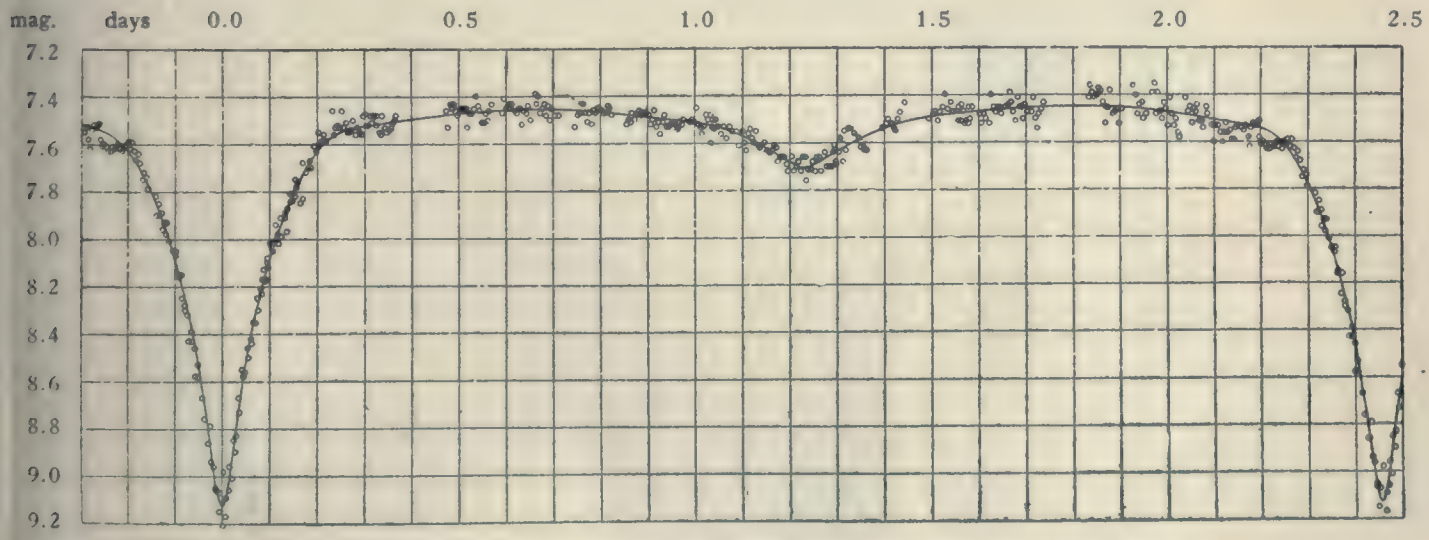

FIGURE 4. OBSERVED MAGNITUDES AND THEORETICAL LIGHT CURVE OF Z VULPECULAE

The most striking difference between the two series of observations is in the position of the secondary minimum. From the visual observations this minimum appears to occur 0.022 days earlier than the time midway between principal minima; from the photographic results it comes 0.005 days later than the middle phase. Another point is to be noted in Table V. The longitude of periastron is 274°, that is, the line of apsides is di-

TABLE V

ELEMENTS OF THE SYSTEM OF Z VULPECULAE
UNIFORM SOLUTION

Period of revolution	P	2.45492 days
Daily motion in longitude	μ	146°.644
Epoch of primary minimum	T	J.D.2419679.673G.M.T.
Displacement of secondary minimum	$t_s - t_z - \frac{1}{2}P$	+0.005 days
Semi-duration of eclipse, primary	.....	0.2559 days
Semi-duration of eclipse, secondary	.....	0.222 days
Eccentricity of orbit	e	0.05
Longitude of periastron	ω	274°
Inclination of orbit	i	88°54′
Least apparent distance of centers	$\cos i$	0.019
Ratio of radii	k	0.930
Ratio of major and minor axes	$\frac{b}{a}$	0.898
Semi-major axis, brighter star	a_b	0.323
Semi-major axis, fainter star	a_f	0.300
Semi-minor axis, brighter star	b_b	0.290
Semi-minor axis, fainter star	b_f	0.270
Density of brighter star*	ρ_b	0.044⊙
Density of fainter star*	ρ_f	0.055⊙
Maximum obscuration during eclipse	a_0	1.00
"Radiation effect" constant	b	0.015
"Periastron effect" constant	f	+0.005
Light of brighter star	L_b	0.855
Light of fainter star, brighter side	L_f	0.145
Light of fainter star, fainter side	$L_f - 2b$	0.115
Ratio of surface intensities	$\frac{J_b}{J_f}$	5.1
Magnitude of system	.....	7.451
Magnitude of brighter star	.....	7.62
Magnitude of fainter star	.....	9.55
Magnitude at primary minimum	.....	9.125
Magnitude at secondary minimum	.....	7.706
Range of magnitude, primary	.....	1.679
Range of magnitude, secondary	.....	0.260

*assuming equal masses

rected nearly toward the earth. Nevertheless the light curve rises higher after secondary minimum than before it; and though the difference is small, the reality of the "periastron effect" is established by the least-squares solution. If anything is to be inferred, it is that the brightening of the stars due to their closer approach reaches a maximum after periastron passage.

Figures 1 and 2 show the arrangement of the rectified normal magnitudes at primary and at secondary minimum relative to the computed rectified curve from the uni-

form solution. The entire theoretical light curve is shown in Figure 3, with the observed normal magnitudes. Asymmetry in the arrangement of the points is clearly exhibited, that is evidently real* and similar around the two minima. Before each eclipse the points fall below the curve, while the recovery of light after eclipse is overdone in each case, the points rising above the curve for a time. This is well shown in Figure 4, wherein the light curve again appears, with the observed magnitudes from the individual exposures.

The residuals for the observed magnitudes, scaled from the theoretical curve, are

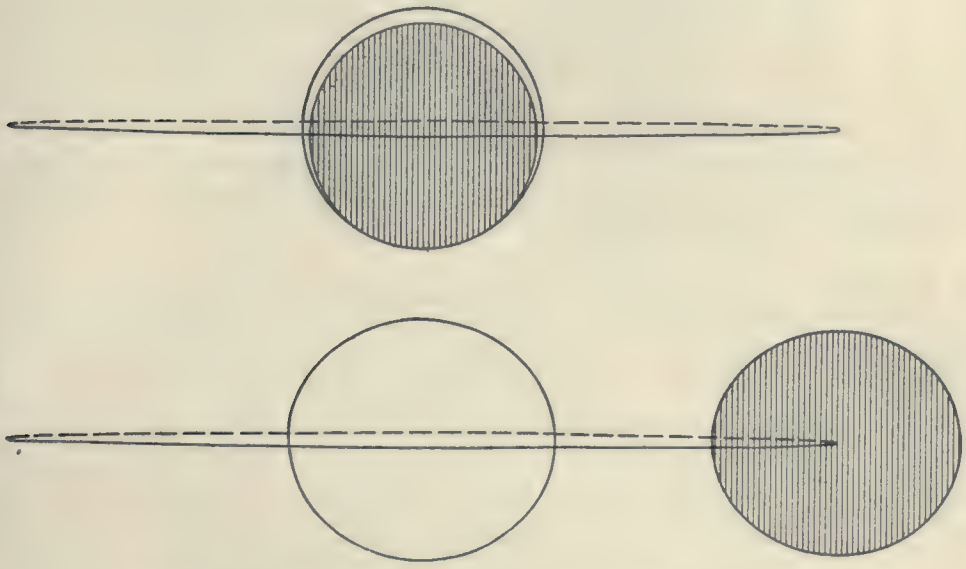

FIGURE 5. THE SYSTEM OF Z VULPECULAE AT PRIMARY MINIMUM AND AT GREATEST ELONGATION

given in the sixth column of Table III. The residuals for the normal magnitudes are found in Table IV. Both are from the uniform solution. The results from the darkened solution are not given, since the observations are not so well represented. The probable error of a normal magnitude, computed from the residuals for all the normals, is $\pm 0^{m}.012$; for the points within eclipse it is $\pm 0^{m}.013$ and without eclipse it is $\pm 0^{m}.009$. The probable error of a single magnitude determination is $\pm 0^{m}.026$. This is the same as the probable error we found for the observations of RX Herculis (*loc. cit.*). It strengthens our confidence in the extrafocal method for the study of variable stars. As before, it is well to remember that the computed light curve does not do entire justice to the observations.

* Since this was written, Professor Russell has kindly pointed out in a letter that the reality of the minor oscillations in the light curve of RX Herculis (*Laws Observatory Bulletin*, No. 25, page 171) is questionable.

The system of Z Vulpeculae is represented in Figure 5, at principal conjunction and at greatest elongation. It is composed of two elongated stars of nearly the same size. The radius of either star is about three-tenths the distance between their centers.

The orbital inclination is 88°54′. The eccentricity of the orbit is 0.05 and periastron passage occurs near secondary minimum. This minimum is 0.005 days later than the phase equal to half the period. At principal conjunction the eclipse is grazing annular. The semi-duration of primary minimum is $6^h\ 08^m$ and of secondary minimum is $5^h\ 20^m$.

The star eclipsed at primary minimum is of magnitude 7.62, and it contributes 85.5 per cent to the light of the system. It is of earlier spectral class than its fainter companion. A radiation effect of the brighter star on the fainter and a slight increase in the light of the system near periastron, is established by the least-squares solution.

The solution that assumes the stars to be uniformly luminous gives the more satisfactory representation of the observed magnitudes. For this reason the results from the alternate solution are omitted. The failure of the darkened solution may be ascribed to the asymmetry in the light variations whose characteristics have been noted. The probable error of a single observed magnitude is $\pm 0^m.026$.

1916 July 8

THE UNIVERSITY OF MISSOURI BULLETIN

ASTRONOMICAL SERIES

LAWS OBSERVATORY BULLETIN NO. 27

THE ECLIPSING BINARY TV CASSIOPEIAE*

By Edith E. Cummings

In 1911 Astbury[1] announced the variability of TV(45.1911) Cassiopeiae (R.A. $=0^h13^m.9$, Decl. $=+58°35'$), and soon afterward he derived[2] the following elements:

Primary Minimum = J.D. 2419338.33 + $1^d.8126 \cdot E$, G.M.T. (Astbury)

Yendell,[3] who observed it in 1911-12, found a range of $0^m.88$ at primary minimum with no constant phase, and no trace of a secondary minimum. The observations of Lazzarino[4] and of Biesbroeck[5] confirm the period and agree in indicating no halt at minimum. Nijland's curve[6] shows a constant phase at minimum of about $0^d.03$, but here again there is no trace of a secondary. McDiarmid[7] has published a visual curve and an orbit computed from the equations of Russell and Shapley. McDiarmid's curve shows a primary minimum of $1^m.05$ with no constant phase, and a secondary of $0^m.09$ coming 17 minutes before mid-period. Ellipticity in the stars and "reflection" are indicated by the manner in which the light continues to vary outside eclipse. In the solution for the elements it was found that the smaller, fainter star, which is eclipsed at secondary minimum, contributes about 15 per cent of the light of the system.

The present discussion is based on 51 plates taken by Dr. Baker and the writer with the 5-inch camera during the period from July 1914 to October 1915. These plates were exposed about 1.5 mm. outside of focus, and were sensitometrized and developed after the procedure described in *Laws Observatory Bulletin* No. 24. All are Seed 27; for plates 251 to 334 the emulsion number is 14540, for 352 to 408 number 14794, for 515 to 606 number 15194. The maximum number of exposures possible without blending of the star images

*Submitted in partial fulfillment of the requirements for the degree of Master of Arts in the University of Missouri.

[1] T. H. Astbury, *Astronomische Nachrichten*, 189, 357, 1911.
[2] T. H. Astbury, *Journal of the British Astronomical Association*, 22, 279, 1912.
[3] P. S. Yendell, *Astronomische Nachrichten*, 190, 430, 1912; 191, 427, 1912.
[4] O. Lazzarino, *Astronomische Nachrichten*, 197, 373, 1914.
[5] G. van Biesbroeck, *Annales de l'observatoire royal de Belgique*, 13, 181, 1913.
[6] H. Shapley, *Princeton Contributions*, No. 3, 132, 1915.
[7] R. J. McDiarmid, *Astrophysical Journal*, 42, 412, 1915.

TABLE I

PLATES OF TV CASSIOPEIAE

Plate No.	Date, G.M.T.	Julian Day	Epoch	Correction to Sun	Helioc. Phase	Tel.	Hour Angle	No. Exp.	Film
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	1914 d. h. m.			days	d. to d.		h.		
251	July 19 21 00	2420333.8750	549	−0.0005	0.4094 0.4428	W	−1.6	9	18
259	23 20 48	337.8667	551	3	0.7796 0.8150	W	−1.4	8	13
262	24 18 04	338.7528	551	2	1.6581 1.6998	W	−4.2	7	14
266	26 17 03	340.7104	553	−0.0001	1.8033 0.0254	W	−5.1	6	12
274	28 20 25	342.8507	554	0	0.3185 0.3643	W	−1.5	9	20
275	28 21 20	342.8889	554	0	0.3692 0.3900	W	−0.6	6	15
281	Aug. 15 15 20	360.6389	564	+0.0011	1.7888 0.0575	W	−5.5	9	13
282	15 17 43	360.7382	564	11	0.0672 0.1422	W	−3.1	12	22
283	16 15 00	361.6250	564	11	0.9679 1.0137	W	−5.8	12	14
284	16 16 15	361.6771	564	11	1.0199 1.0658	W	−4.5	11	15
285	16 17 38	361.7347	564	11	1.0776 1.1234	W	−3.1	12	23
286	16 18 54	361.7875	564	11	1.1304 1.1769	W	−1.8	12	22
291	18 15 25	363.6424	565	12	1.1727 1.2186	W	−5.2	11	16
292	18 16 55	363.7049	565	13	1.2353 1.2812	W	−3.7	12	18
293	18 18 10	363.7569	565	13	1.2874 1.3332	W	−2.4	11	16
294	18 19 32	363.8139	565	13	1.3444 1.3902	W	−1.1	12	18
295	18 21 04	363.8778	565	13	1.4124 1.4499	E	+0.5	10	20
296	20 18 06	365.7542	566	14	1.4687 1.5228	W	−2.4	11	25
310	Sept. 17 21 04	393.8778	582	28	0.5997 0.6372	E	+2.4	10	18
311	17 22 05	393.9201	582	28	0.6421 0.6796	E	+3.5	10	18
320	19 18 12	395.7583	583	29	0.6803 0.6928	W	−0.3	4	clear
321	19 19 14	395.8014	583	29	0.7067 0.7532	E	+0.7	11	19
322	19 20 32	395.8556	583	29	0.7608 0.8067	E	+2.0	12	21
325	20 17 16	396.7194	583	29	1.6285 1.6664	W	−1.2	9	18
326	20 18 11	396.7576	583	29	1.6712 1.7004	W	−0.3	8	12
327	20 19 30	396.8125	583	29	1.7164 1.7664	E	+1.0	10	21
331	22 16 25	398.6840	584	30	1.7698 0.0198	W	−1.9	10	25
332	22 17 56	398.7472	585	30	0.0274 0.0691	W	−0.4	7	21
333	22 19 22	398.8070	585	30	0.0829 0.1406	E	+1.0	10	26
334	22 20 50	398.8680	585	30	0.1482 0.1941	E	+2.5	12	25
352	Oct. 17 15 55	423.6632	598	36	1.3885 1.4135	W	−0.8	7	18
358	17 19 33	423.8146	598	36	1.5316 1.5774	E	+2.9	12	31
359	17 20 55	423.8715	598	36	1.5927 1.6260	E	+4.2	9	26
366	22 13 49	428.5757	601	36	0.8479 0.9035	W	−2.6	11	27
367	22 15 20	428.6389	601	36	0.9111 0.9667	W	−1.0	11	26
383	Nov. 10 13 41	447.5701	611	35	1.7163 1.7718	W	−1.4	9	28
390	1915 14 13 54	451.5792	614	35	0.3069 0.3278	W	−1.0	6	24
408	Jan. 14 17 17	512.7201	647	+0.0007	1.6168 1.6626	E	+6.4	12	27
515	July 15 17 41	694.7368	748	−0.0008	0.5635 0.6010	W	−5.0	10	19
526	Sept. 2 14 31	743.6049	775	+0.0021	0.4943 0.5367	W	−5.0	10	18
527	2 15 49	743.6590	775	21	0.5429 0.5873	W	−3.8	9	19
535	7 14 13	748.5924	778	23	0.0442 0.0775	W	−5.1	7	19
536	7 15 10	748.6319	778	23	0.0838 0.1213	W	−4.0	10	20
538	7 18 07	748.7549	778	23	0.2025 0.2484	W	−1.2	12	20
539	7 19 10	748.7986	778	23	0.2546 0.2796	W	−0.2	7	13
547	11 15 14	752.6347	780	25	0.4615 0.4991	W	−3.6	10	17
559	12 21 23	753.8910	780	26	1.7054 1.7624	E	+2.4	11	26
564	13 19 16	754.8028	781	26	0.8171 0.8546	E	+0.4	10	14
565	13 20 25	754.8507	781	26	0.8609 0.9025	E	+1.5	11	18
605	Oct. 14 16 50	785.7014	798	35	0.9108 0.9274	W	0.0	5	clear
606	14 17 37	785.7340	798	35	0.9351 0.9684	E	+0.7	9	18

was found to be 12; and, since in a sky of normal transparency 6 minutes was sufficient to give the correct density, most of the plates at and near maximum light were exposed for only 72 minutes. At primary minimum the length of a single exposure was 8 or 10 minutes. A description of the plates is found in Table I. The dates in columns (2) and (3) and the hour angle in column (8) are given for the middle exposure. The epoch, column (4), and the heliocentric phases of the first and last exposures, column (6), are based on Astbury's elements corrected by the addition of $+0^d.0001$ to the epoch. The wedge readings on the film, column (10), are frequently high, so that especial care was necessary in order to avoid error in the use of the reduction curves.

The three comparison stars (Table II) are close to the variable and in no case farther than 18 mm. from the center of the plate. Thus the corrections for reduction to center and for atmospheric extinction are small. Star *c* has magnitude 6.54 in *Revised Harvard Photometry*, and since it is of class A this is adopted as its photographic magnitude. In measuring, four settings of the microphotometer wedge were made on each image. These wedge readings were reduced to Δmagnitudes by means of the cor-

TABLE II
COMPARISON STARS FOR TV CASSIOPEIAE

Star	B.D.		R.A.(1900)	Decl.(1900)	Adopted Photographic Magnitude	Harvard	
	No.	Mag.				Visual Mag.	Class
			h m				
v	+58°30		0 13.9	+58°35′			
a	58 28	8.0	13.8	59 10	8.18		
b	58 24	7.8	12.4	58 30	7.68		
c	58 11	7.3	5.3	59 6	6.54	6.54	A

rected reduction curve for the emulsion concerned (See *Bulletin* 24, Table IX), which was derived by the writer from sensitometer images on plates of this star and RX Herculis. When the plate was fogged, the individual curve for that plate was used up to the point where it joins the clear film curve. These Δmagnitudes were then corrected for atmospheric extinction and reduced to the center of the plate, by use of Tables III and IV of *Bulletin* No. 24. The magnitudes of the comparison stars were determined by combining the differences $c - a$, $c - b$, for an equal number of plates taken on each side of the pier. Using these adopted photographic magnitudes of *a*, *b*, and *c* the differences $a - v$, $b - v$, $c - v$, were converted into magnitudes v_a, v_b, v_c, and the mean of these was taken as the magnitude of the variable for each exposure. After this work was done it was discovered that the normal Laws Observatory scale requires multiplication by the factor 1.15 (See *Laws Observatory Bulletin* No. 24, Section VI) to bring it into agreement with the Harvard Sequences. This necessitated a revision of the magnitudes for comparison stars and variable. First the Harvard magnitude was reduced to the normal Laws Observatory scale; assuming that the two scales agree at magnitude 6.00, it was found that on our scale the star *c* would be of magnitude 6.470. Thus all the original

TABLE III

PHOTOGRAPHIC OBSERVATIONS OF TV CASSIOPEIAE

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O–C (darkened)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O–C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag.			2420	days		mag.
251	1	333.8583	0.4094	7.35	−.03	281	11	360.6938	0.0575	7.99	.00
	2	.8625	.4136	7.36	−.02						
	3	.8667	.4178	7.38	.00	282	1	360.7035	0.0672	7.86	−.05
	4	.8708	.4219	7.36	−.02		2	.7104	.0741	7.87	.00
	5	.8750	.4261	7.37	−.01		3	.7174	.0811	7.78	−.03
	6	.8792	.4303	7.36	−.02		4	.7243	.0880	7.70	−.06
	7	.8833	.4344	7.37	−.01		5	.7312	.0949	7.67	−.04
	8	.8875	.4386	7.39	+.01		6	.7382	.1019	7.67	.00
	9	.8917	.4428	7.39	+.01		7	.7451	.1088	7.60	−.04
							8	.7521	.1158	7.59	−.02
259	1	337.8535	0.7796	7.40	−.04		9	.7590	.1227	7.56	−.02
	2	.8576	.7837	7.37	−.07		10	.7660	.1297	7.54	−.01
	3	.8618	.7879	7.47	+.03		11	.7729	.1366	7.53	.00
	4	.8667	.7928	7.50	+.06		12	.7785	.1422	7.48	−.03
	5	.8722	.7983	7.45	+.01						
	6	.8778	.8039	7.41	−.03	283	1	361.6042	0.9679	7.36	.00
	7	.8833	.8094	7.44	.00		2	.6083	.9720	7.45	.00
	8	.8889	.8150	7.45	.00		3	.6125	.9762	7.40	−.05
							4	.6167	.9804	7.38	−.07
262	1	338.7319	1.6581	7.45	−.05		5	.6208	.9845	7.41	−.04
	2	.7389	.6651	7.54	+.02		6	.6250	.9887	7.36	−.09
	3	.7458	.6720	7.52	−.01		7	.6292	.9929	7.44	−.01
	4	.7528	.6790	7.62	+.06		8	.6333	.9970	7.47	+.02
	5	.7597	.6859	7.60	+.02		9	.6375	1.0012	7.45	.00
	6	.7667	.6929	7.59	−.02		10	.6417	.0054	7.41	−.04
	7	.7736	.6998	7.61	−.03		11	.6458	.0095	7.46	+.01
							12	.6500	.0137	7.43	−.02
266	1	340.6896	1.8033	8.33	+.06						
	2	.6965	.8102	8.39	+.12	284	1	361.6562	1.0199	7.44	−.01
	3	.7035	0.0046	8.29	+.02		2	.6604	.0241	7.45	.00
	4	.7104	.0115	8.25	.00		3	.6646	.0283	7.51	+.06
	5	.7174	.0185	8.29	+.06		4	.6688	.0325	7.41	−.04
	6	.7243	.0254	8.23	+.03		5	.6729	.0366	7.41	−.04
							6	.6771	.0408	7.41	−.04
274	1	342.8299	0.3185	7.41	+.01		7	.6812	.0449	7.51	+.06
	2	.8340	.3226	7.40	.00		8	.6854	.0491	7.50	+.05
	3	.8382	.3268	7.44	+.04		9	.6896	.0533	7.48	+.03
	5	.8465	.3351	7.43	+.03		11	.6979	.0616	7.50	+.06
	6	.8507	.3393	7.38	−.02		12	.7021	.0658	7.50	+.06
	9	.8632	.3518	7.37	−.02						
	10	.8674	.3560	7.37	−.02	285	1	361.7139	1.0776	7.44	.00
	11	.8715	.3601	7.39	.00		2	.7180	.0817	7.45	+.01
	12	.8757	.3643	7.35	−.04		3	.7222	.0859	7.47	+.03
							4	.7264	.0901	7.45	+.01
275	1	342.8806	0.3692	7.39	.00		5	.7306	.0943	7.40	−.04
	2	.8847	.3733	7.37	−.02		6	.7347	.0984	7.41	−.03
	3	.8889	.3775	7.35	−.04		7	.7389	.1026	7.40	−.03
	4	.8931	.3817	7.40	+.02		8	.7430	.1067	7.44	+.01
	5	.8972	.3858	7.35	−.03		9	.7472	.1109	7.45	+.02
	6	.9014	.3900	7.37	−.01		10	.7514	.1151	7.44	+.01
							11	.7556	.1193	7.41	−.02
281	1	360.6125	1.7888	8.17	−.04		12	.7597	.1234	7.39	−.04
	2	.6180	.7943	8.29	+.05						
	3	.6250	.8013	8.25	−.01	286	1	361.7667	1.1304	7.35	−.08
	4	.6319	.8082	8.27	.00		2	.7708	.1345	7.41	−.02
	5	.6389	0.0026	8.25	−.02		3	.7750	.1387	7.43	.00
	8	.6729	.0366	8.06	−.07		4	.7792	.1429	7.39	−.04
	9	.6799	.0436	7.99	−.09		5	.7833	.1470	7.41	−.02
	10	.6868	.0505	7.94	−.10		6	.7875	.1512	7.39	−.03

TABLE III—Continued

PHOTOGRAPHIC OBSERVATIONS OF TV CASSIOPEIAE

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O—C (darkened)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O—C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag.			2420	days		mag.
286	7	361.7920	1.1557	7.46	+.04	295	3	363.8694	1.4207	7.41	.00
	8	.7965	.1602	7.45	+.03		4	.8736	.4249	7.39	−.02
	9	.8007	.1644	7.45	+.03		5	.8781	.4294	7.39	−.02
	10	.8049	.1686	7.39	−.03		6	.8823	.4336	7.44	+.02
	11	.8090	.1727	7.38	−.04		7	.8861	.4374	7.38	−.04
	12	.8132	.1769	7.46	+.04		8	.8903	.4416	7.35	−.07
							9	.8944	.4457	7.40	−.02
291	1	363.6215	1.1727	7.40	−.02		10	.8986	.4499	7.37	−.05
	3	.6299	.1811	7.43	+.01						
	4	.6340	.1852	7.39	−.03	296	1	365.7299	1.4687	7.45	+.03
	5	.6382	.1894	7.36	−.06		2	.7340	.4728	7.44	+.01
	6	.6424	.1936	7.43	+.01		3	.7382	.4770	7.44	+.01
	7	.6465	.1977	7.43	+.01		4	.7430	.4818	7.43	.00
	8	.6507	.2019	7.44	+.02		5	.7486	.4874	7.43	.00
	9	.6549	.2061	7.43	+.01		6	.7542	.4930	7.40	−.03
	10	.6590	.2102	7.41	−.01		7	.7597	.4985	7.44	+.01
	11	.6632	.2144	7.40	−.02		8	.7653	.5041	7.44	+.01
	12	.6674	.2186	7.39	−.02		9	.7708	.5096	7.41	−.03
							10	.7771	.5159	7.39	−.05
292	1	363.6840	1.2353	7.47	+.06		11	.7840	.5228	7.41	−.03
	2	.6882	.2395	7.50	+.09						
	3	.6924	.2437	7.44	+.03	310	1	393.8611	0.5997	7.31	−.08
	4	.6965	.2478	7.43	+.02		2	.8653	.6039	7.36	−.03
	5	.7007	.2520	7.44	+.03		3	.8694	.6080	7.41	+.02
	6	.7049	.2562	7.45	+.04		4	.8736	.6122	7.43	+.04
	7	.7090	.2603	7.50	+.09		5	.8778	.6164	7.37	−.02
	8	.7132	.2645	7.50	+.09		6	.8819	.6205	7.33	−.06
	9	.7174	.2687	7.46	+.05		7	.8861	.6247	7.43	+.03
	10	.7215	.2728	7.47	+.06		8	.8903	.6289	7.38	−.02
	11	.7257	.2770	7.39	−.02		9	.8944	.6330	7.40	.00
	12	.7299	.2812	7.41	.00		10	.8986	.6372	7.40	.00
293	1	363.7361	1.2874	7.43	+.02	311	1	393.9035	0.6421	7.46	+.06
	2	.7403	.2916	7.44	+.03		2	.9076	.6462	7.38	−.02
	3	.7444	.2957	7.43	+.02		3	.9118	.6504	7.47	−.03
	4	.7486	.2999	7.47	+.06		4	.9160	.6546	7.47	+.07
	5	.7528	.3041	7.41	.00		5	.9201	.6587	7.36	−.04
	6	.7569	.3082	7.40	.00		6	.9243	.6629	7.38	−.02
	7	.7611	.3124	7.40	.00		7	.9285	.6671	7.38	−.03
	8	.7653	.3166	7.40	.00		8	.9326	.6712	7.41	.00
	9	.7694	.3207	7.41	+.01		9	.9368	.6754	7.39	−.02
	11	.7778	.3291	7.35	−.05		10	.9410	.6796	7.39	−.02
	12	.7819	.3332	7.35	−.05						
						320	1	395.7542	0.6803	7.31	−.10
294	1	363.7931	1.3444	7.40	.00		2	.7583	.6844	7.47	+.06
	2	.7972	.3485	7.41	+.01		3	.7625	.6886	7.46	+.05
	3	.8017	.3530	7.37	−.03		4	.7667	.6928	7.37	−.04
	4	.8059	.3572	7.41	+.01						
	5	.8097	.3610	7.39	−.01	321	1	395.7806	0.7067	7.44	+.02
	6	.8139	.3652	7.30	−.10		2	.7847	.7108	7.37	−.05
	7	.8181	.3694	7.32	−.08		3	.7889	.7150	7.51	+.09
	8	.8222	.3735	7.39	−.02		4	.7931	.7192	7.48	+.06
	9	.8264	.3777	7.39	−.02		6	.8014	.7275	7.46	+.04
	10	.8306	.3819	7.32	−.09		7	.8056	.7317	7.45	+.03
	11	.8347	.3860	7.37	−.04		8	.8101	.7362	7.45	+.03
	12	.8389	.3902	7.39	−.02		9	.8146	.7407	7.50	+.07
							10	.8188	.7449	7.41	−.02
295	1	363.8611	1.4124	7.39	−.02		11	.8229	.7490	7.47	+.04
	2	.8653	.4166	7.39	−.02		12	.8271	.7532	7.43	.00

TABLE III—Continued

PHOTOGRAPHIC OBSERVATIONS OF TV CASSIOPEIAE

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O–C (darkened)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O–C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag.			2420	days		mag.
322	1	395.8347	0.7608	7.35	−.08	332	5	398.7542	0.0552	7.96	−.04
	2	.8389	.7650	7.38	−.05		6	.7611	.0621	7.97	+.02
	3	.8430	.7691	7.44	+.01		7	.7681	.0691	7.87	−.03
	4	.8472	.7733	7.35	−.08						
	5	.8514	.7775	7.50	+.06	333	1	398.7819	0.0829	7.76	−.03
	6	.8556	.7817	7.46	+.02		2	.7903	.0913	7.72	−.01
	7	.8597	.7858	7.51	+.07		3	.7962	.0972	7.72	+.02
	8	.8639	.7900	7.43	−.01		4	.8014	.1024	7.58	−.09
	9	.8680	.7941	7.46	+.02		5	.8070	.1080	7.58	−.06
	10	.8722	.7983	7.44	.00		6	.8167	.1177	7.60	.00
	11	.8764	.8025	7.51	+.07		7	.8222	.1232	7.60	+.02
	12	.8806	.8067	7.43	−.01		8	.8278	.1288	7.53	−.03
							9	.8333	.1343	7.58	+.04
325	1	396.7024	1.6285	7.39	−.07		10	.8396	.1406	7.46	−.05
	3	.7111	.6372	7.40	−.07						
	4	.7153	.6414	7.40	−.07	334	1	398.8472	0.1482	7.60	+.10
	5	.7194	.6455	7.43	−.05		2	.8514	.1524	7.59	+.10
	6	.7236	.6497	7.41	−.07		3	.8556	.1566	7.60	+.12
	7	.7278	.6539	7.40	−.09		4	.8597	.1607	7.55	+.08
	8	.7319	.6580	7.48	−.02		5	.8639	.1649	7.53	+.06
	9	.7361	.6622	7.45	−.06		6	.8680	.1690	7.45	−.01
	10	.7403	.6664	7.45	−.07		7	.8722	.1732	7.48	+.02
							8	.8764	.1774	7.41	−.05
326	1	396.7451	1.6712	7.54	.00		9	.8806	.1816	7.46	+.01
	2	.7493	.6754	7.55	.00		10	.8847	.1857	7.48	+.03
	3	.7535	.6796	7.53	−.03		11	.8889	.1899	7.41	−.04
	4	.7576	.6837	7.59	+.01		12	.8931	.1941	7.52	+.07
	5	.7618	.6879	7.54	−.05						
	6	.7660	.6921	7.58	−.03	352	1	423.6507	1.3885	7.38	−.03
	7	.7701	.6962	7.64	+.02		2	.6549	.3927	7.45	+.04
	8	.7743	.7004	7.59	−.05		3	.6590	.3968	7.41	.00
							4	.6632	.4010	7.43	+.02
327	1	396.7903	1.7164	7.72	.00		5	.6674	.4052	7.52	+.11
	2	.7958	.7209	7.85	+.09		6	.6715	.4093	7.45	+.04
	3	.8014	.7275	7.82	+.02		7	.6757	.4135	7.46	+.05
	4	.8070	.7331	7.83	.00						
	5	.8125	.7386	7.94	+.06	358	1	423.7938	1.5316	7.48	+.04
	6	.8181	.7442	7.87	−.05		2	.7979	.5357	7.46	+.02
	7	.8236	.7497	7.96	.00		3	.8021	.5399	7.50	+.06
	8	.8292	.7553	8.05	+.04		4	.8062	.5440	7.50	+.05
	9	.8347	.7608	8.04	.00		5	.8104	.5482	7.52	+.07
	10	.8403	.7664	8.07	−.01		6	.8146	.5524	7.46	+.01
							7	.8188	.5566	7.46	+.01
331	1	398.6562	1.7698	8.13	+.02		8	.8229	.5607	7.44	−.01
	2	.6632	.7768	8.08	−.07		9	.8271	.5649	7.45	.00
	3	.6705	.7841	8.16	−.04		10	.8312	.5690	7.36	−.09
	4	.6774	.7910	8.17	−.05		11	.8354	.5732	7.39	−.06
	5	.6840	.7976	8.21	−.04		12	.8396	.5774	7.46	+.01
	6	.6910	.8046	8.21	−.06						
	7	.6979	.8115	8.30	+.03	359	1	423.8549	1.5927	7.48	+.02
	8	.7049	0.0059	8.22	−.05		2	.8590	.5968	7.52	+.06
	9	.7118	.0128	8.28	+.03		3	.8632	.6010	7.44	−.02
	10	.7188	.0198	8.22	.00		4	.8674	.6052	7.48	+.02
							5	.8715	.6093	7.46	.00
332	1	398.7264	0.0274	8.21	+.02		6	.8757	.6135	7.44	−.02
	2	.7333	.0343	8.16	+.01		7	.8799	.6177	7.55	+.09
	3	.7403	.0413	8.08	−.01		8	.8840	.6218	7.39	−.07
	4	.7472	.0482	8.04	−.01		9	.8882	.6260	7.44	−.02

TABLE III—Continued

PHOTOGRAPHIC OBSERVATIONS OF TV CASSIOPEIAE

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O–C (darkened)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O–C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag.			2420	days		mag.
366	1	428.5479	0.8479	7.37	−.08	515	6	694.7410	0.5844	7.38	−.01
	2	.5535	.8535	7.43	−.02		7	.7451	.5885	7.33	−.06
	3	.5590	.8590	7.38	−.07		8	.7493	.5927	7.38	−.01
	4	.5646	.8646	7.39	−.06		9	.7535	.5969	7.37	−.02
	5	.5701	.8701	7.39	−.06		10	.7576	.6010	7.30	−.09
	6	.5757	.8757	7.43	−.02						
	7	.5812	.8812	7.43	−.03	526	1	743.5882	0.4943	7.35	−.03
	8	.5868	.8868	7.47	+.01		2	.5924	.4985	7.36	−.02
	9	.5924	.8924	7.44	−.02		3	.5965	.5026	7.39	+.01
	10	.5979	.8979	7.51	+.05		4	.6007	.5068	7.36	−.02
	11	.6035	.9035	7.46	.00		5	.6049	.5110	7.44	+.06
							6	.6090	.5151	7.44	+.06
367	1	428.6111	0.9111	7.44	−.02		7	.6139	.5200	7.45	+.07
	2	.6167	.9167	7.48	+.02		8	.6194	.5255	7.46	+.08
	3	.6222	.9222	7.48	+.02		9	.6250	.5311	7.50	+.12
	4	.6278	.9278	7.46	.00		10	.6306	.5367	7.50	+.12
	5	.6333	.9333	7.58	+.12						
	6	.6389	.9389	7.48	+.02	527	1	743.6368	0.5429	7.33	−.05
	7	.6444	.9444	7.51	+.05		2	.6424	.5485	7.31	−.07
	8	.6500	.9500	7.48	+.02		3	.6479	.5540	7.38	.00
	9	.6556	.9556	7.48	+.02		4	.6535	.5596	7.35	−.03
	10	.6611	.9611	7.41	−.05		5	.6590	.5651	7.35	−.03
	11	.6667	.9667	7.41	−.04		6	.6646	.5707	7.36	−.02
							7	.6701	.5762	7.36	−.02
383	1	447.5424	1.7163	7.69	−.03		8	.6757	.5818	7.37	−.02
	4	.5590	.7329	7.85	+.01		9	.6812	.5873	7.38	−.01
	5	.5646	.7385	7.87	.00						
	6	.5701	.7440	7.92	.00	535	1	748.5757	0.0442	8.08	.00
	7	.5757	.7496	7.92	−.04		2	.5812	.0497	8.08	+.04
	8	.5812	.7551	8.08	+.08		3	.5868	.0553	8.04	+.04
	9	.5868	.7607	8.07	+.02		4	.5924	.0609	8.01	+.05
	10	.5924	.7663	8.10	+.01		5	.5979	.0664	7.98	+.05
	11	.5979	.7718	8.20	+.07		6	.6035	.0720	7.93	+.05
							7	.6090	.0775	7.92	+.08
390	5	451.5708	0.3069	7.44	+.03						
	6	.5750	.3111	7.44	+.03	536	1	748.6153	0.0838	7.81	+.01
	7	.5792	.3153	7.45	+.04		2	.6194	.0879	7.77	.00
	8	.5833	.3194	7.45	+.05		3	.6236	.0921	7.75	+.02
	9	.5875	.3236	7.40	.00		4	.6278	.0963	7.76	+.05
	10	.5917	.3278	7.43	+.03		5	.6319	.1004	7.72	+.03
							6	.6361	.1046	7.69	+.03
408	1	512.6993	1.6168	7.48	+.02		7	.6403	.1088	7.67	+.03
	2	.7035	.6210	7.48	+.02		8	.6444	.1129	7.62	.00
	3	.7076	.6251	7.46	.00		9	.6486	.1171	7.59	−.01
	4	.7118	.6293	7.50	+.04		10	.6528	.1213	7.67	+.09
	5	.7160	.6335	7.47	.00						
	6	.7201	.6376	7.55	+.08	538	1	748.7340	0.2025	7.43	−.01
	7	.7243	.6418	7.51	+.04		2	.7382	.2067	7.41	−.03
	8	.7285	.6460	7.53	+.05		3	.7424	.2109	7.43	−.01
	9	.7326	.6501	7.53	+.05		4	.7465	.2150	7.44	.00
	10	.7368	.6543	7.58	+.09		5	.7507	.2192	7.44	.00
	11	.7410	.6585	7.58	+.08		6	.7549	.2234	7.45	+.02
	12	.7451	.6626	7.58	+.07		7	.7590	.2275	7.47	+.04
							8	.7632	.2317	7.44	+.01
515	1	694.7201	0.5635	7.33	−.05		9	.7674	.2359	7.48	+.05
	2	.7243	.5677	7.28	−.10		10	.7715	.2400	7.41	−.02
	3	.7285	.5719	7.33	−.05		11	.7757	.2442	7.46	+.03
	4	.7326	.5760	7.31	−.07		12	.7799	.2484	7.43	.00
	5	.7368	.5812	7.36	−.03						

TABLE III—Continued
PHOTOGRAPHIC OBSERVATIONS OF TV CASSIOPEIAE

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O—C (darkened)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O—C (darkened)
(1)	(2)	(3)	(4)	(5)	(9)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag.			2420	days		mag.
539	1	748.7861	0.2546	7.39	−.03	564	5	754.8028	0.8338	7.44	−.01
	2	.7903	.2588	7.38	−.04		6	.8070	.8380	7.39	−.06
	3	.7944	.2629	7.39	−.03		7	.8111	.8421	7.41	−.04
	4	.7986	.2671	7.40	−.02		8	.8153	.8463	7.41	−.04
	5	.8028	.2713	7.40	−.02		9	.8194	.8504	7.40	−.05
	6	.8070	.2755	7.43	+.01		10	.8236	.8546	7.40	−.05
	7	.8111	.2796	7.40	−.02						
						565	1	754.8299	0.8609	7.48	+.03
547	1	752.6180	0.4615	7.38	.00		2	.8340	.8650	7.54	+.09
	2	.6222	.4657	7.36	−.02		3	.8382	.8692	7.51	+.06
	3	.6264	.4699	7.39	+.01		4	.8424	.8734	7.51	+.06
	4	.6306	.4741	7.41	+.03		5	.8465	.8775	7.48	+.02
	5	.6347	.4782	7.40	+.02		6	.8507	.8817	7.47	+.01
	6	.6389	.4824	7.39	+.01		7	.8549	.8859	7.48	+.02
	7	.6431	.4866	7.40	+.02		8	.8590	.8900	7.44	−.02
	8	.6472	.4907	7.41	+.03		9	.8632	.8942	7.45	−.01
	9	.6514	.4949	7.44	+.06		10	.8674	.8984	7.47	+.01
	10	.6556	.4991	7.44	+.06		11	.8715	.9025	7.43	−.03
559	1	753.8618	1.7054	7.67	+.01	605	1	785.6931	0.9108	7.46	.00
	2	.8674	.7110	7.74	+.05		2	.6972	.9149	7.44	−.02
	3	.8736	.7172	7.75	+.02		3	.7014	.9191	7.46	.00
	4	.8799	.7235	7.72	−.05		4	.7056	.9233	7.45	−.01
	5	.8854	.7290	7.76	−.04		5	.7097	.9274	7.50	+.04
	6	.8910	.7346	7.84	−.01						
	7	.8965	.7401	7.85	−.04	606	1	785.7174	0.9351	7.54	+.08
	8	.9021	.7457	7.92	−.02		2	.7215	.9392	7.52	+.06
	9	.9076	.7512	7.90	−.08		3	.7257	.9434	7.55	+.09
	10	.9132	.7568	7.98	−.04		4	.7299	.9476	7.53	+.07
	11	.9188	.7624	8.00	−.06		5	.7340	.9517	7.46	.00
							6	.7382	.9559	7.44	−.02
564	1	754.7861	0.8171	7.44	−.01		7	.7424	.9601	7.45	−.01
	2	.7903	.8213	7.43	−.02		8	.7465	.9642	7.46	.00
	3	.7944	.8254	7.48	+.03		9	.7507	.9684	7.37	−.09
	4	.7986	.8296	7.46	+.01						

magnitudes were too great by $0^m.070$. They were converted to the revised scale by use of the formula:

$$\text{Revised magnitude} = [(m - 0.070) - 6.00] \times 1.15 + 6.00$$

where m is the unrevised magnitude.

The revised photographic magnitudes of TV Cassiopeiae, 491 in all, appear in Table III. These points are formed into 51 normal places (Table IV). Numbers 1 to 10 and 41 to 51, which are on the primary minimum, consist of about 6 observations each; for the other normal places 12 observations is the average number. When the normal magnitudes are plotted against phase we have a curve with a secondary minimum about $0^m.08$ in depth, and a slightly asymmetrical primary of about $0^m.90$; the curve shows the "periastron effect", being noticeably higher before than after secondary minimum; it is concave upward, which shows that the stars are elliptical; and a "reflection effect" is evidenced by a slight increase of light on both sides of the secondary.

TABLE IV

NORMAL MAGNITUDES OF TV CASSIOPEIAE

No.	Phase	θ	No. Obs.	Observed Magnitude	Observed Intensity	Rectified Intensity	Computed Magnitude (uniform)	O–C (uniform)	Computed Magnitude (darkened)	O–C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	days									
1	0.0093	1°.85	6	8.263	0.448	0.505	8.263	.000	8.263	.000
2	.0287	5.70	5	8.176	0.485	0.542	8.179	−.003	8.184	−.008
3	.0462	9.18	6	8.036	0.552	0.611	8.063	−.027	8.072	−.036
4	.0607	12.06	7	7.971	0.586	0.646	7.964	+.007	7.971	.000
5	.0748	14.86	5	7.877	0.639	0.700	7.861	+.016	7.867	+.010
6	.0877	17.42	6	7.751	0.718	0.780	7.772	−.021	7.769	−.018
7	.0997	19.80	7	7.687	0.761	0.825	7.700	−.013	7.694	−.007
8	.1138	22.60	8	7.612	0.816	0.879	7.621	−.009	7.621	−.009
9	.1340	26.61	9	7.542	0.870	0.930	7.532	+.010	7.544	−.002
10	.1649	32.75	7	7.517	0.890	0.943	7.456	+.061	7.465	+.052
11	.1983	39.38	8	7.448	0.949	0.999	7.447	+.001		
12	.2428	48.22	12	7.428	0.966	1.001	7.429	−.001		
13	.3082	61.21	12	7.424	0.970	0.984	7.407	+.017		
14	.3653	72.55	12	7.375	1.005	1.012	7.388	−.013		
15	.4360	86.59	12	7.371	1.018	1.007	7.379	−.008		
16	.4933	97.97	12	7.400	0.992	0.979	7.377	+.023		
17	.5441	108.06	12	7.389	1.002	0.991	7.379	+.010		
18	.5824	115.67	11	7.357	1.033	1.028	7.387	−.030		
19	.6190	122.94	12	7.382	1.008	1.012	7.395	−.013		
20	.6683	132.73	12	7.406	0.986	0.999	7.405	+.001		
21	.7226	143.52	10	7.443	0.953	0.980	7.421	+.022		
22	.7693	152.79	10	7.413	0.980	1.017	7.431	−.018		
23	.7960	158.09	10	7.459	0.939	0.984	7.442	+.017		
24	.8257	163.99	9	7.438	0.958	1.008	7.447	−.009		
25	.8571	170.23	10	7.432	0.963	1.017	7.450	−.018		
26	.8850	175.77	13	7.458	0.940	0.998	7.456	+.002		
27	.9177	182.26	12	7.470	0.930	0.990	7.459	+.011		
28	.9531	189.30	16	7.466	0.933	0.992	7.457	+.009		
29	.9996	198.53	14	7.433	0.962	1.017	7.451	−.018		
30	1.0514	208.82	9	7.462	0.937	0.986	7.447	+.015		
31	.1026	218.99	11	7.429	0.966	1.006	7.435	−.006		
32	.1532	229.04	12	7.410	0.983	1.014	7.425	−.015		
33	.2038	239.09	13	7.425	0.969	0.993	7.417	+.008		
34	.2691	252.06	13	7.443	0.953	0.968	7.408	+.035		
35	.3273	263.62	12	7.401	0.991	1.005	7.406	−.005		
36	.3838	274.84	13	7.389	1.002	1.015	7.405	−.016		
37	.4279	283.60	12	7.402	0.990	1.010	7.413	−.011		
38	.4999	297.90	13	7.432	0.963	1.000	7.432	.000		
39	.5646	310.75	12	7.460	0.938	0.992	7.451	+.009		
40	.6179	321.33	12	7.459	0.939	1.006	7.465	−.006		
41	.6404	325.80	7	7.471	0.929	0.996	7.467	+.004	7.470	+.001
42	.6547	328.64	7	7.492	0.911	0.981	7.484	+.008	7.493	−.001
43	.6692	331.52	8	7.531	0.879	0.949	7.516	+.015	7.529	+.002
44	.6898	335.61	7	7.580	0.840	0.912	7.590	−.010	7.593	−.013
45	.7095	339.52	7	7.680	0.766	0.838	7.691	−.011	7.686	−.006
46	.7278	343.16	6	7.806	0.682	0.751	7.801	+.005	7.800	+.006
47	.7400	345.58	6	7.884	0.635	0.701	7.884	.000	7.887	−.003
48	.7519	347.94	7	7.971	0.586	0.652	7.972	−.001	7.980	−.009
49	.7644	350.43	6	8.068	0.536	0.598	8.060	+.008	8.068	.000
50	.7845	354.42	6	8.179	0.484	0.543	8.188	−.009	8.193	−.014
51	.8052	358.53	7	8.280	0.441	0.497	8.268	+.012	8.268	+.012

The solution which follows is based on the theory developed at Princeton by Russell and Shapley.[8] Reference to formulae and tables contained in their papers will be indicated by using italics. First the normal magnitudes were changed into intensities by use of the formula:

$$\log l = 0.4\ (m_0 - m)$$

where m is the normal magnitude and m_0 is the magnitude of the system during the intervals between eclipses. The preliminary value of m_0 was taken to be $7^m.357$. The resulting values of l are expressed in terms of $L_1 + L_2$, the total light of the system. In Table IV, column (6), appear the observed normal intensities read from *Table A*; but for these the final value of m_0 was used. The preliminary values of the normal intensities on the primary minimum were plotted against the phases, computed by using Astbury's elements. The resulting curve showed the necessity for a correction to the epoch of $+0^d.001$, a correction which was not evident from a curve drawn through the normal magnitudes. The adopted elements which were used in computing the phases in column (2), are:

Primary Minimum = J.D. 2419338.331 + $1^d.8126 \cdot E$, G.M.T. (Corrected elements)

It is assumed that the intensities outside of primary minimum can be represented by the formula:

$$l = a - b\cos\theta - c\cos^2\theta - f\sin\theta - nd$$

where θ is the longitude from principal conjunction; a is the intensity of the system aside from the "reflection", ellipticity, and "periastron" effects and eclipse of the fainter star; b is half the amount by which that side of the faint star turned toward the bright star exceeds the side turned away; $2c = z = \epsilon^2\sin^2 i$, ϵ being the eccentricity of a meridian section of the prolate spheroids; f is the maximum amount by which the curve is raised midway between primary and secondary due to a "periastron effect"; d is the depth of secondary minimum, and n is the loss of light at the corresponding phase on the primary minimum, in terms of the loss at maximum obscuration. A subsequent change in the interpretation of the term $b\cos\theta$ is explained later.

The next step was to form an equation of condition for each of the normal places outside eclipse, and to solve these for the constants a, b, c, f, and d. Preliminary values of the constants were found as follows. When l^2 was plotted against $\cos^2\theta$ the slope, $z = 2c$, of the resulting straight line was 0.115; using this preliminary value of z the intensities were rectified by adding to each l the quantity $+c\cos^2\theta$. When the resulting intensities were plotted the secondary entirely disappeared, showing that this drop is due to ellipticity only. Hence we conclude that the light of the fainter star must be practically zero. The points were now examined to determine preliminary values of the other

[8] H. N. Russell, *Astrophysical Journal*, 35, 315, 1912; 36, 54, 1912; H. N. Russell and H. Shapley, *Astrophysical Journal*, 36, 239, 385, 1912; H. Shapley, *Princeton Contributions*, No. 3, 1915.

constants. To find b, comparison was made of l for large values of $\cos\theta$ equal but opposite is sign; for f, comparison was made of l for large values of $\sin\theta$; for a, terms in b, c, and f were combined with l, after changing signs. Since the curve when rectified with the preliminary z showed no secondary minimum the consideration of d was at first omitted in the least-squares solution. The equations of condition are of the form:

$$\delta a - \cos\theta\cdot\delta b - \cos^2\theta\cdot\delta c - \sin\theta\cdot\delta f = l_o - l_c$$

When weight unity is assigned to each observation outside eclipse the following normal equations result:

$$\begin{aligned} +30.000\,\delta a + 9.348\,\delta b - 13.978\,\delta c - 1.161\,\delta f &= +0.009 \\ +13.978\,\delta b - 8.083\,\delta c - 0.755\,\delta f &= -0.002 \\ +10.108\,\delta c + 0.562\,\delta f &= +0.033 \\ +15.969\,\delta f &= -0.022 \end{aligned}$$

The solution gives:

	Preliminary	Correction	Final	Probable Error	Weights
$a =$	$+0.963$	$+0.0060$	$+0.969$	± 0.0031	10.212
$b =$	$+0.0115$	$+0.0048$	$+0.016$	± 0.0037	7.192
$c =$	$+0.0575$	$+0.0158$	$+0.073$	± 0.0064	2.339
$f =$	-0.0115	-0.0013	-0.013	± 0.0025	15.905
$m_0 =$	$7^m.357$	$+0^m.034$	$7^m.391$	$\pm 0^m.0034$	

To get the correction to m_0, the difference between the final a and unity was converted into magnitude. The final m_0 is the magnitude of the system between eclipses, when the effect of ellipticity and the "reflection" and "periastron" effects have been removed.

Now when the curve at primary minimum is solved for the remaining elements of the orbit it appears that the two stars are very little different in size, the larger star being the fainter. Here arises a difficulty concerning b, the "reflection effect" constant, and d, the depth of secondary. For if reflection from the larger star constitutes, in the maximum, 0.032 of the light of the system, then when this reflecting surface is eclipsed at secondary minimum the curve should exhibit a diminution in light from this cause even after the effect of ellipticity is removed. Then too, the visual curve and orbit by McDiarmid indicate that the rectified depth of secondary is $0^m.080$ and that the fainter star, which in his solution is also the smaller, contributes about 0.15 of the light of the system. If the depths of the visual minima are correct, the observed decrease (0.5) in photographic intensity should be accompanied by a secondary minimum about a tenth of a magnitude in depth. For these reasons a second least-squares solution was made which included the nd terms. The function $l + b\cos\theta$ was plotted against $\sin\theta$, and the slope (0.014) of the resulting straight line was taken as the preliminary value of d. When the normal

equations were solved the result was: $\delta d = -0.013$. This is taken as conclusive evidence that no secondary exists on the rectified curve, and the first least-squares solution is adopted as definitive. However, the first question remains: if $b = 0.016$ represents the "reflection" constant, why is there no drop in intensity when the reflecting surface is eclipsed? It appears that in this case $b \cos\theta$ cannot be interpreted as a reflection effect, but must be regarded as merely a cosine oscillation in the curve. A further comparison with McDiarmid's results will be made later.

The remaining elements of the system were determined from the light curve at primary minimum. The first step was to rectify the normal intensities by means of the formula:

$$R = \frac{l + b(1 + \cos\theta) + c \cdot l \cdot \cos^2\theta + f \sin\theta}{a + b}$$

using the final values of the constants. The values of the rectified intensity, R, for all the points both on and outside minimum are given in column (7) of Table IV. Two solutions for the elements were made on two hypotheses: 1st, that the disks are uniformly luminous; 2nd, that they are uniformly darkened toward the limb. The uniform solution follows.

The rectified intensities, R, of the points on the primary minimum were plotted against the corrected phases; the points on the downslope were reflected onto the upslope, and through all a smooth curve was drawn. From this curve the phases, t, were read for 11 fractions, n, of the maximum loss of light, $1 - \lambda_1$. Then the value of the χ function of k and α_0 was obtained by use of *formula g'*:

$$\frac{\sin^2\theta(n)}{1 - z\cos^2\theta(n)} = C\omega_2(n) + D\omega_1(n), \quad \frac{C}{D} = \chi(k, \alpha_0, \tfrac{1}{4})$$

The functions $\omega_1(n)$ and $\omega_2(n)$ are taken from *Table IIIa; k* is the ratio of the radii of the two stars, α_0 is the maximum obscuration of the brighter star, and the χ function is defined by the equation:

$$\chi(k, \alpha_0, n) = \frac{\sin^2\theta\,(n)\ \{\ 1 - z\cos^2\theta\,(\tfrac{1}{2})\ \}}{\sin^2\theta\,(\tfrac{1}{2})\ \{\ 1 - z\cos^2\theta\,(n)\ \}}$$

Now for TV Cassiopeiae most of the formulae assume a simplified form, for (assuming that the dark star is the larger) both L_1 and $1 - \lambda_2$ are zero, and $1 - \lambda_1 = \alpha_0$ can be read at once from the curve. Hence the problem reduces to the finding of a suitable value for k. The preliminary trials for $\frac{C}{D}$ proved at once that the larger star is in front at primary minimum. Consider the equation

$$\alpha_0 = 1 - \lambda_1 + \frac{1 - \lambda_2}{k^2}$$

Suppose that the dark star is the smaller. Then $1-\lambda_1=0$, $1-\lambda_2=0.5$, and $\alpha_0=\frac{1-\lambda_2}{k^2}$. Since α_0 must be equal to or less than 1, k^2 cannot be less than $\sqrt{0.5}$. Thus the limiting values for k are 1.00 and 0.71. But when trials were made assuming various values between these limits it was found that the resulting χ functions were much too great for agreement with the observed curve. This confirms the assumption that $1-\lambda_1=\alpha_0$, and that $L_1=0=1-\lambda_2$. After numerous trials it was found that the value $k=0.96$ most nearly satisfies *equation g'*, but a satisfactory agreement with the observed curve was not obtained by using the ω functions. Therefore a curve was computed using the formula

$$\psi(k,\ a_1)=\frac{\sin^2\theta-A}{B}$$

Given $k=0.96$ and $\alpha_0=0.50$, a satisfactory curve is obtained by letting $A=0.0211$, $B=0.0407$. Next the ordinates of this curve were converted from intensities to magnitudes and these were converted from rectified to natural magnitudes; the ordinates for values of t corresponding to those for the normal places appear in column (8), Table IV, and the residuals of the observed points from this curve appear in column (9). Using the above values of k and α_0 the elements i and a_f were found from *formula h'*:

$$\cos^2 i\cos^2\theta'+\sin^2\theta'=a^2(1-z\cos^2\theta')(1+k)^2$$
$$\cos^2 i=a^2(1-z)[1+kp(k,\alpha_0)]^2$$

where θ' corresponds to the end of eclipse, a_f is the maximum radius of the dark star, and the function $p(k,\ \alpha_0)$ is taken from *Table I*. After a_f is found, the radii b_f and c_f are obtained by using the formulae:

$$b^2=a^2(1-\epsilon^2),\quad \frac{c^2}{a^2}=\frac{1-3.4\,a^3}{1+8.6\,a^3}$$

Multiplying the resulting values by k we find a_b, b_b, c_b. If we assume equal masses, the densities of the two stars are obtained by use of the formula:

$$\rho=\frac{0.00672}{P^2abc}$$

where P is the period in days. The absolute magnitude of the system, the hypothetical parallax, and the distance in light years are computed from formulae given in *Princeton Contributions*, No. 3. The list of elements on the uniform hypothesis appears in Table V, column (3). The unit of length is the radius of the relative orbit, and the unit of light is the total light of the system, $L_b+L_f=L_b$.

For the darkened solution the procedure was somewhat different. The rectified curve was slightly altered, being widened and brought down at the top, while the bottom

was left unchanged. The functions $\sin^2\theta$ and $1 - z_d\cos^2\theta$ were computed for values of θ corresponding to $n = 0$, $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$, using $z_d = 0.091 = \frac{5}{8}z$. A series of approximations were made to get χ functions which would give the most accordant values of k. The final solution gave $k = 0.82$. Using this value a curve was computed; the ordinates were changed from intensities to rectified magnitudes, then to natural magnitudes, and the residuals were taken as before. The ordinates of the computed darkened curve appear in column (10) of Table IV; the residuals, in column (11). As before, *formula h'* was employed to obtain i and a_f, the only difference being that z_d was used in place of z, and the function $p(k, \alpha_0)$ was this time taken from *Table Ix.* From the new values of the radii of the two stars their densities and the parallax of the system were again computed, using the same formulae as before. The elements of the system on the hypothesis of darkened disks are found in Table V.

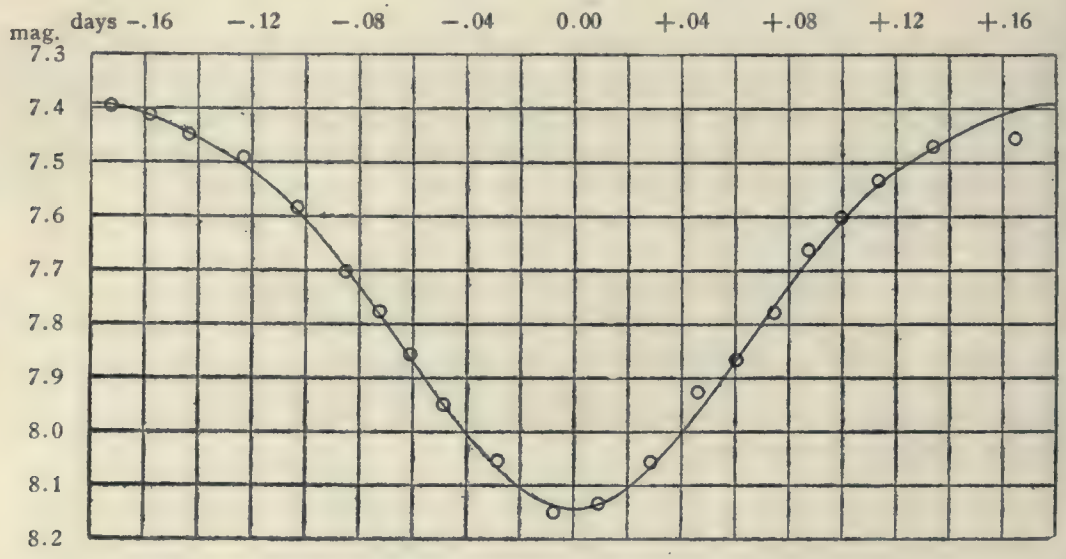

FIGURE 1. RECTIFIED NORMAL MAGNITUDES AND LIGHT CURVE OF TV CASSIOPEIAE DURING PRIMARY MINIMUM (DARKENED SOLUTION)

This table of photographic elements can be compared directly with the visual elements in *Astrophysical Journal* **42**, 432, 1915. The differences are striking. In the visual elements the fainter star is the smaller, and it contributes 14 per cent to the light of the system on the uniform and 16 per cent on the darkened hypothesis. This fundamental difference introduces many others, so that a detailed comparison is useless. However the inclination of the orbit is found to be different by only $3\frac{1}{2}$ degrees (uniform) and 2 degrees (darkened), the greater inclination being derived from the visual curve. When the investigation of this star began it was hoped that the comparison of the depths of the visual and photographic minima would give evidence on the question of a difference of

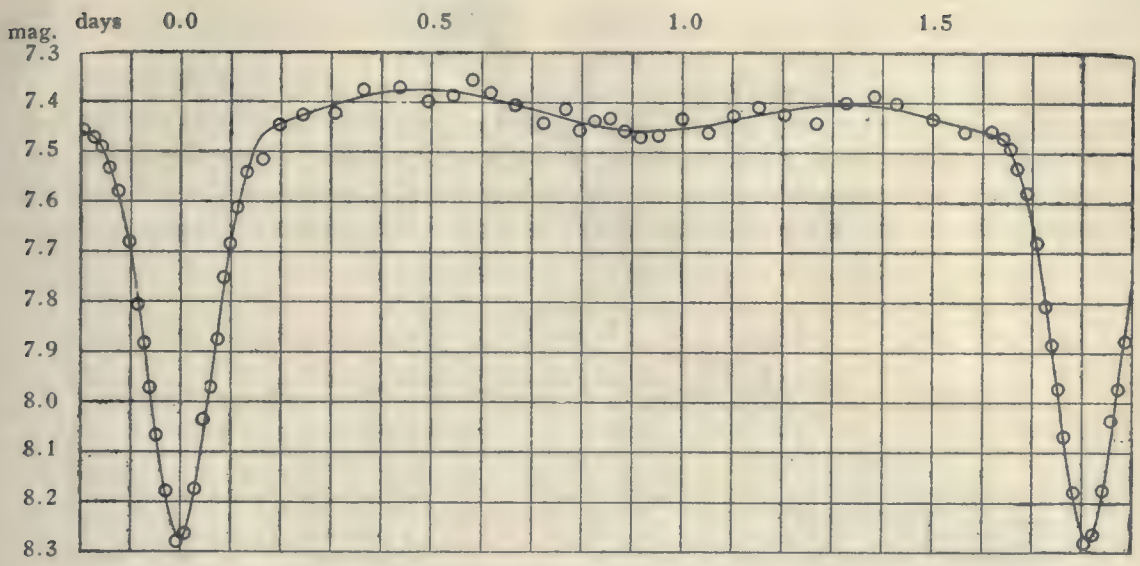

FIGURE 2. NORMAL MAGNITUDES AND THEORETICAL LIGHT CURVE OF TV CASSIOPEIAE

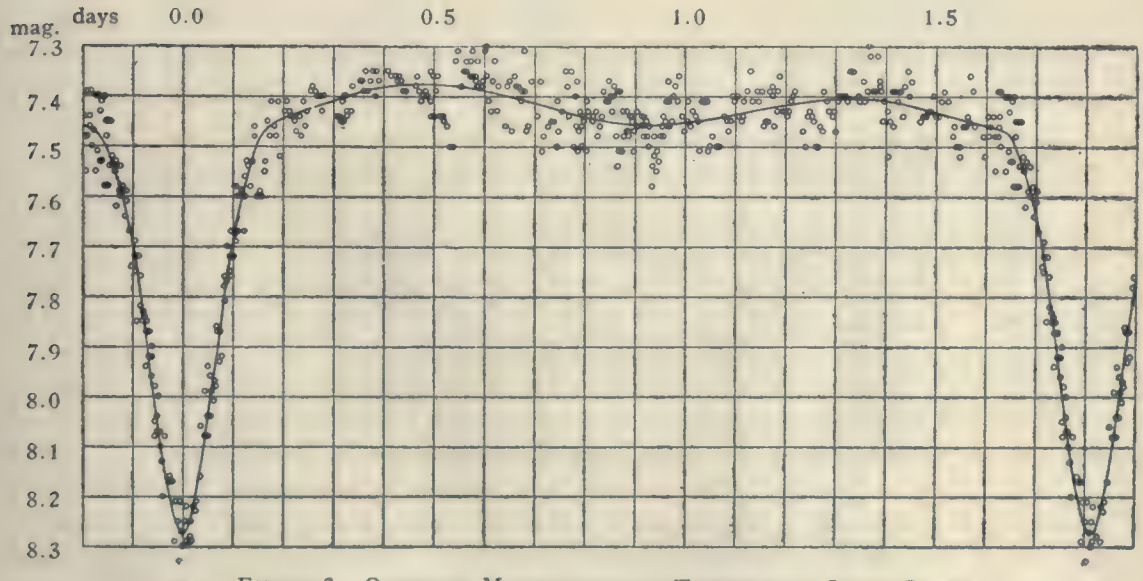

FIGURE 3. OBSERVED MAGNITUDES AND THEORETICAL LIGHT CURVE

spectral type for the two stars. However, since the photographic minima are in both cases shallower than the visual, no conclusions can be drawn.

Figure 1 shows the theoretical light curve and the individual observations on which it is based. The curve appears again in Figure 2, with the normal magnitudes. Figure 3 gives the rectified curve and normal magnitudes during primary minimum. In all these figures the computed curve is derived on the darkened hypothesis; this solution is regarded as definitive, since the residuals of the normal places are slightly less for

TABLE V
ELEMENTS OF THE SYSTEM OF TV CASSIOPEIAE

		Uniform	Darkened
Period of revolution	P	1.8126 days	
Daily motion in longitude	μ	198°.610	
Epoch of primary minimum	T	J.D. 2419338.331 G.M.T.	
Semi-duration of eclipse	.....	0.1662 days	0.1751 days
Inclination of orbit	i	76°.2	72°.2
Least apparent distance of centers	$\cos i$	0.238	0.306
Ratio of radii	k	0.96	0.82
Ratio of major and minor axes	$\frac{a}{b}$	1.087	1.055
Semi-major axis, bright star	a_b	0.300	0.290
Semi-minor axis, bright star	b_b	0.276	0.275
Semi-major axis, dark star	a_f	0.313	0.354
Semi-minor axis, dark star	b_f	0.287	0.335
Density of bright star	ρ_b	0.10	0.10
Density of dark star	ρ_f	0.09	0.06
Maximum obscuration during eclipse	α_0	0.50	0.50
"Radiation effect" constant	b	0.016	0.016
Light of bright star	L_b	1.000	1.000
Light of dark star	L_f	0.000	0.000
Magnitude of system	.....	7.391	7.391
Magnitude at primary minimum	.....	8.273	8.273
Magnitude at secondary minimum	.....	7.459	7.459
Range of magnitude, primary	.....	0.897	0.897
Range of magnitude, secondary	.....	0.083	0.083
Absolute magnitude of bright star	M	−0.2	−0.1
Hypothetical parallax	π	0".0031	0".0031
Distance in light years		1052	1052

the darkened than for the uniform curve. For the curve during eclipse the sum of the squares of the normal magnitude residuals is 0.002675 (uniform) and 0.002595 (darkened); the resulting probable errors of a single normal place are respectively $\pm 0^m.012$ and $\pm 0^m.011$. If the tenth normal place is neglected, the probable error is $\pm 0^m.008$ on both hypotheses. For the portion outside eclipse the curve is the same on both hypotheses; here the probable error is $\pm 0^m.010$. For the entire curve on the darkened solution the probable error of a single observation is $\pm 0^m.029$.

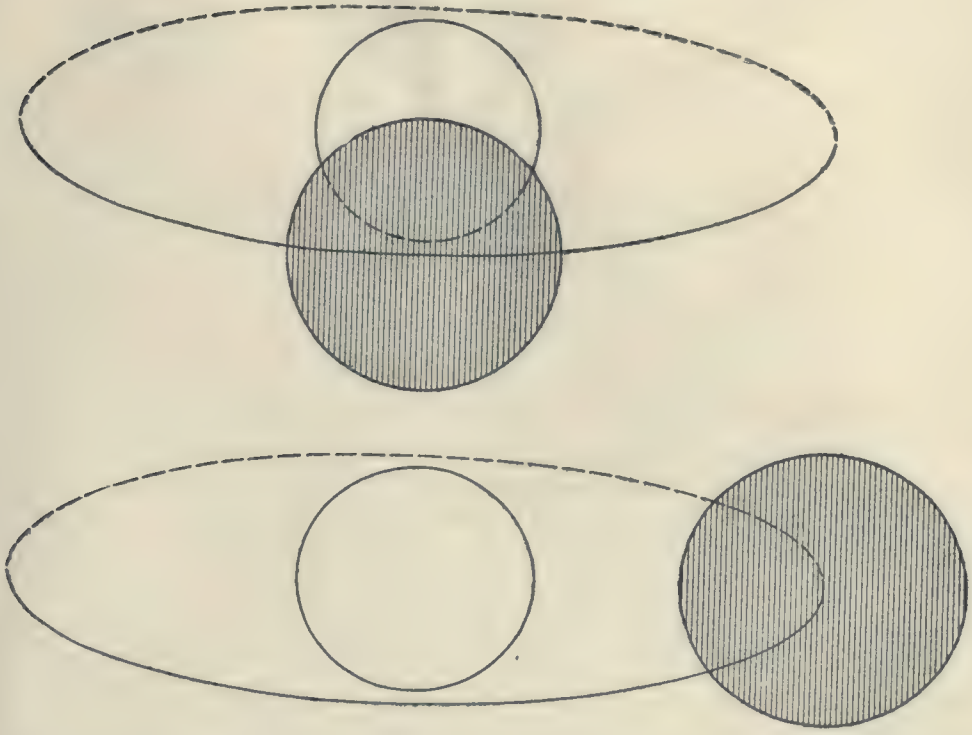

FIGURE 4. THE SYSTEM OF TV CASSIOPEIAE AT PRIMARY MINIMUM AND AT GREATEST ELONGATION

The present photographic light curve and orbit of TV Cassiopeiae are based on 491 images of the variable on 51 plates taken with the 5-inch camera of the Laws Observatory. The system is composed of two ellipsoidal stars which revolve in an orbit inclined 72° to the plane of the sky; the diameter of the relative orbit is 3 times the major axis of the bright star. The fainter star, which is slightly the larger of the two, apparently gives no light; at principal conjunction it occults one half the disk of its bright companion. Eccentricity of the orbit is shown by a "periastron effect", but data are lacking for determining its amount. A peculiarity of this system is that the "reflection" or "radiation effect" cannot be interpreted in the usual way. The probable error of a single magnitude determination is $\pm 0^{m}.029$.

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THE ECLIPSING BINARY u HERCULIS

By Robert H. Baker

From the time of its discovery by Schmidt[1] in 1869 until the detection of the true character of its fluctuations in 1908, the variable star 68 u Herculis (R.A. = $17^h13^m.6$, Decl. = $+33°12'$) presented peculiarities difficult to explain. Its variations appeared to Schmidt and to subsequent observers to be approximately regular with a period of about 40 days, but with rapid changes during minimum light. The binary character of this star was discovered in 1903 by Frost and Adams[2] from an examination of four spectrograms made at the Yerkes Observatory. This fact together with the early class of spectrum, B3, readily cast suspicion on the current interpretation of its light variation. A series of radial velocity determinations of this star was made by the writer in 1908 at the Allegheny Observatory, which indicated a revolution period of 2.05102 days and gave the clue required for completing its photometric study then in progress at the Harvard Observatory. Collecting his observed magnitudes[3] on this period Wendell discovered that u Herculis is an eclipsing variable with a marked secondary minimum. The latter fact might have been inferred from the Allegheny spectrograms which show the spectrum of the fainter star sufficiently well defined for measurement.

Soon after the period was announced, a number of visual light curves were published, notably by Enebo[4] and Ichinohe[5], and by Hertzsprung[6] who discussed more than four thousand observations by Schmidt between the years 1869 and 1879 and found 2.051027 days a more exact value of the period. Series of visual observations of smaller number have been published by Lau[7] and Jost[8].

It is not often possible to obtain absolute dimensions of binary stars and of their orbits. Relative dimensions in terms of the separation of the two stars may be derived from the light curve in the case of eclipsing binaries, and the linear value of the unit is made known by spectroscopic study when the spectra of both components of the binary

[1] J. F. J. Schmidt, *Astronomische Nachrichten*, 74, 229, 1869.
[2] E. B. Frost and W. S. Adams, *Astrophysical Journal*, 17, 381, 1903.
[3] O. C. Wendell, *Harvard Annals*, 69, 127, 1914.
[4] S. Enebo, *Astronomische Nachrichten*, 182, 309, 1909.
[5] N. Ichinohe, *Astronomische Nachrichten*, 183, 193, 1909.
[6] E. Hertzsprung, *Astronomische Nachrichten*, 189, 245, 257, 1911.
[7] H. E. Lau, *Astronomische Nachrichten*, 191, 313, 1912.
[8] E. Jost, *Astronomische Nachrichten*, 194, 201, 1913.

are measurable. But the majority of eclipsing variables are so faint as to be beyond the reach of present spectrographs. Moreover, only a few spectroscopic double stars show the spectrum of the fainter component. In the extensive list of eclipsing binaries only five fulfill the necessary conditions; they are β Aurigae, V Puppis, RX Herculis, β Lyrae and u Herculis. For one of these, V Puppis, the spectroscopic orbit is not yet available. In the case of β Lyrae, studied by Curtiss[9], definite conclusions are restricted by unusual complications in the spectrum. Absolute elements for β Aurigae have been published by Stebbins[10], based on his light curve obtained with the selenium photometer and on the writer's spectroscopic elements. Elements for RX Herculis were recently obtained at the Laws Observatory[11]; the scale of this system depends on only two spectrograms and further spectroscopic support is desirable.

Absolute elements for the system of u Herculis were derived in 1910 at the Allegheny Observatory[12] by combining the spectroscopic elements with information furnished by Wendell's visual photometric observations. After more suitable formulae had been developed at the Princeton Observatory, Shapley[13] recomputed the photometric elements from the same data. Unfortunately the Harvard magnitudes for this star have not the accuracy of Wendell's observations generally; this is probably due to the variability of the comparison star. A general source of inaccuracy in the derivation of absolute elements follows one of the imposed conditions, namely, the presence of both spectra on the plates. This signifies that the two stars have nearly the same brightness. Consequently the decrease in light during eclipses can not greatly exceed $0^m.75$ and the degree of indetermination in some of the photometric elements may be considerable.

The spectroscopic elements [14] are the following:

$P = 2.05102$ days		Spectroscopic elements of u Herculis
$T =$ J.D.2418125.80		
$e = 0.053$	± 0.010	
$\omega = 66°.15$	$\pm 0°.54$	
$\gamma = -21.16$ km.		
$K_b = 99.50$ km.	± 0.99 km.	
$K_f = 253$ km.	± 12 km	
$a_b \sin i = 2{,}800{,}000$ km.		
$a_f \sin i = 7{,}120{,}000$ km.		
$m_b \sin i = 6.8 \odot$		
$m_f \sin i = 2.6 \odot$		

In an orbit so nearly circular as this, the longitude of periastron is uncertain within wide limits. The small probable error of ω is due to the assumption of the time of periastron passage before entering the least-squares solution. The spectra are of class B3 and close-

[9] R. H. Curtiss, *Allegheny Publications*, 2, 73, 1911.
[10] J. Stebbins, *Astrophysical Journal*, 34, 112, 1911.
[11] R. H. Baker and E. E. Cummings, *Laws Observatory Bulletin*, No. 25, 1916.
[12] F. Schlesinger and R. H. Baker, *Allegheny Publications*, 2, 51, 1910.
[13] H. Shapley, *Princeton Contributions*, No. 3, 67, 1915.
[14] R. H. Baker, *Allegheny Publications*, 1, 77, 1909.

ly resemble those of α Virginis. The lines are broad and diffuse, and do not permit accurate measurement. The spectrum of the secondary component is exactly similar to that of the primary, but the lines are of the vanishing order of intensity and exceedingly difficult to measure.

In previous solutions for the photometric elements the two stars are supposed to differ in brightness by an even magnitude. This assumption is based largely on an incomplete experiment made by the writer at the Allegheny Observatory. The spectrum of α Cygni was photographed twice on the same plate, the plate being moved slightly between exposures, so that the spectrogram resembled that of a binary showing the lines of both components. By varying the exposure time or by diaphragming the mirror the relative brightness of the components of the artificial binary was varied. The secondary spectrum was not detected when the difference was made greater than one magnitude. But the experiment was not carried out for spectra showing lines of various degrees of definition. It is reasonable to suppose that the difference of magnitude necessary for extinction of the secondary spectrum decreases as the lines become more diffuse and that the difference, considerably less than one magnitude, required in the present solution is enough to make the secondary spectrum difficult to detect.

The photographic light curve of u Herculis is based on 53 extrafocal plates which are listed in Table I. The times in columns (2) and (3) and the hour angle in column (8) are given for the middle exposure. The heliocentric phases for the first and last exposures, column (6), are referred to primary minimum; these and the epoch number in column (4) are based on the elements:

$$\text{Primary Minimum} = \text{J.D. } 2419928.790 + 2^{d}.05102 \cdot E, \text{G.M.T. (Adopted Elements)}$$

These plates are among the first taken with the five inch camera, while the work was in the experimental stage; on this account the procedure differs greatly from that finally adopted and described in *Laws Observatory Bulletin*, No. 24. The plates were exposed 5mm. outside focus, giving images about 1mm. in diameter near the center of the plate, whose illumination is much less uniform than that of the smaller images now obtained near focus. In measuring them it was the practice to make four settings on different parts of the images: above, below, to the right and left of center, in each case half-way from the center to the edge, except where images overlap. The separate exposures vary from two to four minutes and give images frequently less dense than are now required for the best results. The plates are all Seed 27, but no record of the emulsion number was kept, since it was not then understood that the character of the reduction curve varies with the emulsion used. The plates of 1913 were not exposed in the sensitometer; this instrument was not under control until June, 1914. In addition, the developing tank used for these plates was later discarded because it gave poor temperature control and was replaced by the present one. Also the practice of fixing the plates on edge was soon discontinued, although its effect on the measures was never more than suspected. Taken altogether, so many details for proper control were lacking, that I had fully decided to credit these plates

TABLE I

PLATES OF U HERCULIS

Plate No.	Date, G.M.T.	Julian Day	Epoch	Correction to Sun	Heliоc. Phase	Tel.	Hour Angle	No. Exp.	Film
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	1913 d. h. m.				d. to d.		h		
5	April 28 17 10	2419886.715	− 21	+0.0027	0.979 1.018	W	−3.8	19	clear
10	May 1 17 02	889.710	20	28	1.912 1.974	W	−3.7	16	clear
11	1 18 22	889.766	20	28	1.978 2.018	W	−2.4	20	13
17	26 15 49	914.659	7	32	0.203 0.255	W	−3.3	30	21
18	26 17 11	914.716	7	32	0.261 0.313	W	−1.9	31	23
19	26 18 41	914.778	7	32	0.324 0.373	W	−0.6	29	23
20	27 15 16	915.636	7	32	1.186 1.227	W	−3.8	13	clear
21	27 16 22	915.685	7	32	1.231 1.272	W	−2.7	13	clear
22	27 17 39	915.735	7	32	1.284 1.320	W	−1.6	11	clear
23	27 19 11	915.799	7	32	1.331 1.379	W	+0.2	7	clear
26	30 15 48	918.658	5	32	0.115 0.139	W	−3.0	15	clear
29	June 3 16 18	922.682	3	33	0.039 0.054	W	−2.2	8	12
32	8 17 47	927.741	1	33	0.984 1.026	W	−0.5	21	14
33	8 19 10	927.799	1	33	1.044 1.080	E	+1.0	18	19
34	8 20 10	927.840	1	33	1.085 1.123	E	+1.9	19	19
35	8 21 07	927.880	1	33	1.134 1.157	E	+2.9	12	22
36	9 15 24	928.642	1	33	1.887 1.925	W	−2.8	19	26
37	9 16 23	928.683	− 1	33	1.928 1.966	W	−1.8	19	23
38	9 17 38	928.735	0	33	1.975 2.023	W	−0.6	22	21
39	9 18 52	928.786	0	33	2.032 0.018	E	+0.7	19	18
40	9 20 04	928.836	0	33	0.031 0.068	E	+1.9	19	19
41	9 20 57	928.873	0	33	0.072 0.103	E	+2.8	16	22
42	25 15 36	944.650	+ 7	30	1.484 1.525	W	−1.6	17	22
43	25 16 41	944.695	7	30	1.530 1.575	W	−0.5	20	20
44	25 17 50	944.743	7	30	1.591 1.606	E	+0.7	7	18
45	26 15 27	945.644	8	30	0.429 0.471	W	−1.6	16	25
46	26 16 34	945.690	8	30	0.476 0.517	W	−0.5	16	18
47	26 18 03	945.752	8	30	0.532 0.582	E	+0.9	19	22
48	28 15 32	947.647	9	30	0.379 0.429	W	−1.4	19	20
51	July 1 17 11	950.716	10	29	1.399 1.438	E	+0.4	15	18
52	1 18 14	950.760	10	29	1.443 1.482	E	+1.6	15	20
54	2 16 59	951.708	11	29	0.340 0.379	E	+0.2	14	24
58	24 15 24	973.642	21	21	1.763 1.802	E	+0.2	15	21
59	24 16 26	973.685	21	21	1.806 1.845	E	+1.2	15	23
61	27 15 14	976.635	23	20	0.654 0.692	E	+0.2	13	21
62	27 16 21	976.681	23	20	0.703 0.736	E	+1.3	13	20
63	28 15 31	977.646	23	20	1.663 1.710	E	+0.5	16	20
64	28 16 44	977.697	23	20	1.717 1.758	E	+1.8	14	22
65	31 16 31	980.688	25	19	0.600 0.650	E	+1.7	17	24
68	Aug. 1 16 43	981.696	25	18	1.620 1.649	E	+2.0	10	27
70	2 16 30	982.688	26	18	0.560 0.589	E	+1.8	10	16
76	6 14 58	2419986.624	28	16	0.391 0.425	E	+0.6	12	clear
78	22 15 15	2420002.635	35	8	2.038 0.034	E	+1.9	18	22
79	23 15 52	003.661	36	7	1.021 1.052	E	+2.6	12	clear
80	25 14 58	005.624	37	+0.0006	0.918 0.974	E	+1.8	19	16
81	Sept. 24 13 28	035.561	52	−0.0011	0.117 0.161	E	+2.8	15	19
82	1914 26 13 56	037.581	53	−0.0012	0.060 0.113	E	+2.9	18	16
239	July 15 15 18	329.638	195	+0.0025	0.889 0.914	W	−0.7	7	clear
513	1915 15 15 31	694.646	373	25	0.808 0.848	W	−0.4	15	12
514	July 15 16 30	694.688	373	25	0.853 0.889	E	+0.6	14	21
520	20 18 13	699.759	375	22	1.819 1.858	E	+2.7	15	19
521	20 18 54	699.788	375	+0.0022	1.862 1.876	E	+3.4	6	clear
546	Sept. 11 14 20	752.597	401	−0.0003	1.334 1.362	E	+2.3	11	13

to experience. However, the measures and computations were finally carried through with very satisfactory results.

In the absence of sensitometer images on most of the plates the reduction curve for each plate must be defined by the measured densities and known photographic magnitudes of a sufficient number of comparison stars. This requirement and the brightness of the variable star necessitates the selection of comparison stars farther from the center of the plate than usual. The seven stars given in Table II, all within 55mm. of the center, are found to be the most useful for this purpose.

Relative photographic magnitudes of the comparison stars are derived from plate 239, which contains sensitometer images. After the addition of a constant correction to bring the magnitudes of the Class A stars into closest agreement with the Harvard visual magnitudes, we have the adopted photographic magnitudes in the last column of Table II. The effective magnitudes of the comparison stars on each plate may now be derived, with proper allowance for distance from center and for differential atmospheric extinction. These, with the measured densities of the images, defined the reduction curve for each

TABLE II
COMPARISON STARS FOR u HERCULIS

Star (Herculis)	B.D.	R.A. (1900)	Decl. (1900)	Harvard		Adopted Photographic Magnitude
				Visual Mag.	Class	
		h m				
ε 58	+31°2947	16 56.5	+31°04′	3.92	A	3.96
d 59	33 2817	16 57.9	33 43	5.27	A2	5.25
c	36 2827	17 04.5	36 04	5.38	A	5.60
π 67	36 2844	17 11.6	36 55	3.36	K2	4.76
e 69	37 2864	17 14.2	37 24	4.80	A	4.68
w 72	32 2896	17 16.9	32 36	5.36	G	5.86
	34 2971	17 23.2	34 47	5.91	A	5.74

plate. The microphotometer wedge readings on the extrafocal images of the variable star and of the comparison stars ε, d, π and e are converted to Δ magnitudes by use of the reduction curve for the plate. After correction for atmospheric extinction and reduction to center, the magnitude differences ε − u, d − u, π − u and e − u are taken for each exposure. Employing the adopted photographic magnitudes of the comparison stars, with slight corrections due to partial measurement of the non-uniform millimeter images, we obtain the magnitude of the variable star for each comparison; and finally the mean of the four values as the observed magnitude of the variable star for each exposure. When two or more plates were obtained on the same night, slight adjustments of entire plates were subsequently made to secure better accordance between the plates.

The observed photographic magnitudes are given in Table III. The exposures are unnecessarily numerous on some of the plates, so that only part of them have been measured. The coordinates of 839 points are now available for determination of the light curve. They are first assembled into 45 normal places, Table IV, of which numbers 12 to 17 and 32 to 36 occur outside eclipse.

TABLE III

PHOTOGRAPHIC OBSERVATIONS OF U HERCULIS

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O—C (darkened)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O—C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2419	days		mag.			2419	days		mag.
5	1	886.6948	0.9789	4.69	−.04	17	2	914.6347	0.2050	4.55	−.04
	2	.6969	.9810	4.71	−.02		3	.6361	.2064	4.54	−.03
	3	.7007	.9848	4.75	+.02		4	.6382	.2085	4.54	−.03
	4	.7024	.9865	4.73	−.01		5	.6396	.2099	4.54	−.03
	5	.7045	.9886	4.76	+.02		6	.6417	.2120	4.54	−.03
	6	.7066	.9907	4.78	+.04		7	.6431	.2134	4.56	−.01
	7	.7087	.9928	4.74	.00		8	.6451	.2154	4.56	−.01
	8	.7108	.9949	4.73	−.01		9	.6465	.2168	4.55	−.02
	9	.7132	.9973	4.73	−.02		10	.6486	.2189	4.57	.00
	10	.7153	0.9994	4.76	+.01		11	.6500	.2203	4.55	−.01
	11	.7174	1.0015	4.74	−.01		12	.6521	.2224	4.57	+.01
	12	.7194	.0035	4.67	−.08		13	.6535	.2238	4.53	−.03
	13	.7215	.0056	4.73	−.03		14	.6556	.2259	4.53	−.03
	14	.7236	.0077	4.74	−.02		15	.6569	.2272	4.56	+.01
	15	.7257	.0098	4.67	−.09		16	.6590	.2293	4.52	−.03
	16	.7278	.0119	4.66	−.10		17	.6604	.2307	4.54	−.01
	17	.7299	.0140	4.70	−.06		18	.6625	.2328	4.56	+.01
	18	.7319	.0160	4.70	−.06		19	.6639	.2342	4.56	+.02
	19	.7340	.0181	4.69	−.08		20	.6660	.2363	4.57	+.03
							21	.6674	.2377	4.52	−.02
10	1	889.6785	1.9117	4.72	−.02		22	.6694	.2397	4.56	+.02
	3	.6826	.9158	4.78	+.03		23	.6708	.2411	4.54	.00
	5	.6868	.9200	4.78	+.02		24	.6729	.2432	4.53	−.01
	7	.6910	.9242	4.81	+.03		25	.6743	.2446	4.58	+.04
	9	.6951	.9283	4.84	+.04		26	.6764	.2467	4.54	+.01
	11	.6993	.9325	4.80	−.01		28	.6799	.2502	4.51	−.02
	13	.7035	.9367	4.80	−.03		29	.6812	.2515	4.50	−.03
	15	.7076	.9408	4.82	−.02		30	.6833	.2536	4.52	−.01
	17	.7115	.9447	4.78	−.07		31	.6847	.2550	4.52	−.01
	19	.7153	.9485	4.88	+.02						
	21	.7194	.9526	4.86	−.02	18	1	914.6903	0.2606	4.50	−.02
	23	.7236	.9568	4.88	−.01		2	.6917	.2620	4.52	.00
	25	.7278	.9610	4.90	.00		3	.6938	.2641	4.49	−.03
	27	.7319	.9651	4.90	−.02		4	.6951	.2654	4.51	−.01
	29	.7361	.9693	4.90	−.03		5	.6972	.2675	4.51	−.01
	31	.7403	.9735	4.91	−.03		6	.6986	.2689	4.53	+.01
							7	.7007	.2710	4.53	+.01
11	1	889.7448	1.9780	4.92	−.04		8	.7021	.2724	4.51	−.01
	2	.7469	.9801	4.89	−.08		9	.7042	.2745	4.50	−.01
	3	.7490	.9822	4.91	−.06		10	.7056	.2759	4.51	.00
	4	.7510	.9842	4.89	−.09		11	.7076	.2779	4.48	−.03
	5	.7531	.9863	4.89	−.10		12	.7090	.2793	4.51	.00
	6	.7552	.9884	4.90	−.09		13	.7111	.2814	4.52	+.01
	7	.7573	.9905	4.90	−.10		14	.7128	.2831	4.45	−.06
	8	.7594	.9926	4.92	−.09		15	.7146	.2849	4.53	+.02
	9	.7615	.9947	4.94	−.08		16	.7160	.2863	4.45	−.06
	10	.7635	.9967	4.97	−.05		17	.7180	.2883	4.49	−.02
	11	.7656	1.9988	4.93	−.10		18	.7194	.2897	4.48	−.03
	12	.7677	2.0009	4.95	−.09		19	.7215	.2918	4.49	−.02
	13	.7698	.0030	4.96	−.08		20	.7229	.2932	4.51	.00
	14	.7719	.0051	4.98	−.07		21	.7250	.2953	4.46	−.05
	15	.7740	.0072	4.97	−.08		22	.7264	.2967	4.51	.00
	16	.7764	.0096	4.95	−.11		23	.7285	.2988	4.52	+.02
	17	.7785	.0117	5.00	−.06		24	.7299	.3002	4.53	+.03
	18	.7806	.0138	5.00	−.07		25	.7319	.3022	4.55	+.05
	19	.7826	.0158	4.99	−.09		26	.7333	.3036	4.53	+.03
	20	.7847	.0179	5.03	−.05		27	.7354	.3057	4.51	+.01
							28	.7368	.3071	4.53	+.03
17	1	914.6326	0.2029	4.50	−.09		29	.7389	.3092	4.51	+.01

TABLE III—Continued
PHOTOGRAPHIC OBSERVATIONS OF U HERCULIS

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O–C (darkened)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O–C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2419	days		mag.			2419	days		mag.
18	30	914.7403	0.3106	4.56	+.06	21	31	915.7021	1.2724	4.53	+.01
	31	.7424	.3127	4.51	+.01						
						22	1	915.7142	1.2845	4.50	−.01
19	1	914.7542	0.3245	4.49	−.01		4	.7188	.2891	4.55	+.04
	2	.7562	.3265	4.51	+.01		7	.7229	.2932	4.50	.00
	3	.7576	.3279	4.50	.00		10	.7271	.2974	4.47	−.03
	4	.7597	.3300	4.51	+.01		11	.7285	.2988	4.52	+.02
	5	.7611	.3314	4.49	−.01		14	.7326	.3029	4.49	−.01
	6	.7632	.3335	4.50	.00		16	.7354	.3057	4.48	−.02
	7	.7646	.3349	4.51	+.01		18	.7382	.3085	4.52	+.02
	8	.7667	.3370	4.51	+.01		21	.7424	.3127	4.46	−.03
	9	.7681	.3384	4.51	+.01		23	.7451	.3154	4.50	+.01
	10	.7701	.3404	4.51	+.01		26	.7493	.3196	4.47	−.02
	11	.7715	.3418	4.49	.00						
	12	.7736	.3439	4.49	.00	23	1	915.7604	1.3307	4.44	−.04
	13	.7750	.3453	4.49	.00		4	.7646	.3349	4.52	+.04
	14	.7771	.3474	4.51	+.02		6	.7951	.3654	4.44	−.04
	15	.7785	.3488	4.52	+.03		9	.7993	.3696	4.43	−.05
	16	.7806	.3509	4.49	.00		11	.8021	.3724	4.49	+.02
	17	.7819	.3522	4.49	.00		14	.8062	.3765	4.45	−.02
	18	.7840	.3543	4.48	−.01		16	.8090	.3793	4.52	+.05
	19	.7854	.3557	4.52	+.03						
	20	.7875	.3578	4.49	.00	26	16	918.6465	0.1148	4.80	−.02
	21	.7889	.3592	4.50	+.01		17	.6479	.1162	4.83	+.01
	22	.7910	.3613	4.52	+.03		18	.6500	.1183	4.83	+.02
	23	.7924	.3627	4.49	.00		19	.6514	.1197	4.80	.00
	24	.7944	.3647	4.50	+.01		20	.6535	.1218	4.83	+.03
	25	.7958	.3661	4.49	.00		21	.6549	.1232	4.80	+.01
	26	.7979	.3682	4.49	.00		22	.6569	.1252	4.83	+.05
	27	.7993	.3696	4.45	−.04		23	.6583	.1266	4.83	+.06
	28	.8014	.3717	4.45	−.04		24	.6604	.1287	4.81	+.04
	29	.8028	.3731	4.46	−.03		25	.6618	.1301	4.80	+.03
							26	.6639	.1322	4.79	+.03
20	1	915.6153	1.1856	4.58	−.03		27	.6653	.1336	4.78	+.02
	4	.6194	.1897	4.61	.00		28	.6674	.1357	4.85	+.10
	6	.6222	.1925	4.60	.00		29	.6688	.1371	4.82	+.07
	9	.6264	.1967	4.62	+.02		30	.6708	.1391	4.82	+.08
	11	.6292	.1995	4.59	.00						
	14	.6333	.2036	4.59	.00	29	1	922.6726	0.0390	5.09	+.02
	16	.6361	.2064	4.57	−.01		2	.6747	.0411	5.05	−.01
	19	.6403	.2106	4.59	+.01		3	.6767	.0431	5.04	−.02
	21	.6431	.2134	4.57	−.01		4	.6788	.0452	5.03	−.02
	24	.6472	.2175	4.56	−.01		5	.6809	.0473	5.00	−.05
	26	.6500	.2203	4.57	.00		6	.6830	.0494	5.02	−.02
	29	.6542	.2245	4.53	−.03		7	.6851	.0515	5.03	.00
	31	.6569	.2272	4.54	−.02		8	.6872	.0536	4.94	−.09
21	1	915.6604	1.2307	4.55	.00	32	1	927.7201	0.9844	4.73	.00
	4	.6646	.2349	4.57	+.02		2	.7222	.9865	4.78	+.04
	6	.6674	.2377	4.55	.00		3	.7243	.9886	4.78	+.04
	9	.6715	.2418	4.57	+.03		4	.7264	.9907	4.78	+.04
	11	.6743	.2446	4.55	+.01		5	.7285	.9928	4.80	+.06
	14	.6785	.2488	4.55	+.01		6	.7306	.9949	4.79	+.04
	16	.6816	.2519	4.55	+.01		7	.7326	.9969	4.80	+.05
	19	.6854	.2557	4.57	+.04		8	.7347	0.9990	4.77	+.02
	21	.6882	.2585	4.49	−.04		9	.7368	1.0011	4.76	+.01
	24	.6924	.2627	4.53	.00		10	.7389	.0032	4.78	+.03
	26	.6951	.2654	4.49	−.03		11	.7410	.0053	4.80	+.04
	29	.6993	.2696	4.51	−.01		12	.7430	.0073	4.78	+.02

TABLE III—Continued

PHOTOGRAPHIC OBSERVATIONS OF U HERCULIS

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O–C (darkened)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O–C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2419	days		mag.			2419	days		mag.
32	13	927.7451	1.0094	4.73	−.03	35	11	927.8903	1.1546	4.68	+.02
	14	.7472	.0115	4.78	+.02		12	.8924	.1567	4.67	+.01
	15	.7493	.0136	4.78	+.02						
	16	.7514	.0157	4.75	−.01	36	1	928.6229	1.8872	4.72	+.05
	17	.7535	.0178	4.80	+.03		2	.6250	.8892	4.77	+.09
	18	.7556	.0199	4.76	−.01		3	.6271	.8914	4.74	+.05
	19	.7576	.0219	4.80	+.03		4	.6292	.8935	4.78	+.09
	20	.7597	.0240	4.76	−.01		5	.6312	.8955	4.75	+.06
	21	.7618	.0261	4.72	−.05		6	.6333	.8976	4.75	+.05
							7	.6354	.8997	4.76	+.05
33	1	927.7799	1.0442	4.79	+.02		8	.6375	.9018	4.79	+.08
	2	.7819	.0462	4.79	+.02		9	.6396	.9039	4.74	+.02
	3	.7840	.0483	4.77	.00		10	.6417	.9060	4.77	+.04
	4	.7861	.0504	4.78	+.01		11	.6438	.9081	4.77	+.04
	5	.7882	.0525	4.74	−.03		12	.6458	.9101	4.73	−.01
	6	.7903	.0546	4.78	+.01		13	.6479	.9122	4.77	+.03
	7	.7924	.0567	4.76	−.01		14	.6500	.9143	4.74	−.01
	8	.7944	.0587	4.80	+.03		15	.6521	.9164	4.73	−.02
	9	.7965	.0608	4.78	+.01		16	.6542	.9185	4.75	.00
	10	.7986	.0629	4.76	−.01		17	.6562	.9205	4.77	.00
	11	.8007	.0650	4.77	.00		18	.6583	.9226	4.76	−.01
	12	.8028	.0671	4.75	−.02		19	.6604	.9247	4.74	−.04
	13	.8049	.0692	4.75	−.01						
	14	.8070	.0713	4.77	+.01	37	1	928.6639	1.9282	4.82	+.03
	15	.8090	.0733	4.73	−.03		2	.6660	.9303	4.83	+.03
	16	.8111	.0754	4.72	−.04		3	.6681	.9324	4.85	+.04
	17	.8132	.0775	4.72	−.04		4	.6701	.9344	4.83	+.02
	18	.8153	.0796	4.69	−.07		5	.6722	.9365	4.83	+.01
							6	.6743	.9386	4.83	.00
34	1	927.8208	1.0851	4.72	−.03		7	.6764	.9407	4.87	+.03
	2	.8229	.0872	4.75	.00		8	.6785	.9428	4.85	+.01
	3	.8252	.0895	4.75	.00		9	.6806	.9449	4.88	+.03
	4	.8278	.0921	4.71	−.03		10	.6826	.9469	4.89	+.03
	5	.8299	.0942	4.77	+.03		11	.6847	.9490	4.88	+.02
	6	.8320	.0963	4.74	.00		12	.6868	.9511	4.91	+.03
	7	.8340	.0983	4.73	−.01		13	.6889	.9532	4.86	−.02
	8	.8361	.1004	4.75	+.01		14	.6910	.9553	4.91	+.03
	9	.8382	.1025	4.69	−.04		15	.6931	.9574	4.90	+.01
	10	.8403	.1046	4.71	−.02		16	.6951	.9594	4.87	−.03
	11	.8424	.1067	4.73	.00		17	.6972	.9615	4.89	−.02
	12	.8444	.1087	4.73	.00		18	.6993	.9636	4.95	+.04
	13	.8465	.1108	4.72	.00		19	.7014	.9657	4.89	−.03
	14	.8486	.1129	4.73	+.01						
	15	.8507	.1150	4.70	−.02	38	1	928.7111	1.9754	4.93	−.02
	16	.8528	.1171	4.70	−.02		2	.7132	.9775	4.95	−.01
	17	.8549	.1192	4.69	−.03		3	.7153	.9796	4.91	−.06
	18	.8569	.1212	4.71	.00		4	.7174	.9817	4.95	−.02
	19	.8590	.1233	4.71	.00		5	.7194	.9837	5.00	+.02
							6	.7215	.9858	4.97	−.02
35	1	927.8694	1.1337	4.70	+.01		7	.7246	.9889	4.96	−.04
	2	.8715	.1358	4.68	−.01		8	.7278	.9921	4.99	−.02
	3	.8736	.1379	4.71	+.02		9	.7299	.9942	4.94	−.07
	4	.8757	.1400	4.70	+.02		10	.7319	.9962	4.99	−.03
	5	.8778	.1421	4.70	+.02		11	.7347	1.9990	5.01	−.02
	6	.8799	.1442	4.68	.00		12	.7375	2.0018	5.05	+.01
	7	.8819	.1462	4.69	+.02		13	.7396	.0039	5.01	−.03
	8	.8840	.1483	4.69	+.02		14	.7417	.0060	5.03	−.02
	9	.8861	.1504	4.65	−.02		15	.7438	.0081	5.05	.00
	10	.8882	.1525	4.67	+.01		16	.7458	.0101	5.01	−.05

TABLE III—Continued

PHOTOGRAPHIC OBSERVATIONS OF U HERCULIS

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O–C (darkened)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O–C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2419	days		mag.			2419	days		mag.
38	17	928.7479	2.0122	5.03	−.06	41	13	928.8833	0.0966	4.84	−.04
	18	.7500	.0143	5.06	−.01		14	.8854	.0987	4.89	+.02
	19	.7521	.0164	5.03	−.05		15	.8875	.1008	4.87	+.01
	20	.7542	.0185	5.11	+.03		16	.8896	.1029	4.85	−.01
	21	.7562	.0205	5.07	−.01						
	22	.7583	.0226	5.06	−.03	42	1	944.6285	1.4844	4.50	+.03
							2	.6319	.4878	4.52	+.05
39	1	928.7674	2.0317	5.09	−.01		3	.6354	.4913	4.50	+.03
	2	.7694	.0337	5.09	−.02		4	.6375	.4934	4.47	.00
	3	.7715	.0358	5.06	−.05		5	.6403	.4962	4.53	+.06
	4	.7736	.0379	5.08	−.03		6	.6424	.4983	4.50	+.03
	5	.7757	.0400	5.09	−.02		7	.6451	.5010	4.50	+.03
	6	.7778	.0421	5.14	+.02		8	.6472	.5031	4.51	+.04
	7	.7799	.0442	5.11	−.01		9	.6500	.5059	4.44	−.03
	8	.7819	.0462	5.12	.00		10	.6521	.5080	4.47	.00
	9	.7840	.0483	5.10	−.02		11	.6549	.5108	4.51	+.04
	10	.7861	2.0504	5.10	−.02		12	.6569	.5128	4.48	+.01
	11	.7882	0.0015	5.15	+.03		13	.6597	.5156	4.46	−.01
	12	.7903	.0036	5.13	+.01		14	.6618	.5177	4.46	−.01
	13	.7924	.0057	5.13	+.01		15	.6646	.5205	4.48	+.01
	14	.7944	.0077	5.11	−.01		16	.6667	.5226	4.44	−.03
	15	.7965	.0098	5.14	+.02		17	.6694	.5253	4.41	−.06
	16	.7986	.0119	5.12	.00						
	17	.8007	.0140	5.17	+.05	43	1	944.6736	1.5295	4.46	−.01
	18	.8028	.0161	5.18	+.07		2	.6757	.5316	4.50	+.03
	19	.8049	.0182	5.19	+.08		3	.6785	.5344	4.48	+.01
							4	.6806	.5365	4.46	−.01
40	1	928.8177	0.0310	5.20	+.11		5	.6833	.5392	4.48	+.01
	2	.8194	.0327	5.14	+.05		6	.6854	.5413	4.46	−.01
	3	.8215	.0348	5.10	+.02		7	.6882	.5441	4.44	−.03
	4	.8236	.0369	5.10	+.03		8	.6903	.5462	4.48	+.01
	5	.8257	.0390	5.06	−.01		9	.6931	.5490	4.49	+.02
	6	.8278	.0411	5.08	+.01		10	.6951	.5510	4.48	+.01
	7	.8299	.0432	5.07	+.01		11	.6979	.5538	4.46	−.01
	8	.8320	.0453	5.05	.00		12	.7000	.5559	4.48	+.01
	9	.8340	.0473	5.03	−.01		13	.7028	.5587	4.45	−.02
	10	.8361	.0494	4.99	−.05		14	.7049	.5608	4.49	+.02
	11	.8382	.0515	5.01	−.02		15	.7076	.5635	4.49	+.02
	12	.8403	.0536	5.00	−.03		16	.7097	.5656	4.44	−.03
	13	.8424	.0557	5.01	−.01		17	.7125	.5684	4.43	−.04
	14	.8444	.0577	4.99	−.02		18	.7146	.5705	4.47	.00
	15	.8465	.0598	5.00	−.01		19	.7174	.5733	4.48	+.01
	16	.8486	.0619	5.02	+.02		20	.7194	.5753	4.49	+.02
	17	.8507	.0640	4.95	−.05						
	18	.8528	.0661	4.98	−.01	44	1	944.7354	1.5913	4.44	−.04
	19	.8549	.0682	4.95	−.03		2	.7382	.5941	4.49	+.01
							3	.7403	.5962	4.49	+.01
41	1	928.8583	0.0716	4.98	+.01		4	.7430	.5989	4.50	+.02
	2	.8604	.0737	4.97	+.01		5	.7451	.6010	4.47	−.01
	3	.8625	.0758	4.95	−.01		6	.7479	.6038	4.48	.00
	4	.8646	.0779	4.93	−.01		7	.7500	.6059	4.50	+.02
	5	.8667	.0800	4.91	−.03						
	6	.8688	.0821	4.89	−.04	45	1	945.6243	0.4291	4.54	+.06
	7	.8708	.0841	4.93	.00		2	.6271	.4319	4.53	+.05
	8	.8729	.0862	4.89	−.03		3	.6299	.4347	4.55	+.07
	9	.8750	.0883	4.86	−.05		4	.6326	.4374	4.52	+.04
	10	.8771	.0904	4.89	−.01		5	.6354	.4402	4.55	+.07
	11	.8792	.0925	4.86	−.03		6	.6382	.4430	4.54	+.06
	12	.8812	.0945	4.87	−.02		7	.6410	.4458	4.53	+.05

TABLE III—Continued

PHOTOGRAPHIC OBSERVATIONS OF U HERCULIS

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O—C (darkened)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O—C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2419	days		mag.			2419	days		mag.
45	8	945.6438	0.4486	4.56	+.08	48	13	947.6583	0.4121	4.50	+.01
	9	.6465	.4513	4.53	+.05		14	.6611	.4149	4.49	.00
	10	.6493	.4541	4.52	+.04		15	.6639	.4177	4.50	+.01
	11	.6521	.4569	4.51	+.03		16	.6667	.4205	4.53	+.04
	12	.6549	.4597	4.49	+.01		17	.6694	.4232	4.53	+.04
	13	.6576	.4624	4.50	+.02		18	.6722	.4260	4.57	+.08
	14	.6604	.4652	4.47	−.01		19	.6750	.4288	4.49	.00
	15	.6632	.4680	4.48	.00						
	16	.6660	.4708	4.44	−.04	51	1	950.6965	1.3992	4.50	+.03
							2	.6993	.4020	4.50	+.03
46	1	945.6708	0.4756	4.50	+.02		3	.7021	.4048	4.49	+.02
	2	.6736	.4784	4.51	+.03		4	.7049	.4076	4.49	+.02
	3	.6764	.4812	4.49	+.01		5	.7076	.4103	4.48	+.01
	4	.6792	.4840	4.47	−.01		6	.7104	.4131	4.45	−.02
	5	.6820	.4868	4.45	−.03		7	.7132	.4159	4.49	+.02
	6	.6847	.4895	4.46	−.02		8	.7160	.4187	4.50	+.03
	7	.6875	.4923	4.46	−.02		9	.7188	.4215	4.48	+.01
	8	.6903	.4951	4.46	−.02		10	.7215	.4242	4.48	+.01
	9	.6931	.4979	4.48	.00		11	.7243	.4270	4.44	−.03
	10	.6958	.5006	4.47	−.01		12	.7271	.4298	4.46	−.01
	11	.6986	.5034	4.47	−.01		13	.7299	.4326	4.45	−.02
	12	.7014	.5062	4.49	+.01		14	.7326	.4353	4.47	.00
	13	.7042	.5090	4.46	−.02		15	.7354	.4381	4.46	−.01
	14	.7070	.5118	4.48	.00						
	15	.7097	.5145	4.47	−.01	52	1	950.7403	1.4430	4.44	−.03
	16	.7125	.5173	4.45	−.03		2	.7430	.4457	4.46	−.01
							3	.7458	.4485	4.44	−.03
47	1	945.7271	0.5319	4.41	−.07		4	.7486	.4513	4.47	.00
	2	.7299	.5347	4.43	−.05		5	.7514	.4541	4.44	−.03
	3	.7326	.5374	4.44	−.04		6	.7542	.4569	4.45	−.02
	4	.7354	.5402	4.47	−.01		7	.7569	.4596	4.34	−.13
	5	.7382	.5430	4.44	−.04		8	.7597	.4624	4.40	−.07
	6	.7410	.5458	4.48	.00		9	.7625	.4652	4.44	−.03
	7	.7438	.5486	4.49	+.01		10	.7653	.4680	4.48	+.01
	8	.7465	.5513	4.44	−.04		11	.7681	.4708	4.52	+.05
	9	.7493	.5541	4.46	−.02		12	.7708	.4735	4.48	+.01
	10	.7521	.5569	4.44	−.04		13	.7736	.4763	4.45	−.02
	11	.7549	.5597	4.43	−.05		14	.7764	.4791	4.51	+.04
	12	.7576	.5624	4.52	+.04		15	.7792	.4819	4.46	−.01
	13	.7604	.5652	4.44	−.04						
	14	.7632	.5680	4.39	−.09	54	1	951.6882	0.3399	4.45	−.04
	15	.7660	.5708	4.46	−.02		2	.6910	.3427	4.48	−.01
	16	.7688	.5736	4.39	−.09		3	.6938	.3455	4.42	−.07
	17	.7715	.5763	4.50	+.02		4	.6965	.3482	4.44	−.05
	18	.7743	.5791	4.48	.00		5	.6993	.3510	4.44	−.05
	19	.7771	.5819	4.46	−.02		6	.7021	.3538	4.46	−.03
							7	.7049	.3566	4.47	−.02
48	1	947.6250	0.3788	4.49	.00		8	.7076	.3593	4.44	−.05
	2	.6278	.3816	4.52	+.03		9	.7101	.3618	4.46	−.03
	3	.6306	.3844	4.54	+.05		10	.7132	.3649	4.47	−.02
	4	.6333	.3871	4.53	+.04		11	.7160	.3677	4.47	−.02
	5	.6361	.3899	4.53	+.04		12	.7191	.3708	4.49	.00
	6	.6389	.3927	4.51	+.02		14	.7243	.3760	4.42	−.07
	7	.6417	.3955	4.51	+.02		15	.7271	.3788	4.46	−.03
	8	.6444	.3982	4.51	+.02						
	9	.6472	.4010	4.47	−.02	58	1	973.6222	1.7629	4.48	−.02
	10	.6500	.4038	4.48	−.01		2	.6250	.7657	4.51	+.01
	11	.6528	.4066	4.47	−.02		3	.6278	.7685	4.49	−.01
	12	.6556	.4094	4.50	+.01		4	.6306	.7713	4.55	+.04

TABLE III—Continued
PHOTOGRAPHIC OBSERVATIONS OF u HERCULIS

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O–C (darkened)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O–C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2419	days		mag.			2419	days		mag.
58	5	973.6333	1.7740	4.52	+.01	63	4	977.6340	1.6725	4.52	+.04
	6	.6361	.7768	4.60	+.09		5	.6368	.6753	4.54	+.06
	7	.6389	.7796	4.48	−.03		6	.6403	.6788	4.54	+.06
	8	.6417	.7824	4.63	+.12		7	.6431	.6816	4.54	+.05
	9	.6444	.7851	4.62	+.11		8	.6465	.6850	4.52	+.03
	10	.6472	.7879	4.49	−.02		9	.6493	.6878	4.46	−.03
	11	.6500	.7907	4.51	−.01		10	.6528	.6913	4.49	.00
	12	.6528	.7935	4.54	+.02		11	.6556	.6941	4.48	−.01
	13	.6556	.7963	4.49	−.03		12	.6590	.6975	4.51	+.02
	14	.6583	.7990	4.50	−.02		13	.6618	.7003	4.52	+.03
	15	.6611	.8018	4.49	−.03		14	.6653	.7038	4.46	−.03
							15	.6681	.7066	4.40	−.09
59	1	973.6653	1.8060	4.51	−.02		16	.6715	.7100	4.46	−.03
	2	.6681	.8088	4.48	−.05						
	3	.6708	.8115	4.51	−.02	64	1	977.6785	1.7170	4.40	−.09
	4	.6736	.8143	4.49	−.04		2	.6820	.7205	4.45	−.04
	5	.6764	.8171	4.48	−.06		3	.6847	.7232	4.45	−.04
	6	.6792	.8199	4.54	.00		4	.6882	.7267	4.53	+.04
	7	.6820	.8227	4.55	.00		5	.6910	.7295	4.50	+.01
	8	.6847	.8254	4.52	−.03		6	.6944	.7329	4.49	.00
	9	.6875	.8282	4.53	−.02		7	.6972	.7357	4.46	−.03
	10	.6903	.8310	4.54	−.02		8	.7007	.7392	4.50	+.01
	11	.6931	.8338	4.53	−.03		9	.7035	.7420	4.47	−.03
	12	.6958	.8365	4.51	−.05		10	.7070	.7455	4.48	−.02
	13	.6986	.8393	4.52	−.05		11	.7097	.7482	4.46	−.04
	14	.7014	.8421	4.52	−.05		12	.7132	.7517	4.49	−.01
	15	.7042	.8449	4.50	−.08		13	.7160	.7545	4.45	−.05
							14	.7194	.7579	4.45	−.05
61	1	976.6160	0.6545	4.55	+.07						
	2	.6194	.6579	4.51	+.03	65	1	980.6632	0.5996	4.47	−.01
	3	.6222	.6607	4.48	.00		2	.6667	.6031	4.48	.00
	4	.6257	.6642	4.50	+.02		3	.6694	.6058	4.44	−.04
	5	.6285	.6670	4.51	+.03		4	.6729	.6093	4.46	−.02
	6	.6319	.6704	4.49	+.01		5	.6757	.6121	4.43	−.05
	7	.6347	.6732	4.50	+.02		6	.6792	.6156	4.49	+.01
	8	.6382	.6767	4.48	.00		7	.6820	.6184	4.58	+.10
	9	.6410	.6795	4.46	−.02		8	.6854	.6218	4.51	+.03
	10	.6444	.6829	4.45	−.03		9	.6882	.6246	4.48	.00
	11	.6472	.6857	4.50	+.02		10	.6917	.6281	4.47	−.01
	12	.6507	.6892	4.43	−.05		11	.6944	.6308	4.45	−.03
	13	.6535	.6920	4.47	−.01		12	.6979	.6343	4.52	+.04
							13	.7007	.6371	4.48	.00
62	1	976.6646	0.7031	4.47	−.01		14	.7042	.6406	4.54	+.06
	2	.6674	.7059	4.52	+.04		15	.7070	.6434	4.50	+.02
	3	.6701	.7086	4.50	+.02		16	.7104	.6468	4.54	+.06
	4	.6729	.7114	4.50	+.02		17	.7132	.6496	4.47	−.01
	5	.6757	.7142	4.47	−.01						
	6	.6785	.7170	4.48	.00	68	11	981.6840	1.6203	4.44	−.04
	7	.6812	.7197	4.52	+.04		12	.6868	.6231	4.42	−.06
	8	.6840	.7225	4.52	+.04		13	.6903	.6266	4.44	−.04
	9	.6868	.7253	4.50	+.02		14	.6931	.6294	4.46	−.02
	10	.6896	.7281	4.43	−.05		15	.6965	.6328	4.40	−.08
	11	.6924	.7309	4.47	−.01		16	.6993	.6356	4.41	−.07
	12	.6951	.7336	4.50	+.02		17	.7028	.6391	4.46	−.02
	13	.6979	.7364	4.44	−.04		18	.7062	.6425	4.43	−.05
							19	.7097	.6460	4.43	−.05
63	1	977.6243	1.6628	4.49	+.01		20	.7125	.6488	4.44	−.04
	2	.6278	.6663	4.53	+.05						
	3	.6306	.6691	4.49	+.01	70	1	982.6750	0.5603	4.48	.00

TABLE III—Continued
PHOTOGRAPHIC OBSERVATIONS OF U HERCULIS

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O—C (darkened)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O—C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2419	days		mag.			2420	days		mag.
70	2	982.6785	0.5638	4.50	+.02	80	5	005.6076	0.9305	4.65	−.01
	3	.6812	.5665	4.50	+.02		6	.6111	.9340	4.70	+.03
	4	.6847	.5700	4.49	+.01		7	.6139	.9368	4.66	−.01
	5	.6875	.5728	4.45	−.03		8	.6174	.9403	4.68	.00
	6	.6910	.5763	4.49	+.01		9	.6201	.9430	4.71	+.03
	7	.6938	.5791	4.48	.00		10	.6236	.9465	4.72	+.04
	8	.6972	.5825	4.48	.00		11	.6264	.9493	4.70	+.01
	9	.7000	.5853	4.47	−.01		12	.6299	.9528	4.71	+.02
	10	.7035	.5888	4.46	−.02		13	.6326	.9555	4.72	+.02
							14	.6361	.9590	4.69	−.01
76	1	986.6083	0.3913	4.53	+.04		15	.6389	.9618	4.68	−.03
	2	.6111	.3941	4.48	−.01		16	.6424	.9653	4.70	−.01
	3	.6139	.3969	4.51	+.02		17	.6451	.9680	4.71	.00
	4	.6174	.4004	4.49	.00		18	.6486	.9715	4.72	.00
	5	.6201	.4031	4.52	+.03		19	.6514	.9743	4.73	+.01
	6	.6236	.4066	4.46	−.03						
	7	.6264	.4094	4.48	−.01	81	1	035.5611	0.1170	4.80	−.01
	8	.6299	.4129	4.53	+.04		2	.5646	.1205	4.73	−.07
	9	.6326	.4156	4.51	+.02		3	.5674	.1233	4.71	−.08
	10	.6361	.4191	4.49	.00		4	.5708	.1267	4.75	−.03
	11	.6389	.4219	4.53	+.04		5	.5736	.1295	4.71	−.06
	12	.6424	.4254	4.47	−.02		6	.5771	.1330	4.72	−.04
		2420					7	.5799	.1358	4.73	−.02
78	1	002.6132	2.0383	5.11	.00		8	.5833	.1392	4.69	−.05
	2	.6160	.0411	5.12	.00		9	.5861	.1420	4.78	+.05
	3	.6188	.0439	5.14	+.02		10	.5896	.1455	4.70	−.02
	4	.6215	.0466	5.13	+.01		11	.5924	.1483	4.67	−.04
	5	.6243	2.0494	5.11	−.01		12	.5958	.1517	4.65	−.05
	6	.6271	0.0012	5.13	+.01		13	.5986	.1545	4.65	−.04
	7	.6299	.0040	5.17	+.05		14	.6021	.1580	4.67	−.02
	8	.6326	.0067	5.16	+.04		15	.6049	.1608	4.67	−.01
	9	.6354	.0095	5.15	+.03						
	10	.6382	.0123	5.14	+.02	82	1	037.5556	0.0603	5.06	+.05
	11	.6410	.0151	5.16	+.04		2	.5583	.0630	5.04	+.04
	12	.6438	.0179	5.15	+.04		3	.5618	.0665	5.01	+.02
	13	.6465	.0206	5.16	+.05		4	.5646	.0693	5.02	+.04
	14	.6493	.0234	5.14	+.04		5	.5681	.0728	5.04	+.07
	15	.6521	.0262	5.14	+.04		6	.5708	.0755	4.99	+.03
	16	.6549	.0290	5.12	+.03		7	.5743	.0790	5.02	+.07
	17	.6576	.0317	5.11	+.02		8	.5771	.0818	5.02	+.09
	18	.6604	.0345	5.10	+.02		9	.5806	.0853	4.98	+.05
							10	.5833	.0880	4.98	+.07
79	1	003.6472	1.0212	4.78	+.01		11	.5868	.0915	4.96	+.06
	2	.6500	.0240	4.79	+.02		12	.5896	.0943	4.94	+.05
	3	.6528	.0268	4.80	+.03		13	.5930	.0977	4.92	+.04
	4	.6556	.0296	4.82	+.05		14	.5958	.1005	4.92	+.06
	5	.6583	.0323	4.81	+.04		15	.5993	.1040	4.92	+.07
	6	.6611	.0351	4.79	+.02		16	.6021	.1068	4.92	+.08
	7	.6639	.0379	4.78	+.01		17	.6056	.1101	4.86	+.02
	8	.6667	.0407	4.79	+.02		18	.6083	.1130	4.87	+.04
	9	.6694	.0434	4.82	+.05						
	10	.6722	.0462	4.80	+.03	239	1	329.6250	0.8886	4.58	−.02
	11	.6750	.0490	4.80	+.03		2	.6292	.8928	4.59	−.02
	12	.6778	.0518	4.80	+.03		3	.6333	.8969	4.62	+.01
							4	.6375	.9011	4.63	+.01
80	1	005.5951	0.9180	4.64	.00		5	.6417	.9053	4.63	.00
	2	.5986	.9215	4.67	+.02		6	.6458	.9094	4.60	−.03
	3	.6014	.9243	4.68	+.03		7	.6500	.9136	4.62	−.02
	4	.6049	.9278	4.71	+.06						

TABLE III—Continued
PHOTOGRAPHIC OBSERVATIONS OF u HERCULIS

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O—C (darkened)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O—C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag.			2420	days		mag.
513	1	694.6264	0.8084	4.48	−.04	520	3	699.7451	1.8248	4.62	+.07
	2	.6292	.8112	4.50	−.02		4	.7479	.8276	4.61	+.06
	3	.6319	.8139	4.48	−.04		5	.7507	.8304	4.65	+.09
	4	.6347	.8167	4.52	.00		6	.7535	.8332	4.62	+.06
	5	.6375	.8195	4.49	−.04		7	.7562	.8359	4.65	+.08
	6	.6403	.8223	4.51	−.02		8	.7590	.8387	4.61	+.04
	7	.6431	.8251	4.51	−.02		9	.7618	.8415	4.61	+.03
	8	.6465	.8285	4.47	−.06		10	.7646	.8443	4.64	+.06
	9	.6493	.8313	4.54	.00		11	.7674	.8471	4.62	+.04
	10	.6521	.8341	4.54	.00		12	.7701	.8498	4.60	+.01
	11	.6549	.8369	4.51	−.03		13	.7729	.8526	4.64	+.04
	12	.6576	.8396	4.52	−.02		14	.7757	.8554	4.61	+.01
	13	.6604	.8424	4.50	−.05		15	.7785	.8582	4.61	.00
	14	.6632	.8452	4.50	−.05						
	15	.6660	.8480	4.49	−.06	521	1	699.7819	1.8616	4.58	−.03
							2	.7847	.8644	4.57	−.05
514	1	694.6708	0.8528	4.56	.00		3	.7875	.8672	4.56	−.06
	2	.6736	.8556	4.56	.00		4	.7903	.8700	4.58	−.05
	3	.6764	.8584	4.59	+.03		5	.7931	.8728	4.57	−.06
	4	.6792	.8612	4.58	+.01		6	.7958	.8755	4.58	−.06
	5	.6820	.8640	4.55	−.02						
	6	.6847	.8667	4.55	−.02	546	1	752.5833	1.3340	4.47	−.01
	7	.6875	.8695	4.58	.00		2	.5861	.3368	4.50	+.02
	8	.6903	.8723	4.55	−.03		3	.5889	.3396	4.47	−.01
	9	.6931	.8751	4.60	+.02		4	.5917	.3424	4.47	−.01
	10	.6958	.8778	4.61	+.02		5	.5944	.3451	4.45	−.03
	11	.6986	.8806	4.59	.00		6	.5972	.3479	4.46	−.02
	12	.7014	.8834	4.57	−.02		7	.6000	.3507	4.49	+.01
	13	.7042	.8862	4.59	.00		8	.6028	.3535	4.45	−.03
	14	.7070	.8890	4.58	−.02		9	.6056	.3563	4.45	−.03
							10	.6083	.3590	4.49	+.01
520	1	699.7396	1.8193	4.57	+.03		11	.6111	.3618	4.43	−.05
	2	.7424	.8221	4.60	+.05						

The time of principal minimum is established by drawing a symmetrical curve to represent the normal places. The light elements employed in the present discussion are given above. They assume the correctness of the spectroscopic period. But by comparison of all available observations of the star Hertzsprung (*loc. cit.*) derived the elements:

Primary Minimum. = J.D. 2410102.321 + 2^d.051027·E,G.M.T. (Hertzsprung)

The former period was used here because it appeared to satisfy Enebo's observations a little better and because the longer period, derived by comparing recent observations with Schmidt's which began nearly fifty years ago, might involve a variation of the period. However, Hertzsprung found no evidence of a varying period. It appears now that the longer period might better have been used; but the difference for the present series is negligible. The light elements are therefore:

Primary Minimum = J.D. 2419928.790 + 2^d.051027·E,G.M.T. (Definitive Elements)

TABLE IV

NORMAL MAGNITUDES OF U HERCULIS

No.	Phase	θ	No. Obs.	Observed Magnitude	Observed Intensity	Rectified Intensity	Computed Magnitude (uniform)	O–C (uniform)	Computed Magnitude (darkened)	O–C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	days							mag.		mag.
1	0.0002	0°.04	18	5.130	0.548	0.587	5.120	+.010	5.120	+.010
2	0.0214	3.76	14	5.151	0.537	0.576	5.105	+.046	5.108	+.043
3	0.0424	7.44	15	5.054	0.587	0.628	5.056	−.002	5.061	−.007
4	0.0623	10.94	18	5.000	0.617	0.657	4.995	+.005	5.000	.000
5	0.0851	14.94	17	4.934	0.656	0.696	4.917	+.017	4.927	+.007
6	0.1086	19.06	16	4.853	0.707	0.746	4.839	+.014	4.844	+.009
7	0.1290	22.64	14	4.781	0.755	0.795	4.778	+.003	4.778	+.003
8	0.1476	25.91	10	4.712	0.805	0.843	4.723	−.011	4.719	−.007
9	0.2187	38.39	19	4.546	0.938	0.972	4.571	−.025	4.567	−.021
10	0.2573	45.16	22	4.522	0.958	0.988	4.524	−.002	4.526	−.004
11	0.3036	53.29	26	4.501	0.978	1.000	4.501	.000	4.501	.000
12	0.3523	61.84	31	4.484	0.993	1.010	4.495	−.011		
13	0.4018	70.52	39	4.499	0.980	0.991	4.489	+.010		
14	0.4814	84.50	33	4.486	0.991	0.996	4.482	+.004		
15	0.5753	100.98	33	4.468	1.007	1.009	4.478	−.010		
16	0.6554	115.04	22	4.492	0.986	0.988	4.479	+.013		
17	0.7324	128.55	20	4.479	0.997	1.004	4.483	−.004		
18	0.8328	146.18	18	4.515	0.964	0.976	4.545	−.030	4.538	−.023
19	0.8789	154.27	14	4.581	0.908	0.921	4.592	−.011	4.587	−.006
20	0.9316	163.52	18	4.673	0.835	0.849	4.663	+.010	4.662	+.011
21	0.9830	172.54	19	4.742	0.782	0.797	4.730	+.012	4.733	+.009
22	1.0060	176.58	19	4.744	0.781	0.794	4.751	−.007	4.754	−.010
23	1.0282	180.47	17	4.777	0.758	0.771	4.770	+.007	4.771	+.006
24	1.0560	185.35	15	4.775	0.759	0.772	4.771	+.004	4.771	+.004
25	1.0850	190.44	12	4.733	0.790	0.801	4.750	−.017	4.751	−.018
26	1.1119	195.16	12	4.714	0.803	0.814	4.718	−.004	4.723	−.009
27	1.1452	201.01	12	4.685	0.825	0.836	4.677	+.008	4.677	+.008
28	1.2050	211.50	12	4.582	0.907	0.913	4.594	−.012	4.587	−.005
29	1.2467	218.82	12	4.542	0.941	0.945	4.547	−.005	4.540	+.002
30	1.2922	226.81	10	4.507	0.971	0.974	4.507	.000	4.503	+.004
31	1.3326	233.90	11	4.474	1.002	1.001	4.782	−.008	4.783	−.009
32	1.3881	243.64	20	4.475	1.001	0.998	4.473	+.002		
33	1.4535	255.12	19	4.453	1.021	1.015	4.469	−.016		
34	1.5133	265.62	25	4.477	0.999	0.994	4.471	+.006		
35	1.5742	276.31	20	4.475	1.001	0.999	4.474	+.001		
36	1.6647	292.19	25	4.473	1.003	1.010	4.484	−.011		
37	1.7485	306.90	23	4.491	0.987	1.004	4.495	−.004	4.495	−.004
38	1.8138	318.36	24	4.451	0.942	0.968	4.448	+.003	4.447	+.004
39	1.8475	324.28	15	4.586	0.904	0.934	4.595	−.009	4.588	−.002
40	1.8906	331.84	15	4.709	0.807	0.840	4.688	+.021	4.684	+.025
41	1.9221	337.37	18	4.781	0.755	0.791	4.773	+.008	4.773	+.008
42	1.9476	341.85	18	4.861	0.702	0.739	4.852	+.009	4.861	.000
43	1.9759	346.81	19	4.918	0.666	0.705	4.947	−.029	4.955	−.037
44	1.9989	350.85	18	4.975	0.631	0.670	5.023	−.048	5.028	−.053
45	2.0223	354.96	18	5.048	0.590	0.630	5.084	−.036	5.087	−.039

Inasmuch as the system of u Herculis is approaching the earth at the rate of 21.16 km. a second, the true period is somewhat greater, as Hertzsprung has pointed out, in fact 2.051172 days.

In Table V I have given the times of primary minimum established from six series of observations. Schmidt's results were assembled by Hertzsprung and evidently also those of Enebo and Wendell; for Enebo gives the time of principal minimum J.D. 2416538.46, and Schlesinger and Baker have found the time J.D. 2418125.938 for Wendell's observations. The number of observations in each set appears in the table; the first four series are visual determinations and the last two are photographic. Hertzsprung's observations were made to establish a single minimum. The fourth column of the table contains differences between the observed times and those computed from the definitive elements, and it shows the accuracy of the definitive period.

There is a marked difference between the photographic light curve and the curve that represents Wendell's visual observations. The photographic curve has the primary minimum $0^m.15$ less in depth and the secondary minimum somewhat deeper than the visual curve. This implies that the fainter star is the earlier in spectral class and that the difference in color index between the two stars is nearly a half magnitude. Consequently a difference in the spectra ought to be noticed on the spectrograms. But the spectra of the two stars on the Allegheny plates are certainly identical. The latter part of Table V is interesting in this connection. It appears that Wendell's range at primary minimum, $0^m.71$, exceeds that of the other observers. Schmidt's curve is not photometric; Lau's curve drops $0^m.59$ at primary minimum, but his determinations are not numerous. Accordingly we conclude that the magnitude of primary minimum varies or that too great a loss of light is given by the Harvard observations. In either event there is little to be gained by comparing the Harvard and Laws Observatory curves.

TABLE V

COMPARISON OF OBSERVATIONS

Observer	No. Obs.	Primary Minimum	O–C	Range primary	Range secondary	$\frac{R_p}{R_s}$
		J.D., G.M.T.	days	mag.	mag.	
Schmidt	4148	2405830.032	+.002			1.9
Enebo	438	2416846.105	+.009	0.48	0.18	2.7
Ichinohe	154	2418125.93	−.007	0.55	0.20	2.8
Wendell	138	2418125.933	−.006	0.71	0.24	3.0
Hertzsprung	18	2418751.500	−.001			
Baker	839	2419928.790	.000	0.56	0.26	2.2

An inspection of the light curve defined by the normal places shows that the two stars are close together compared with their diameters; for the duration of the eclipses is more than half the period of revolution. The secondary minimum occurs $0^d.020$ after the time midway between primary minima. Outside eclipse the curve rises toward secondary minimum and it is slightly higher after this minimum than before it; also its upward convexity suggests a degree of ellipticity in the stars. This part of the curve must first

receive attention. The solution is made according to the method developed at the Princeton Observatory[15]. References to tables and formulae in this series of papers will be distinguished as usual by italics.

We have the expression:

$$l = a - b\cos\theta - c\cos^2\theta - f\sin\theta$$

in which l is the intensity of the light, a is the rectified intensity which is constant between eclipses, $2b$ is the difference in luminosity between the hemispheres of the fainter star turned toward and away from the brighter star, c is the ellipticity constant that determines the ratio between the major and minor axes of the stars, $2f$ is that part of the difference in intensity between periastron and apastron which is not taken up in the reflection constant and θ is the longitude reckoned from the position of the stars at primary minimum. After conversion from magnitudes to intensities, each normal place furnishes an equation of condition of the form obtained by differentiating the above expression. These, when equally weighted, yield the following normal equations:

$$\begin{aligned} +17.000\,\delta a + 0.766\,\delta b - 3.997\,\delta c - 0.520\,\delta f &= -0.012 \\ +3.997\,\delta b - 0.435\,\delta c + 1.017\,\delta f &= +0.012 \\ +1.509\,\delta c + 0.529\,\delta f &= 0.000 \\ +13.002\,\delta f &= -0.079 \end{aligned}$$

On solving these equations we have:

	Preliminary	Correction	Final	Probable Error	Weight
a =	0.980	−0.0008 =	0.979	±0.0029	6.290
b =	0.01	+0.0048 =	0.015	±0.0038	3.731
c =	0.03	+0.0019 =	0.032	±0.010	0.535
f =	+0.01	−0.0065 =	+0.004	±0.0020	12.307
m_0 =	$4^m.453$	$+0^m.023$ =	$4^m.476$	$\pm 0^m.0031$	

The final m_0 is the magnitude of the system between eclipses, after the radiation, ellipticity and periastron effects have been eliminated; its correction is the magnitude difference corresponding to the intensity difference between the final value of a and unity. The probable errors of the constants are found from their weights in the solution. The probable error of a normal magnitude, weight unity, is $0^m.0078$, and of a normal intensity is 0.0072. This completes the discussion of the light curve independent of the eclipses.

The effects of ellipticity, radiation and brightening at periastron are now removed from the normal intensities during eclipse by use of the formula:

$$R = \frac{l + b(1+\cos\theta) + c \cdot l \cdot \cos^2\theta + f\sin\theta}{a+b}$$

[15]H. N. Russell, *Astrophysical Journal*, 35, 315, 1912; 36, 54, 1912; H. N. Russell and H. Shapley, *Astrophysical Journal*, 36, 239, 385, 1912.

where R is the rectified intensity. These values, which appear in the seventh column of Table IV, are now plotted against corresponding phases and symmetrical curves are drawn to represent the points. The spectroscopic results should be used so far as possible to guide the solution of the light curve. These give for the eccentricity and the longitude of periastron respectively $e = 0.053$ and $\omega = 66°.15$. The time of periastron is given by the formula:

$$T = \text{J.D. } 2418125.80 + 2^d.051027 \cdot E, \text{G.M.T.}$$

On computing the times of the two conjunctions following the Laws Observatory epoch, putting $u = 90°$ and $270°$ and letting $i = 75°$, we have:

Principal Conjunction = J.D. 2419928.775, G.M.T.
Secondary Conjunction = J.D. 2419929.8295

Two discrepancies between the spectroscopic and photometric results are to be noted. First, the time of primary minimum is $0^d.015$ later than that of principal conjunction. Schlesinger and Baker found from Wendell's observations a difference of $0^d.016$ in the same direction. Other eclipsing variables have shown a peculiarity of this character. Second, the interval between the two conjunctions exceeds one-half the period by $0^d.029$; but the photographic light curve shows the secondary minimum displaced only $0^d.020$. Shapley appears to have found a satisfactory agreement at this point between the requirements of the spectroscopic elements and the Harvard magnitudes. But on plotting his computed light curve from the data in *Princeton Contributions*, No. 3, page 68, I find that the secondary minimum is not symmetrical relative to phase $1^d.0255 + 0^d.029 = 1^d.0545$, as it should be for such agreement; the displacement appears to be one-half the required value, placing the secondary minimum at phase $1^d.040$*. It is probable that a division by two crept in some way into the computation of this quantity. It is, of course, arbitrary to suppose that secondary minimum follows conjunction by the same interval as primary minimum, since we can not interpret the delay in either case. I have made this assumption and, keeping the eccentricity unchanged, have adopted the value of ω required by the displacement of the secondary minimum. These photometric elements become: $e = 0.053$, $\omega = 73°.8$. Inasmuch as the light curve computed from the spectroscopic elements gives a satisfactory representation of Wendell's visual observations, with the exception noted, it is not to be supposed that the same elements will lead to a light curve suitable for the present photographic observations, except by modifying the assumptions, especially since the two stars are of the same spectral class.

The distances between the centers of the two stars at primary and secondary minimum has the following ratio:

$$\frac{1 + kp(k, a_{op})}{1 + kp(k, a_{os})} = \frac{1 - e \sin \omega}{1 + e \sin \omega} = \frac{0.949}{1.051} = 0.903$$

*In a recent letter Dr. Shapley writes that he used the value $1^d.040$.

The rectified light curve gives the corresponding losses of light respectively $1-\lambda_p=0.408$ and $1-\lambda_s=0.225$ relative to the total light of the system. If we assume a difference of one magnitude between the two stars, or $L_b=0.715$ and $L_f=0.285$, the brighter star is the larger; the ratio of the radii and the maximum obscurations at the minima are $k=0.82$, $a_{op}=0.84$ and $a_{os}=0.81$. The light curve thus defined represents very well the photographic magnitudes during primary minimum, but it does not serve for the secondary minimum. Moreover, preliminary solutions of the light curve indicate a satisfactory value of k near unity. In view of the considerable indetermination in its value, it is proper to adopt $k=1$. The intensities of the two stars are now chosen so that the quantities a_{op} and a_{os}, defined by the formula:

$$a_{op}L_b=1-\lambda_p,\ a_{os}L_f=1-\lambda_s$$

will satisfy the distance relation. A few trials give:

$$L_b=0.631,\ L_f=0.369,\ a_{op}=0.647,\ a_{os}=0.610$$

By use of *Tables III* and *IIIa* and of *formula g'*:

$$\frac{\sin^2\theta(n)}{1-z\cos^2\theta(n)}=C\omega_2(n)+D\omega_1(n),\ \frac{C}{D}=\chi(k, a_o, \tfrac{1}{4})$$

given the value of D_p from the preliminary solution, we find:

$$\chi(k, a_{op}, \tfrac{1}{4}) = 1.920,\ C_p=0.2976,\ D_p=0.155$$
$$\chi(k, a_{os}, \tfrac{1}{4}) = 1.894,\ C_s=0.358\ ,\ D_s=0.189$$

It remains to compute the phases corresponding to different fractions of the greatest loss of light at the two minima. The light curve for uniformly luminous disks is now completely defined.

In the solution for disks completely darkened at the limb, the depths of minima and the ratio of the radii are taken as before. In this case we have:

$$L_b=0.626,\ L_f=0.374,\ a_{op}=0.652,\ a_{os}=0.602$$

Entering *Table IIIx* we find:

$$\chi(k, a_{op}, 0) = 4.33,\ \chi(k, a_{op}, \tfrac{1}{4}) = 1.870,\ \chi(k, a_{op}, \tfrac{3}{4}) = 0.421$$
$$\chi(k, a_{os}, 0) = 4.24,\ \chi(k, a_{os}, \tfrac{1}{4}) = 1.855,\ \chi(k, a_{os}, \tfrac{3}{4}) = 0.427$$

The phase in each case is obtained by *formula f'*:

$$\frac{\sin^2\theta(n)[1-z\cos^2\theta(\frac{1}{2})]}{\sin^2\theta(\frac{1}{2})[1-z\cos^2\theta(n)]}=\chi(k, a_o, n)$$

in which z has five-eights of its value in the uniform solution, or 0.040. The values of $\sin^2\theta(\frac{1}{2})$ for the two minima, taken unchanged from the previous solution, are 0.1465 and 0.1790; in addition, of course, $\sin^2\theta(1) = 0$. The theoretical curve for darkened stars may now be drawn from the five points for each minimum. It differs only slightly from the curve for the uniform solution.

If the orbits were circular, the inclination, i, and the major semi-axis of either star, a, would be derived by *formula h'*:

$$\cos^2 i \cos^2\theta' + \sin^2\theta' = a_b^2 (1 - z\cos^2\theta')(1+k)^2$$
$$\cos^2 i = a_b^2 (1-z)[1 + kp(k, a_0)]^2$$

in which $\theta' = \theta(0)$ and the p functions are taken from *Tables I* and *Ix*. These circular elements are:

	Primary	Secondary
Uniform	$a' = 0.396$, $i' = 77°.6$;	$a'' = 0.431$, $i'' = 75°.5$
Darkened	$= 0.413$, $= 74.9$;	$= 0.449$, $= 72.4$

The corresponding elliptical elements are deduced by Russell's *formula (30)*.

The elements of the system appear in Table VI. The photometric elements are given for both uniform and darkened solutions in terms of the mean distance between the centers of the two stars and of the total light of the system. The densities are derived by the formulae:

$$\rho_b = 0.01344 \cdot \frac{y}{P^2 a_b b_b c_b}, \quad \rho_f = 0.01344 \cdot \frac{1-y}{P^2 a_f b_f c_f}$$

where y is the mass of the brighter star in terms of the total mass of the system, and the polar semi-axis of either star is found to be: $c = 0.284$ and 0.283 for the uniform and darkened solutions respectively. From the photometric elements alone the densities may be determined only by assuming equal masses for the two stars, that is, $y = 0.5$. On combining the photometric and spectroscopic elements we have at once the absolute value of the linear unit $a_b + a_f$, and so of the elements expressed in terms of it. The absolute masses of the two stars and their densities, given $y = 0.723$, are now available. The absolute elements are given in the fifth column of Table VI in terms of the sun's radius (695,553 km.), mass and density. The last two columns of the table contain the elements derived by Schlesinger and Baker from the spectroscopic results and from Wendell's visual observations. It is interesting that the assumption at Allegheny of a greater magnitude difference between the two stars has compensated the greater depth of the visual primary minimum and therefore that the other details of the system are on the whole not much different in the two determinations. The present linear scale is slightly the larger. This comparison implies that the brighter star is the redder and that the difference in color index between the two stars is $0^m.44$, in other words, that the two spectra differ by more

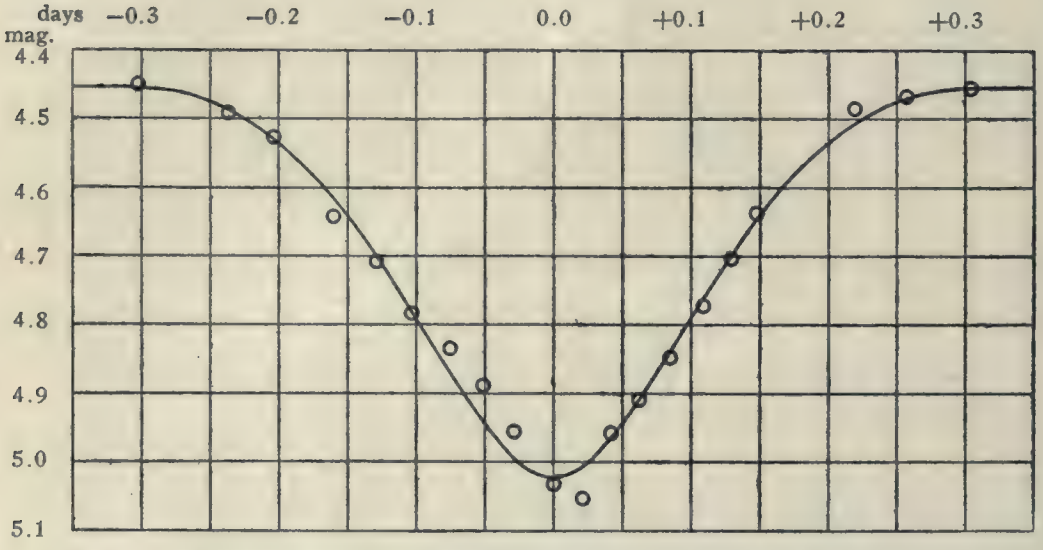

FIGURE 1. RECTIFIED NORMAL MAGNITUDES AND LIGHT CURVE OF u HERCULIS DURING PRIMARY MINIMUM (DARKENED SOLUTION)

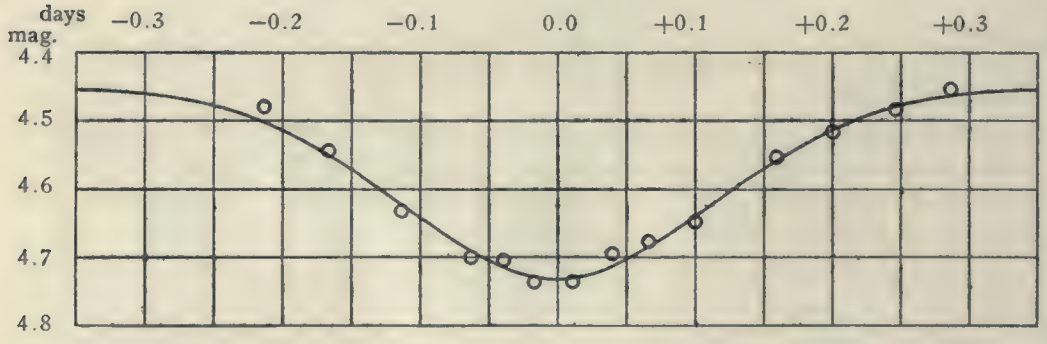

FIGURE 2. RECTIFIED NORMAL MAGNITUDES AND LIGHT CURVE OF u HERCULIS DURING SECONDARY MINIMUM (DARKENED SOLUTION)

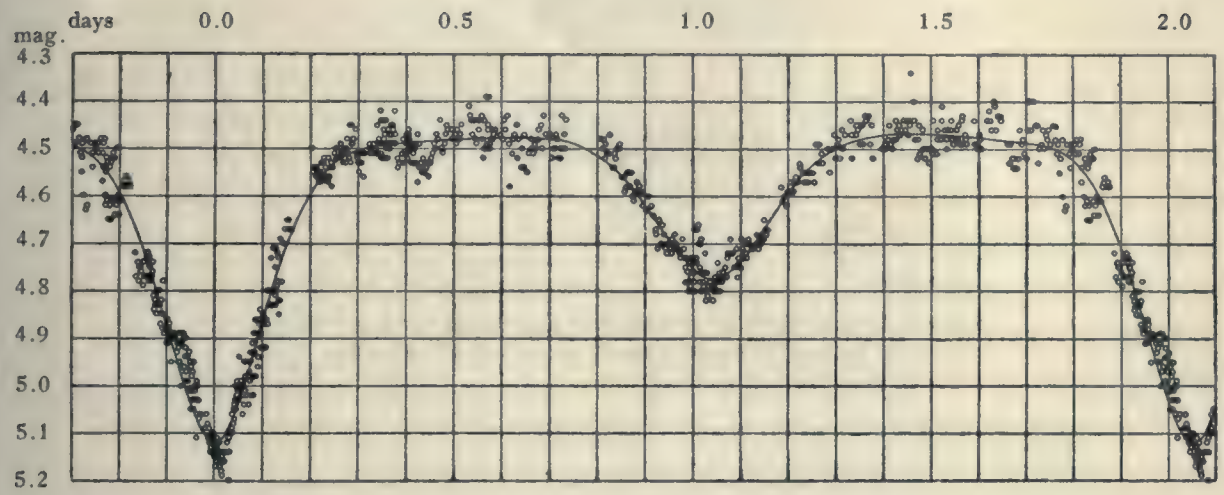

FIGURE 3. OBSERVED MAGNITUDES AND THEORETICAL LIGHT CURVE OF u HERCULIS

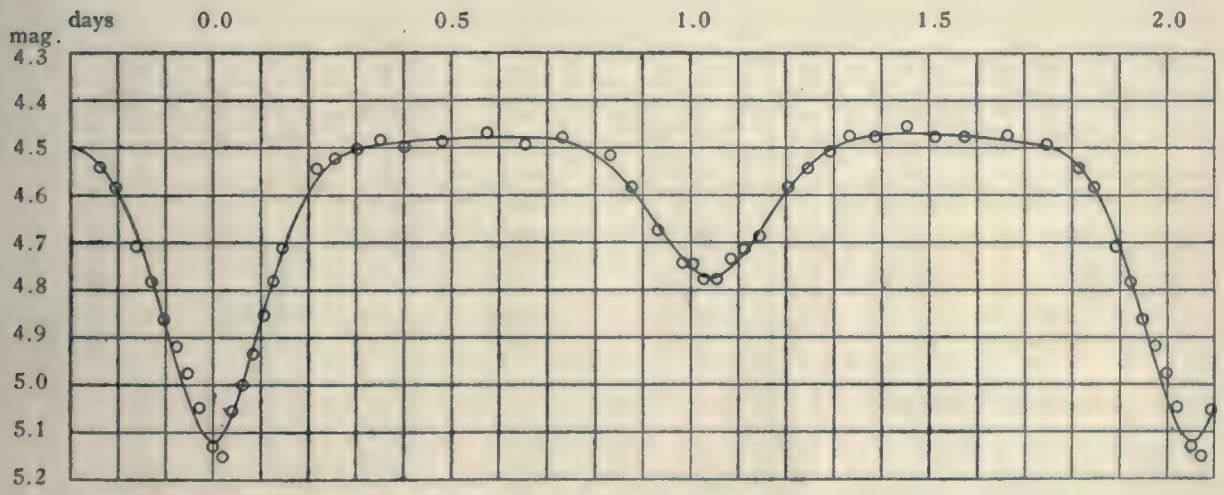

FIGURE 4. NORMAL MAGNITUDES AND THEORETICAL LIGHT CURVE OF u HERCULIS

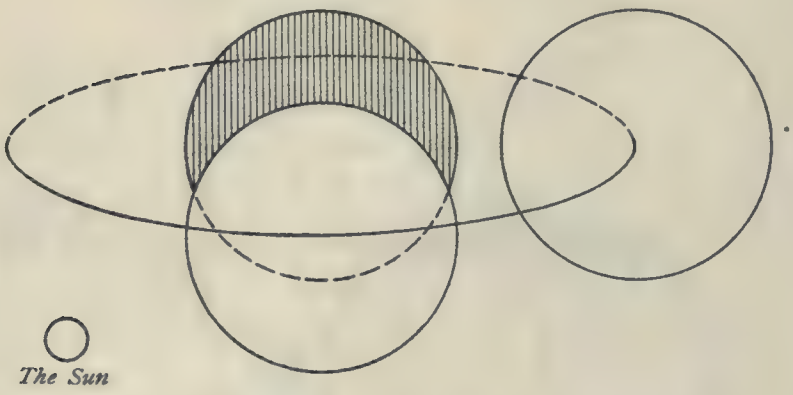

FIGURE 5. THE SYSTEM OF U HERCULIS COMPARED WITH THE SUN, AT SECONDARY MINIMUM AND AT GREATEST ELONGATION

TABLE VI
ELEMENTS OF THE SYSTEM OF U HERCULIS

		Uniform	Darkened	Absolute darkened	Schl. and Baker uniform	Schl. and Baker absolute
Period of revolution	P	2.051027 days			2.05102	
Daily motion in longitude	μ	175°.5224				
Epoch of primary minimum	T	J.D. 2419928.790 G.M.T.			928.785	
Displacement of secondary minimum	Δ	+0.020 days			+0.029	
Semi-duration of eclipse, primary	.....	0.287 days	0.306 days			
Semi-duration of eclipse, secondary	.....	0.327 days	0.343 days			
Eccentricity of orbit	e	0.053	0.053		0.053	
Longitude of periastron	ω	73°.8	73°.8		66°.15	
Inclination of orbit	i	76°.5	73°.5		75°.4	
Least apparent distance of centers	$\cos i$	0.234	0.284	4.23 ⊙	0.252	3.69 ⊙
Radius of relative orbit	$a_b + a_f$	1	1	14.88 ⊙	1	14.66 ⊙
Ratio of radii	k	1.00	1.00		1.00	
Ratio of major and minor axes	$\frac{b}{a}$	0.966	0.979		[1]	
Major semi-axis	$a_b = a_f$	0.416	0.432	6.43 ⊙	} 0.40	5.89 ⊙
Minor semi-axis	$b_b = b_f$	0.402	0.423	6.29 ⊙		
Mass of brighter star	m_b			7.71 ⊙		7.5 ⊙
Mass of fainter star	m_f			2.95 ⊙		2.9 ⊙
Density of brighter star	ρ_b	0.034⊙	0.031⊙	0.049⊙		0.037⊙
Density of fainter star	ρ_f	0.034⊙	0.031⊙	0.017⊙		0.014⊙
Maximum obscuration, primary	a_{op}	0.647	0.652		0.62	
Maximum obscuration, secondary	a_{os}	0.610	0.602		0.59	
"Radiation effect" constant	b	0.015	0.015			
"Periastron effect" constant	f	+0.004	+0.004			
Light of brighter star	L_b	0.631	0.626		0.715	
Light of fainter star, brighter side	L_f	0.369	0.374		0.285	
Light of fainter star, fainter side	$L_f - 2b$	0.339	0.344			
Ratio of surface intensities	$\frac{J_b}{J_f}$	1.71	1.67		2.6	
Magnitude of system	.....	4.476	4.476		4.64	
Magnitude of brighter star	.....	4.976	4.985		5.0	
Magnitude of fainter star	.....	5.558	5.544		6.0	
Magnitude at primary minimum	.....	5.023	5.023		5.29	
Magnitude at secondary minimum	.....	4.731	4.731		4.83	
Range of magnitude, primary	.....	0.555	0.555		0.65	
Range of magnitude, secondary	.....	0.263	0.263		0.19	

than a whole spectral class. But the observed identity of the spectra distinctly contradicts such conclusion. It is therefore unsafe to draw this inference from the comparison of the photographic and visual determinations.

Figures 1 and 2 show the arrangement of the rectified normal magnitudes at the two minima relative to the theoretical curve from the darkened solution. The curves are symmetrical in accordance with the theory; but it appears that the observations during primary minimum could be better represented by an asymmetrical curve whose ascending branch is the steeper. This would increase the epoch of primary minimum and delay it a longer time after principal conjunction. During secondary minimum the asymmetry is in the opposite direction. The sum of the squares of the residuals for the normal places during eclipse is 0.010587 for the uniform solution and 0.010565 for the darkened solution; the latter appears to be as satisfactory as the former and is taken as the definitive solution. The entire theoretical light curve is shown in Figure 3, with the observed magnitudes; each point represents an exposure. The curve is repeated in Figure 4 and is here accompanied by the normal magnitudes.

The residuals for the normal magnitudes from both uniform and darkened curves are given in Table IV. The probable error of a normal magnitude computed from these residuals is $\pm 0^{m}.012$ for points within eclipse on either solution and $\pm 0^{m}.0078$ without eclipse. The smaller value of the latter probable error may be due to the failure of the theory to account for the asymmetry of the curve during eclipses. The residuals for the observed magnitudes, scaled from the final theoretical curve, appear in the sixth column of Table IV. The probable error of a single observed magnitude is $\pm 0^{m}.024$. This is derived from the individual residuals for the entire set of 839 observations.

The present discussion completes one-third of the program of eclipsing binaries undertaken with the 5-inch camera and described in *Laws Observatory Bulletin*, No. 24. The accuracy attained is consistent, as the following table shows.

Star	Number of Plates	Number of Exposures	Probable Error of a Single Observation
RX Herculis	46	480	$\pm 0^{m}.026$
Z Vulpeculae	48	495	± 0.026
TV Cassiopeiae	51	491	± 0.029
u Herculis	53	839	± 0.024

The study of the photographic light curve of u Herculis derived from extrafocal plates made at the Laws Observatory, in combination with the writer's spectroscopic elements, leads to the following conclusions:

The system is composed of two slightly ellipsoidal stars of the same size, each having a major semi-axis 6.43 times the sun's radius. The mass of the brighter star is 7.71 and of the fainter star 2.95 times the sun's mass; the densities are respectively 0.049 and 0.017 times the sun's density. The system is shown in Figure 5, neglecting the small

ellipticity, at greatest elongation and at secondary minimum.

The star eclipsed at primary minimum is of photographic magnitude 4.98 and is brighter by $0^{m}.56$ than its companion. It contributes 62.6 per cent to the light of the system. When the photographic and visual light curves are compared, the brighter star appears to be considerably the redder; but the spectra of the two stars are identical on the Allegheny plates. Radiation and periastron effects are indicated by the least-squares solution.

The eccentricity of the orbits is 0.053 and their inclination is $73^{\circ}.5$. The mean distance between the centers of the two stars is 10,350,000 km. or only 2.3 times the radius of either star, so that the stars are undergoing eclipse 63 per cent of the time. The semi-durations of the eclipses are $7^{h}21^{m}$ and $8^{h}14^{m}$. At primary and secondary minimum respectively 65 and 60 per cent of the disk of one star is eclipsed by the other. The time of primary minimum is $0^{d}.015$ later than that of conjunction of the stars according to the spectroscopic elements.

The solution assuming the stars to be completely darkened at the limb is taken as definitive. It is practically no improvement over the solution that assumes uniformly luminous stars. The probable error of a single observed magnitude is $\pm 0^{m}.024$.

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ASTRONOMICAL SERIES

LAWS OBSERVATORY BULLETIN NO. 29

THE ECLIPSING BINARY U CORONAE

By Robert H. Baker

The early completion of the program of eclipsing binaries, described in Bulletin No. 24, was prevented by the dismantling of the Laws Observatory and the storing of its instruments, awaiting the erection of the new observatory. For six of these binaries the plates already accumulated are sufficient to establish the characteristics of the light variations, although the curves are not always so well covered by observations as those which are already published. In view of possible changes in such systems and of consequent difficulties in combining two sets of observations differing considerably in time, it is purposed to discuss the six binaries without waiting for additional plates.

In the meantime a number of changes have been made in methods of measurement and reduction. The silver dot of the microphotometer cube is now smaller than the original and a diaphragm is placed over the objective of the vertical microscope, reducing its aperture from 8 mm. to 3 mm.; the purpose is to remove any disturbing effect on the measures which may originate in the dark ring bordering the star images on our plates.

New reduction curves are employed for converting wedge readings to magnitude differences; these were made from sets of extrafocal plates of the Pleiades, the exposure times varying from 30 seconds to 30 minutes. Hertzsprung's magnitudes are used for the brighter stars and the Harvard photographic magnitudes for the fainter stars. The use of the sensitometer is discontinued, and it is no longer considered necessary to redetermine the reduction curves for each new emulsion number. The effect of sky-fog is now eliminated by an auxiliary photographic wedge which is placed in front of the usual wedge and moved until the wedge reading on the plate is the same as that for a clear film. While this procedure may not be entirely defensible, it appears to give satisfactory results. No correction is now made to the scale of magnitudes.[1]

ELEMENTS OF THE LIGHT VARIATION

Since the discovery of its light variation, by Winnecke in 1869, U Coronae (1900, R.A. = 15^h14^m1, Decl. = + 32° 01′) has been observed infrequently. Records of 45 minima by various observers between the years 1856 and 1889 were represented in Chandler's[2] analysis by the elements:

[1] *Laws Observatory Bulletin*, No. 24, 149, 1916.
[2] S. C. Chandler, *Astronomical Journal*, 9, 97, 1889.

$$\text{J.D. } 2404147.4434 + 3^d.4522269 \cdot E - 0^d.000000021 \cdot E^2. \text{ (Chandler, I)}$$

The period at that time appeared to be diminishing by 0.0036 seconds from minimum to minimum. Additional evidence, however, indicated an oscillating period and the elements:[3]

$$\text{J.D. } 2404147.396 + 3^d.452219 \cdot E + 0^d.056 \sin (0^\circ.06 \cdot E + 78^\circ). \text{ (Chandler, II)}$$

Pickering,[4] in 1896, made 56 observations of the light near primary minimum. Wendell's[5] observations, 121 in number and mostly in 1899, form the most complete visual record. According to Shapley[6] the time of primary minimum from Wendell's observations is 0.109 days later than that from Chandler's elements, I, neglecting the second order term. But a variation of the period is by no means certain, since the times of well-established minima may be represented with negligible error by the elements:

$$\text{Min.} = \text{J.D. } 2420990.723 + 3^d.452210 \cdot E, \text{ G.M.T. (Definitive Elements)}$$

Authority	Epoch	Observed Minimum	O−C
Chandler, II	− 4879	J.D. 2404147.396 G.M.T.	+.006 days
Pickering	− 2112	2413699.654	−.001
Wendell	− 1754	2414935.545	−.002
Baker	0	2420990.723	.000

These elements are definitive in the sense that they appear to be the most satisfactory for the prediction of the times of minima. The present observations, however, certainly require a shorter period, and the epoch of principal conjunction from Plaskett's spectroscopic results is not obtained from the definitive elements. The matter of the constancy of the period is therefore left open. In view of this uncertainty, it seems appropriate to assemble the Missouri results on the period which best represents them. The following elements are used in discussing these observations:

$$\text{Min.} = \text{J.D. } 2420990.723 + 3^d.45210 \cdot E, \text{ G.M.T. (Adopted Elements)}$$

THE SPECTROSCOPIC ORBIT

Plaskett's[7] recent study of the spectra of U Coronae adds this star to the limited number of eclipsing binaries for which absolute dimensions are available. Contrary to what one might have predicted from the light curve, the spectra of both stars appear on the Victoria spectrograms. According to Plaskett the spectra of the two components appear

[3] *Harvard College Observatory Annals*, **55**, part 1.
[4] E. C. Pickering, *Harvard College Observatory Annals*, **46**, 177.
[5] O. C. Wendell, *Harvard College Observatory Annals*, **69**, 76.
[6] H. Shapley, *Princeton Observatory Contributions*, No. **3**, 17, 1915.
[7] J. S. Plaskett, *Dominion Astrophysical Observatory Publications*, **1**, 11, 1920.

to be of the same class, approximately B3. The lines of the second spectrum are faint and difficult to measure, but the number of lines available and the apparent accuracy of the radial velocities turn out to be nearly as great as for the stronger spectrum. The spectroscopic orbit was derived from measures of 17 plates which were secured in 1919–20. Plaskett apparently found difficulty in bringing the radial velocities into accordance with Chandler's elements, I, as corrected by Shapley, and, deciding that the statement of the correction must be an error or a misprint, he returned to the original elements neglecting the last term. The elements of the orbit were then computed by a least squares solution.

Plaskett's solution assumes a circular orbit and an exact agreement between photometric and spectroscopic phases. But the light elements which were used are not accordant with the more recent photometric observations; the initial epoch required a correction of − 0.109 days in 1899 and of − 0.135 days in 1916. Thus we have an actual difference of 3^h23^m in 1920, if the increase is constant. Since the discrepancy is greater than any hitherto encountered, it is interesting to inquire whether it may not be eliminated or greatly reduced by a different treatment of the spectroscopic observations. Accordingly the phases for the individual spectrograms were recomputed with my definitive elements as a basis; the normal places were then formed as before. A number of graphical solutions were made, assuming various values for eccentricity of orbit and longitude of periastron. While these trials gave results less satisfactory than the original solution, it was found possible to select a velocity curve in better agreement with the velocities of the brighter star alone. However, if principal conjunction is allowed to follow primary minimum by 0.080 days, an acceptable solution for both components is readily accomplished. Plaskett's elements and those from the alternate solution are as follows:

	Plaskett	Alternate	
P =	3.4522269 days	3.452210 days	Spectroscopic elements of U Coronae
T^* =	J.D. 2420990.864	J.D. 2420990.803	
e =	0	0.08	
ω =		60°	
γ =	− 7.5 km.	− 6.5 km.	
K_b =	69.5 km.	70 km.	
K_f =	181.9 km.	185 km.	
$a_b \sin i$ =	3,278,000 km.	3,312,000 km.	
$a_f \sin i$ =	8,662,000 km.	8,754,000 km.	
$m_b \sin^3 i$ =	4.14 ⊙	4.29 ⊙	
$m_f \sin^3 i$ =	1.56 ⊙	1.62 ⊙	

The residuals for the normal places from both solutions are found in Table I; the phases are referred to the time of principal conjunction of the two stars, that is, when $\omega + v = 90°$ for the brighter component. A degree of system in my solution in the arrangement of the residuals for the fainter star might be remedied by a least squares solution.

The conclusion is that the principal conjunction of the two components of U

* Time of principal conjunction.

Coronae occurs nearly two hours later than the primary minimum of the light curve. Uncertainty as to the period may account for some of the difference, but the greater part of it is probably real. Schlesinger[8] has called attention to similar discrepancies in other eclipsing systems, but in the opposite direction. Principal conjunction occurs earlier than primary minimum in the systems of Algol, δ Librae and u Herculis by intervals between 20 minutes and 2 hours.

THE PHOTOGRAPHIC OBSERVATIONS

The present study of the light variations of U Coronae is based on 38 extrafocal plates obtained with the 5-inch camera during the years 1914 to 1917. The instruments and methods employed in this work are described in *Laws Observatory Bulletin*, No. 24, but certain changes from the original procedure are noted at the beginning of this paper. The plates were exposed about 1.5 mm. outside focus, with separate exposure times varying from 5 to 10 minutes. The comparison stars are given in Table II. In *Revised Harvard Photometry* the visual magnitude of star *c* is 6.14 and its spectrum is of class A2. Its

TABLE I

RADIAL VELOCITIES OF U CORONAE

NORMAL PLACES

No.	Plaskett's Solution Δ Phase = 0.141 days					Alternate Solution Δ Phase = 0.080 days		
	Phase	Observed		O−C		Phase	O−C	
		Bright	Faint	Bright	Faint		Bright	Faint
	days	km.	km.	km.	km.	days	km.	km.
1	0.198	−47.4		−15.4		0.258	− 8.5	
2	0.604	−66.0	+161.6	+ 3.4	+ 7.0	0.664	+ 2.8	+3.1
3	0.742	−76.4	+170.8	− 1.1	+ 0.7	0.804	− 2.9	−0.1
4	1.181	−52.5	+135.8	+13.2	− 9.1	1.241	+ 5.8	+5.2
5	1.660	−24.4		− 8.6		1.722	−10.6	
6	1.872	+19.5		+ 8.7		1.931	+10.0	
7	2.246	+53.4	−144.0	+ 4.5	+11.1	2.307	+ 5.7	+6.0
8	2.695	+62.8	−193.8	+ 2.1	− 7.7	2.757	− 3.2	+4.1
9	3.045	+41.5	−142.6	+ 2.1	−12.3	3.108	− 1.8	−4.7
Σvv				603	466		380	112

TABLE II

COMPARISON STARS FOR U CORONAE

Star	B.D.		R.A. (1900)	Decl. (1900)	Adopted Photg. Mag.
	No.	Mag.			
			h m		
v	+32°2569		15 14.1	+32°01′	
a	31 2724	7.3	16.8	31 50	7.02
b	32 2575	7.8	19.3	32 11	7.96
c	33 2574	6.8	15.4	32 54	6.22

[8] F. Schlesinger, *Allegheny Observatory Publications*, 1, 32, 1908; 1, 129, 1909; 2, 59, 1910. See also *Laws Observatory Bulletin*, No. 28, 225, 1917.

photographic magnitude is taken to be 6.22, and from this value the adopted magnitudes of the other stars are found from observed differences. Stars *a* and *b* are used for comparison with the variable star.

The observed photographic magnitudes of U Coronae are given in Table III. The times in the third column are uncorrected. The phases in column (4) are corrected to the sun and are computed from the adopted elements of the light variation. The coordinates of 225 points are now available for determining the light curve. They are first assembled into 39 normal magnitudes, of which numbers 14 to 18 and 23 to 31 occur outside eclipse.

SOLUTIONS FOR ELEMENTS OF THE SYSTEM

The discussion of the light curve follows the method of Russell and Shapley.[9] References to tables and formulae in their series of papers will be given in italics.

The light curve, as defined by the normal magnitudes in Table IV, indicates a partial eclipse at primary minimum with a decrease in brightness amounting to about 1.2 mag. compared with maximum light. No evidence of a secondary minimum is to be noticed, a surprising fact in view of the appearance of the secondary spectrum on the Victoria plates. Outside eclipse the curve rises toward the secondary minimum, showing that the light of the star eclipsed at this time is increased by the radiation of its brighter companion. A small amount of ellipticity in the stars is also inferred.

The intensity l of the light outside eclipse is represented by the expression:

$$l = a - b \cos \theta - c \cos^2 \theta$$

in which a is the intensity which is constant between eclipses, after correction for radiation and ellipticity, $2b$ is the difference in luminosity between the hemispheres of the fainter star which are turned toward and away from the brighter star, c is the ellipticity constant that determines the ratio between the major and minor axes of the stars and θ is the longitude reckoned from principal conjunction. After conversion from magnitudes to intensities, each normal place furnishes an equation of condition of the form obtained by differentiating the above expression. Equal weights are assigned to the normal places. Least squares solutions were made with and without the normal magnitudes within the secondary minimum, with the following results:

	Excluding Secondary Min.				Including Secondary Min.		
	Preliminary	Correction	Final	Probable Error	Correction	Final	Probable Error
a	0.965	+ 0.0145	= 0.9795	± 0.0063	+ 0.0123	= 0.9773	± 0.0059
b	0.030	+ 0.0009	= 0.0309	± 0.0050	+ 0.0043	= 0.0343	± 0.0042
c	0.010	+ 0.0261	= 0.0361	± 0.0121	+ 0.0182	= 0.0282	± 0.0094
m_0	$7^m.395$	$- 0^m.017$	$= 7^m.378$	$\pm 0^m.0068$	$- 0^m.004$	$= 7^m.381$	$\pm 0^m.0064$

[9] H. N. Russell, *Astrophysical Journal*, **35**, 315, 1912; **36**, 54, 1912; H. N. Russell and H. Shapley, *Astrophysical Journal*, **36**, 239, 385, 1912.

TABLE III

PHOTOGRAPHIC OBSERVATIONS OF U CORONAE

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O−C (darkened)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O−C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag.			2420	days		mag.
168	1	271.6493	2.4188	7.29	−.08	457	1	626.6351	1.8385	7.39	.00
	2	.6562	.4257	7.32	−.05		2	.6385	.8419	7.36	−.03
	3	.6632	.4327	7.35	−.02		3	.6420	.8454	7.34	−.05
							4	.6455	.8489	7.40	+.01
172	1	273.6351	0.9524	7.26	−.11		5	.6493	.8527	7.33	−.06
	2	.6420	.9593	7.32	−.05		6	.6535	.8569	7.39	.00
	3	.6490	.9663	7.34	−.03		7	.6576	.8610	7.39	.00
	4	275.6285	2.9458	7.39	−.02		8	.6618	.8652	7.41	+.02
	5	.6354	.9527	7.43	+.02		9	.6660	.8694	7.35	−.04
	6	.6424	.9597	7.34	−.07		10	.6701	.8735	7.39	.00
							11	.6743	.8777	7.34	−.05
181	1	277.6201	1.4852	7.39	+.01		12	.6785	.8819	7.37	−.02
	2	.6257	.4908	7.33	−.05						
	3	.6312	.4963	7.24	−.14	459	1	626.7340	1.9374	7.35	−.03
							2	.7382	.9416	7.28	−.10
429	1	600.6750	0.0431	8.35	−.05		3	.7424	.9458	7.39	+.01
	2	.6833	.0514	8.30	−.04		4	.7465	.9499	7.41	+.03
	3	.6917	.0598	8.26	−.02		5	.7507	.9541	7.29	−.09
	4	.7000	.0681	8.18	−.04		6	.7549	.9583	7.35	−.03
	5	.7083	.0764	8.12	−.03		7	.7590	.9624	7.40	+.02
	6	.7167	.0848	8.11	+.02		8	.7632	.9666	7.35	−.03
	7	.7250	.0931	8.02	.00		9	.7674	.9708	7.39	+.01
430	1	600.7333	0.1014	7.93	−.03	462	1	627.6660	2.8694	7.50	+.09
	2	.7403	.1084	7.96	+.05		2	.6701	.8735	7.53	+.12
	3	.7472	.1153	7.78	−.07		3	.6743	.8777	7.42	+.01
	4	.7535	.1216	7.75	−.05		4	.6785	.8819	7.39	−.02
	5	.7590	.1271	7.77	+.01		5	.6826	.8860	7.42	+.01
	6	.7646	.1327	7.74	+.01						
	7	.7701	.1382	7.74	+.04	474	1	630.6309	2.3821	7.32	−.05
	8	.7757	.1438	7.71	+.05		2	.6351	.3863	7.30	−.07
	9	.7812	.1493	7.64	.00		3	.6385	.3897	7.34	−.03
							4	.6420	.3932	7.34	−.03
431	1	600.7938	0.1617	7.53	−.04		5	.6455	.3967	7.37	.00
	2	.7993	.1674	7.54	−.02						
	3	.8049	.1730	7.50	−.03	508	1	686.7056	3.2208	7.40	−.04
	4	.8104	.1785	7.55	+.03		2	.7097	.2249	7.44	.00
	5	.8160	.1841	7.48	−.02		3	.7139	.2291	7.47	+.02
	6	.8215	.1896	7.48	.00		4	.7194	.2346	7.46	+.01
	7	.8271	.1952	7.51	+.04		5	.7236	.2388	7.44	−.01
	9	.8382	.2063	7.51	+.05		6	.7278	.2430	7.40	−.06
							7	.7319	.2471	7.41	−.05
435	1	603.7115	3.0796	7.51	+.08		8	.7361	.2513	7.48	+.01
	2	.7174	.0855	7.52	+.09		9	.7403	.2555	7.40	−.07
	3	.7229	.0910	7.43	.00		10	.7444	.2596	7.37	−.10
	4	.7285	.0966	7.54	+.11						
	5	.7347	.1028	7.47	+.04	509	2	686.7535	3.2687	7.47	−.02
							3	.7576	.2728	7.44	−.06
446	1	621.7017	0.3573	7.42	−.01		4	.7618	.2770	7.47	−.04
	2	.7052	.3608	7.39	−.04		5	.7660	.2812	7.55	+.03
	3	.7087	.3643	7.39	−.04		6	.7708	.2860	7.55	+.02
	4	.7122	.3678	7.41	−.02		7	.7750	.2902	7.58	+.03
	5	.7156	.3712	7.44	+.01		8	.7792	.2944	7.58	.00
							9	.7833	.2985	7.61	+.01
452	1	625.6754	0.8788	7.43	+.05		10	.7875	.3027	7.66	+.02
	2	.6788	.8822	7.39	+.01						
	3	.6823	.8857	7.37	−.01	625	1	976.6677	3.2091	7.54	+.10
	4	.6858	.8892	7.35	−.03		2	.6712	.2126	7.41	−.03
	5	.6892	.8926	7.39	+.01		3	.6747	.2161	7.45	+.01

TABLE III—Continued

PHOTOGRAPHIC OBSERVATIONS OF U CORONAE

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O−C (darkened)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O−C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag.			2421	days		mag.
625	4	976.6781	3.2195	7.43	−.01	653	5	014.8833	3.4509	8.53	−.02
630	1	989.6684	2.4014	7.38	+.01	664	1	040.6733	1.6229	7.28	−.11
	2	.6719	.4049	7.41	+.04		2	.6767	.6263	7.35	−.04
	3	.6754	.4084	7.42	+.05		3	.6802	.6298	7.34	−.05
	4	.6788	.4118	7.42	+.05		4	.6837	.6333	7.28	−.11
	5	.6823	.4153	7.42	+.05		5	.6872	.6368	7.32	−.07
							6	.6906	.6402	7.31	−.08
632	1	990.7438	0.0247	8.50	.00		7	.6941	.6437	7.35	−.04
	2	.7521	.0330	8.47	+.02		8	.6976	.6472	7.31	−.08
	3	.7590	.0399	8.54	+.12		9	.7010	.6506	7.40	+.01
	4	.7660	.0469	8.41	+.04		10	.7045	.6541	7.39	.00
	5	.7729	.0538	8.39	+.06		11	.7080	.6576	7.37	−.02
	6	.7799	.0608	8.31	+.03		12	.7115	.6611	7.36	−.04
633	1	990.7903	0.0712	8.16	−.04	667	1	043.6837	1.1810	7.46	+.09
	2	.7979	.0788	8.13	.00		2	.6872	.1845	7.41	+.04
	3	.8056	.0865	8.03	−.04		3	.6906	.1879	7.44	+.07
	4	.8125	.0934	7.98	−.04		4	.6941	.1914	7.43	+.06
	5	.8194	.1003	7.88	−.09		5	.6976	.1949	7.44	+.07
	7	.8333	.1142	7.85	−.01						
						672	1	044.6927	2.1900	7.46	+.09
634	1	990.8403	0.1212	7.81	.00		2	.6969	.1942	7.49	+.12
	2	.8458	.1267	7.83	+.07		3	.7004	.1977	7.39	+.02
	3	.8514	.1323	7.74	.00		4	.7038	.2011	7.39	+.02
	4	.8569	.1378	7.72	+.02		5	.7073	.2046	7.45	+.08
	6	.8771	.1580	7.55	−.05						
		2421				684	1	046.7500	0.7951	7.30	−.08
635	1	006.6149	2.0870	7.38	.00		2	.7542	.7993	7.39	+.01
	2	.6184	.0905	7.41	+.03		3	.7583	.8034	7.34	−.04
	3	.6219	.0940	7.33	−.05		4	.7625	.8076	7.37	−.01
	4	.6254	.0975	7.39	+.01		5	.7667	.8118	7.34	−.04
	5	.6288	.1009	7.45	+.07						
						689	1	047.6517	1.6967	7.39	−.01
639	1	006.7684	2.2405	7.38	+.01		3	.6608	.7058	7.37	−.03
	2	.7719	.2440	7.33	−.04		4	.6642	.7092	7.38	−.02
	3	.7750	.2471	7.39	+.02		5	.6677	.7127	7.43	+.03
641	1	007.6802	3.1522	7.31	−.13	697	1	050.6767	1.2695	7.40	+.03
	2	.6844	.1564	7.38	−.06		2	.6816	.2744	7.46	+.09
	3	.6878	.1598	7.39	−.05		3	.6851	.2779	7.40	+.03
	4	.6913	.1633	7.43	−.01		4	.6886	.2814	7.35	−.02
	5	.6948	.1668	7.43	−.01		5	.6920	.2848	7.36	−.01
644	1	008.7330	0.7529	7.41	+.03	705	1	051.7524	2.3451	7.33	−.04
	2	.7378	.7577	7.40	+.02		2	.7559	.3486	7.37	.00
	3	.7420	.7619	7.41	+.03		3	.7594	.3521	7.44	+.07
	4	.7455	.7654	7.35	−.03		4	.7628	.3555	7.39	+.02
	5	.7490	.7689	7.41	+.03		5	.7663	.3590	7.41	+.04
652	1	014.7521	3.3197	7.75	+.01	713	1	066.6229	3.4062	8.38	.00
	2	.7583	.3259	7.78	+.01		2	.6306	.4139	8.36	−.07
	3	.7653	.3329	7.81	−.01		3	.6375	.4208	8.54	+.07
	4	.7722	.3398	7.85	−.02		4	.6444	.4277	8.51	+.01
							5	.6514	.4347	8.55	+.03
653	1	014.8514	3.4190	8.45	−.01		6	.6583	.4416	8.59	+.05
	2	.8597	.4273	8.47	−.02		7	.6660	.4493	8.60	+.05
	3	.8680	.4356	8.48	−.04		8	.6729	0.0041	8.47	+.03
	4	.8764	.4440	8.42	−.12						

TABLE III—Continued

PHOTOGRAPHIC OBSERVATIONS OF U CORONAE

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O−C (darkened)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O−C (darkened)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2421	days		mag.			2421	days		mag.
714	1	066.6806	0.0117	8.65	+.11	735	1	394.6326	0.0165	8.57	+.05
	2	.6875	.0186	8.61	+.09		2	.6396	.0235	8.48	−.02
	3	.6951	.0262	8.53	+.04		3	.6465	.0304	8.36	−.11
	4	.7021	.0332	8.45	.00		4	.6535	.0374	8.40	−.03
	5	.7090	.0401	8.51	+.09		5	.6604	.0443	8.41	+.02
	6	.7160	.0471	8.39	+.02						
	7	.7236	.0547	8.26	−.05	736	1	395.6882	1.0720	7.31	−.06
	8	.7312	.0623	8.30	+.03		2	.6931	.0769	7.24	−.13
							3	.6972	.0810	7.32	−.05
723	1	070.6299	0.5088	7.46	+.04		4	.7021	.0859	7.36	−.01
	2	.6340	.5129	7.49	+.07		5	.7062	.0900	7.38	+.01
	3	.6382	.5171	7.51	+.09						
	4	.6431	.5220	7.49	+.07						

TABLE IV

NORMAL MAGNITUDES OF U CORONAE

No.	Helioc. Phase	Observed Magnitude	No. Obs.	Computed Magnitude (darkened)	O−C	No.	Helioc. Phase	Observed Magnitude	No. Obs.	Computed Magnitude (darkened)	O−C
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
	days				mag.		days				mag.
1	0.0030	8.562	4	8.544	+.018	21	1.8474	7.368	6	7.390	−.022
2	0.0212	8.548	4	8.513	+.035	22	1.8714	7.375	6	7.389	−.014
3	0.0357	8.455	6	8.440	+.015	23	1.9541	7.357	9	7.381	−.024
4	0.0466	8.372	5	8.372	.000	24	2.0940	7.392	5	7.376	+.016
5	0.0583	8.304	5	8.299	+.005	25	2.2149	7.410	8	7.373	+.037
6	0.0736	8.148	4	8.170	−.022	26	2.3708	7.361	10	7.371	−.010
7	0.0895	8.035	4	8.048	−.013	27	2.4149	7.376	8	7.372	+.004
8	0.1034	7.923	3	7.940	−.017	28	2.9058	7.428	8	7.410	+.018
9	0.1198	7.804	5	7.821	−.017	29	3.1254	7.441	10	7.436	+.005
10	0.1336	7.742	5	7.722	+.020	30	3.2156	7.446	5	7.445	+.001
11	0.1504	7.633	3	7.630	+.003	31	3.2241	7.430	9	7.446	−.016
12	0.1701	7.530	4	7.545	−.015	32	3.2728	7.460	3	7.483	−.023
13	0.1938	7.495	4	7.476	+.029	33	3.2879	7.565	4	7.545	+.020
14	0.4314	7.444	9	7.424	+.020	34	3.3006	7.635	2	7.631	+.004
15	0.6215	7.372	10	7.400	−.028	35	3.3228	7.765	2	7.757	+.008
16	0.9133	7.356	8	7.376	−.020	36	3.3363	7.830	2	7.845	−.015
17	1.1345	7.379	10	7.371	+.008	37	3.4150	8.432	4	8.439	−.007
18	1.3575	7.366	8	7.376	−.010	38	3.4275	8.490	2	8.498	−.008
19	1.6419	7.341	12	7.394	−.053	39	3.4390	8.510	4	8.530	−.020
20	1.7061	7.392	4	7.399	−.007						

The final m_0 is the magnitude of the system between eclipses, excluding the effects of radiation and ellipticity. The probable error of a normal magnitude is $0^m.012$ in both solutions; the probable errors of the constants are slightly reduced in the second solution on account of the greater number of points included. Since the changes in the constants are within their probable errors, it makes little difference which set we adopt. The results of the first solution have been used. This completes the discussion of the light curve independent of the eclipses.

The effects of radiation and ellipticity are now removed from the normal intensities during eclipse in the usual way. In the rectification the light of the fainter side of the faint (larger) star is increased to equality with that of the brighter side. The disks of the two stars are first supposed to be uniformly luminous. Since the light curve shows no evidence of a secondary minimum, we have for the light of the large and small star respectively:

$$L_1 = 0 + 2b = 0.062, \quad L_2 = 1 - L_1 = 0.938$$

The loss of light at primary minimum, $1 - \lambda_1$, is 0.595. Then by *formula* (k), $\alpha_0 = 0.634$.

From the curve representing the normal intensities and by use of *formula* (g') we have:

$$C = 0.0764, \quad D = 0.0447, \quad \chi(k, \alpha_0, \tfrac{1}{4}) = \frac{C}{D} = 1.709$$

By *Table III* k is found to be 0.50. In accordance with the theory a slight decrease in light is inevitable when the larger star is eclipsed, because its more luminous hemisphere is then turned toward us. By *formula* (j) the loss of light at this time, $1 - \lambda_2$, is 0.011 mag.

On the assumption that the stars are completely darkened at the limb, the depth of primary minimum is taken the same as before. From the light curve and by use of *formula* (f) we find:

$$\chi(k, \alpha_0, 0) = 3.029, \quad \chi(k, \alpha_0, \tfrac{1}{4}) = 1.749, \quad \chi(k, \alpha_0, \tfrac{3}{4}) = 0.457$$

A satisfactory value, $k = 0.60$, is now derived from *Table IIIx*. The loss of light at secondary minimum is 0.014 mag. The theoretical curve for darkened stars differs only slightly from the curve for the uniform solution. The semi-duration of primary minimum is 26 minutes longer and the depth of the secondary minimum is 0.003 mag. greater.

ELEMENTS OF THE SYSTEM

The elements of the system appear in Table V. The photometric elements are given for both uniform and darkened solutions in terms of the mean distance between the centers of the two stars and the total light of the system. A circular orbit is assumed, although the spectroscopic observations indicate a degree of eccentricity; but its value and the position of periastron are uncertain, and no assistance in these respects is obtained from the light curve.

On combining the photometric and spectroscopic elements we have at once the

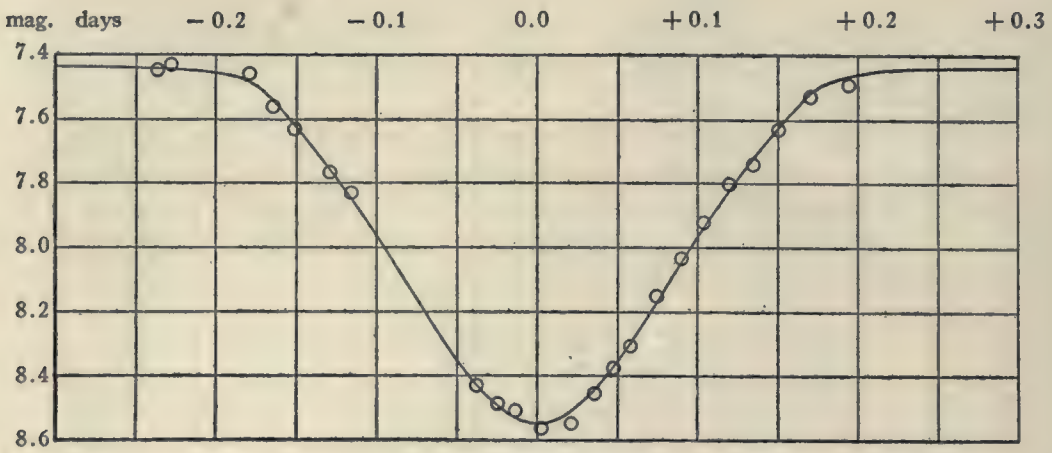

FIGURE 1. NORMAL MAGNITUDES AND THEORETICAL LIGHT CURVE OF U CORONAE AT PRIMARY MINIMUM

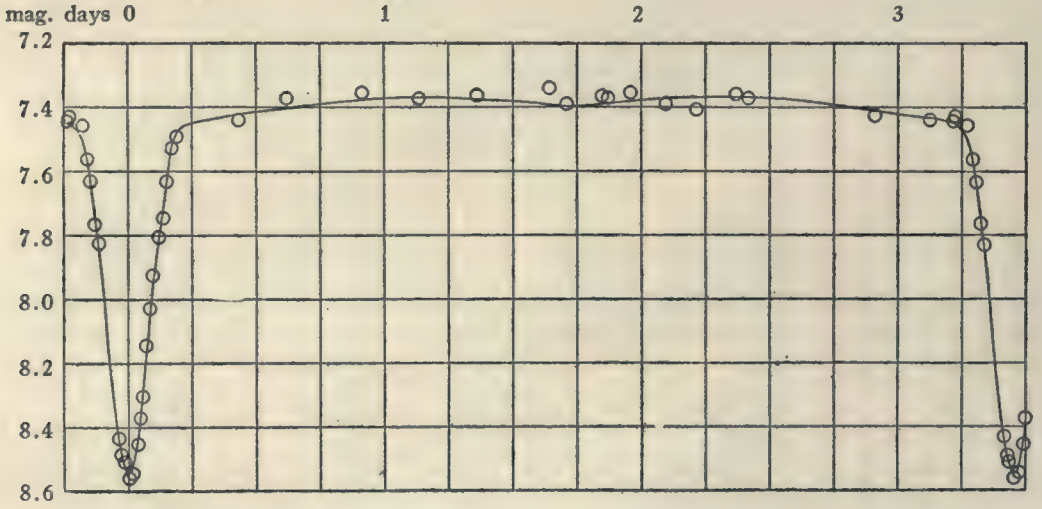

FIGURE 2. NORMAL MAGNITUDES AND THEORETICAL LIGHT CURVE OF U CORONAE

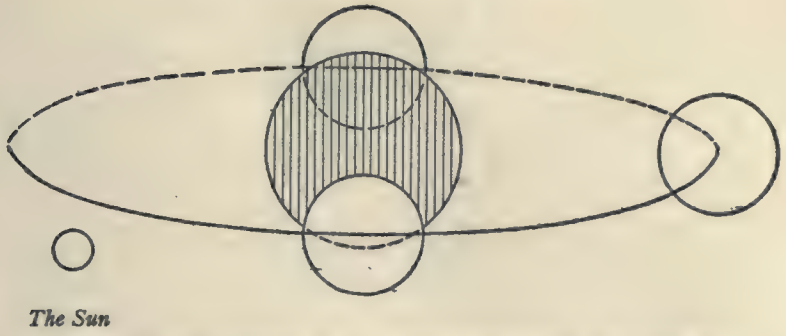

FIGURE 3. THE SYSTEM OF U CORONAE COMPARED WITH THE SUN

TABLE V

ELEMENTS OF THE SYSTEM OF U CORONAE

		Relative		Absolute	Shapley
		Uniform	Darkened	Darkened	Absolute Darkened
Period of revolution	P	3.45210 days			3.4522269
Daily motion in longitude	μ	104°.2843			
Epoch of primary minimum	T	J.D. 2420990.723 G.M.T.			04147.3344
Semi-duration of eclipse	.....	0.201 days	0.219 days	0.219	0.257
Inclination of orbit	i	76°.1	76°.5	76°.5	81°.9
Least apparent distance of centers	$\cos i$	0.240	0.234	4.14 ⊙	2.46 ⊙
Radius of relative orbit	$a_1 + a_2$	1	1	17.66 ⊙	17.34 ⊙
Ratio of radii	k	0.50	0.60	0.60	0.75
Ratio of major and minor axes	b/a	0.962	0.989	0.989	
Semi-major axis, larger star	a_1	0.291	0.279	4.93 ⊙	4.65 ⊙
Semi-major axis, smaller star	a_2	0.146	0.167	2.95 ⊙	3.49 ⊙
Semi-minor axis, larger star	b_1	0.280	0.276	4.87 ⊙	
Semi-minor axis, smaller star	b_2	0.140	0.166	2.93 ⊙	
Mass of smaller star	m_2			4.49 ⊙	4.40 ⊙
Mass of larger star	m_1			1.71 ⊙	1.68 ⊙
Density of smaller star	ρ_2			0.179⊙	0.101⊙
Density of larger star	ρ_1			0.015⊙	0.016⊙
Maximum obscuration	α_0	0.634	0.634	0.634	0.75
"Radiation effect" constant	b	0.031	0.031	0.031	
Light of smaller star	L_2	0.938	0.938	0.938	0.884
Light of larger star, brighter side	L_1	0.062	0.062	0.062	0.116
Light of larger star, fainter side	$L_1 - 2b$	0.000	0.000	0.000	
Ratio of surface intensities	J_b/J_f	∞	∞	∞	13.6
Magnitude of system	.....	7.378	7.378	7.378	7.516
Magnitude of smaller star	.....	7.412	7.412	7.412	7.65
Magnitude of larger star, brighter side	.....	10.43	10.43	10.43	9.86
Magnitude at primary minimum	.....	8.546	8.546	8.546	8.70
Magnitude at secondary minimum	.....	7.396	7.399	7.399	7.571

absolute value of the linear unit, $a_1 + a_2$, and so of the elements expressed in terms of it. Plaskett's spectroscopic elements are used for this purpose. While the elements from the alternate solution should perhaps have been substituted, the differences are within the uncertainties of the elements. The absolute masses of the two stars and their densities, given $y = 0.724$, are now available. The absolute elements from the darkened solution are given in the fifth column of Table V in terms of the sun's radius (695,553 km.), mass and density.

The last column of the table contains corresponding elements which I have derived by combining Shapley's[10] photometric elements with Plaskett's results. In his analysis of Wendell's visual observations Shapley computed five sets of elements, assuming for the loss of light at secondary minimum values between 0.0 and 0.1 mag. The lower limit he considers improbable and the upper limit too large to be consistent with Wendell's observations which, however, give little information about the secondary minimum and which might allow a small decrease in light. In his single darkened solution Shapley assumed a range of 0.05 mag. at secondary minimum.

The presence of the spectrum of the faint component on the Victoria plates would not have been predicted from the light variation. Plaskett does not give an estimate of the relative intensities of the two sets of lines, but he states that nearly as many lines of the second spectrum were measured as of the primary and that the probable error for a plate is not much greater. It is frequently assumed that the appearance of a double spectrum signifies a difference in the brightness of the two stars not greater than one magnitude. Plaskett accordingly adopted the set of Shapley's photometric elements which gives the maximum range at secondary minimum and a difference of 1.1 magnitudes between the two stars. But it was Shapley's opinion that so great a loss of light is not to be inferred from Wendell's observations. Moreover, the Missouri observations give no evidence at all of a secondary minimum.

We are not justified in jumping at once to the conclusion that the larger star is not appreciably self-luminous and that its spectrum is produced by the reflected light of the smaller star. This is, of course, the construction of the evidence at hand concerning U Coronae; and it should be added that another eclipsing binary, RS Vulpeculae, whose photometric orbit is now completed here but not yet published, presents a similar case. Our extrafocal plates show no evidence of a secondary minimum, although Plaskett has observed both spectra on his plates of this star. If this conclusion could be extended to include spectroscopic binaries in general whose secondary spectra are faintly visible, it would explain the preponderance of similar spectra from the two components; and it would be especially welcome, if it could be applied to the visual system of Sirius whose faint component has the same class of spectrum as the bright star, but relatively insignificant surface luminosity.

Geometrical considerations are apparently opposed to this reflection theory in the case under discussion. Using the absolute elements from the darkened solution we find that the larger component of U Coronae receives about 3.6 per cent of the radiation from one hemisphere of the smaller star. But we receive from the larger star 6.6 per cent

[10] H. Shapley, *Princeton Observatory Contributions*, No. 3, 47, 1915.

compared with the light of the larger star. Moreover, if the two stars differ in brightness by from 3 to 4 magnitudes, depending on the phase of the dark star, the visibility of the secondary spectrum is difficult to understand.

The theoretical light curve, Figures 1 and 2, is in accordance with the darkened solution. The residuals for the observed magnitudes, scaled from this curve, appear in the sixth column of Table III; those for the normal magnitudes are given in Table IV, column (5). The probable error of a single observed magnitude of the variable star is $\pm 0^m.034$; that of a normal magnitude is $\pm 0^m.012$.

The study of the photographic light variations of U Coronae, in combination with Plaskett's elements of the spectrographic orbit, leads to the following conclusions:

The system is composed of two ellipsoidal stars whose major semi-axes are 4.93 and 2.95 times the sun's radius. The mass of the larger star is 1.71 and of the smaller star 4.49 times the sun's mass; their densities are respectively 0.015 and 0.179 in terms of the sun's density. The system is shown in Figure 3, at elongation and at the two conjunctions.

The mean distance between the centers of the two stars is 12,280,000 km. The orbit is inclined $76^\circ.5$ and is somewhat eccentric.

At primary minimum 63 per cent of the smaller star is eclipsed, which results in a drop of 1.17 mag. in the light of the system. Photographic and visual ranges of magnitude are practically the same.

Principal conjunction of the two stars, according to the spectroscopic evidence, occurs nearly 2 hours later than the primary minimum. A discrepancy of this kind has been found in other eclipsing systems, but in the opposite direction.

Absence of a diminution in light when the larger star is eclipsed indicates that this star is not appreciably self-luminous, at least in photographic light. The presence of a marked "radiation effect" suggests that the secondary spectrum is produced by reflected light. One of the reasons for hesitancy in advancing this conclusion is that the larger star appears to send us twice as much radiation as it receives from its brighter companion.

1921 June 29

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ASTRONOMICAL SERIES

LAWS OBSERVATORY BULLETIN NO. 30

THE ECLIPSING BINARY U CEPHEI

By Robert H. Baker

At the time of its discovery, by Ceraski in 1880, U Cephei (1900, R.A. = 0^h53^m4, Decl. = + 81°20′) was the sixth known eclipsing binary and the first to be found for more than a decade. It accordingly received the attention of many observers who generally found a difference in the depth of primary minimum in opposite seasons and marked asymmetry in the light variations during the principal eclipse. Other irregularities in the light curve were mentioned by some of the early observers, but they apparently did not persist from one set of observations to another.

Chandler's[1] discussion of these early observations is instructive. He satisfactorily explained the difference in the depth of primary minimum from the spring and autumn observations, which for Wilsing amounted to nearly half a magnitude, as a physiological effect having its origin in the reversal of position of the variable star with respect to the comparison stars. Chandler was not so fortunate in extending his explanation to the asymmetry at primary minimum. Most observers had found a more rapid increase than decrease of light at this time. But this characteristic is conspicuous in the recent light curves of U Cephei which are little affected by this hour angle error; it is evidently real. Wendell's[2] visual observations of this star were made during the interval 1895 to 1912. Most of them are of the primary minimum, especially the steepest part; for his special object was the investigation of variations in the light elements. Of the 695 observations which Shapley[3] used in determining the orbit, only 37 occur outside the eclipses.

The most complete study of U Cephei was made by Dugan[4] and was based on his visual observations of 1914–17 which cover the entire curve. Dugan further developed the theory of eclipsing binaries, taking up the case of two stars rotating and revolving in different periods, and he was partially successful in representing the asymmetry of the light curve. In Dugan's analysis the disks of the stars are assumed to be three times more luminous at the center than at the limb.

Chandler (*loc. cit.*) believed that he had demonstrated a lengthening of the period,

[1] S. C. Chandler, *Astronomical Journal*, 9, 49, 1889.
[2] O. C. Wendell, *Harvard Observatory Annals*, 69: I, 58, 1909; II, 135, 1914.
[3] H. Shapley, *Princeton Observatory Contributions*, No. 3, 33, 1915.
[4] R. S. Dugan, *Princeton Observatory Contributions*, No. 5, 1920.

and he pointed to a single observation by Schwerd in 1828 as confirmatory evidence. The variation of the period appeared to be complex. In a later paper Chandler[5] abandoned the attempt to formulate the variation and adopted linear light elements. Mrs. Shapley's[6] study of the period leaves no doubt as to its variation. She concludes that the variation is very complex and that Wendell's[7] linear elements are most serviceable at present, though they probably predict the minima too early. These elements are:

$$\text{Min.} = \text{J.D. } 2407890.3007 + 2^{d}4928840 \cdot E, \text{G.M.T. (Wendell)}$$

THE PHOTOGRAPHIC OBSERVATIONS

The present study of U Cephei is based on 32 extrafocal plates containing 305 exposures, which were obtained by Miss Cummings and the writer with the 5-inch camera. The plates were exposed about 1.5 mm. outside focus. The separate exposure times vary from 4 to 16 minutes. The instruments and methods employed in this work are described in Bulletin No. 24 and some recent changes in the procedure are mentioned at the beginning of Bulletin No. 29.

TABLE I

COMPARISON STARS FOR U CEPHEI

Star	B.D.	R.A. (1900)	Decl. (1900)	Adopted Photographic Magnitude
		h m		
v	+81°25	0 53.4	+81°20′	
a	81 30	0 56.3	81 26	7.92
c	81 13	0 32.2	81 56	6.92
e	82 23	0 48.5	82 09	8.25

The three stars which were measured in connection with the variable are listed in Table I with their adopted photographic magnitudes. The photographic magnitude and spectral class of star *a* is given in *The Henry Draper Catalogue* as 7.89, A. Because of its nearness to the variable and its convenient brightness, star *a* has been used for comparison by many observers. But there are reasons for suspecting that its light is not constant; in fact, the variability of this comparison star has been given as the cause of unexplainable irregularities in some of the earlier light curves. I have used star *a* only to establish the photographic magnitude of *c*. The adopted magnitude of *e* is obtained by comparison with *c*.

The observed photographic magnitudes appear in Table II. The phases are based on Wendell's elements with the initial epoch corrected according to the requirements of the

[5] *Astronomical Journal*, **23**, 227, 1903.
[6] M. B. Shapley, *Astrophysical Journal*, **44**, 51, 1916.
[7] O. C. Wendell, *Harvard Observatory Annals*, **69**, I, 96, 1909.

light curve. The corrected elements are as follows:

$$\text{Min.} = \text{J.D. } 2407890.3182 + 2.4928840 \cdot E, \text{G.M.T. (Adopted Elements)}$$

The observed magnitudes are assembled in Table III into 50 normal magnitudes, of which numbers 16 to 21 and 33 to 40 occur outside eclipse.

SOLUTIONS FOR ELEMENTS OF THE SYSTEM

The light curve of U Cephei, as it is defined by the normal magnitudes, shows more pronounced asymmetry than that of the average eclipsing binary, a fact so frequently stated by observers of this star that we can not doubt its reality. The loss of light at the principal eclipse is less rapid than the recovery, and the light of the system is fainter by nearly 0.1 mag. at the beginning of the eclipse than at the end, although in general it is brighter before the eclipse than afterward. We are concerned evidently with a persistence of the asymmetry within eclipse combined with a brightening of the system in the part of the orbit near periastron.

It follows that the orbit is eccentric and that a displacement of the secondary minimum in the direction of the higher part of the curve is to be anticipated. Owing to the scarcity of observations on the ascending branch of the secondary minimum, the amount of the displacement is somewhat uncertain. The adopted value, 0.1 day, is probably not greatly in error. Shapley states that Wendell's observations allow the zero phase of secondary minimum to be as much as a tenth of a day after the mid-epoch, and Dugan finds some indication of a displacement in the same direction. We have, therefore, $e \cos \omega = 0.06$, but the duration of secondary minimum is not established well enough to permit a separation of these two elements.

In Dugan's analysis of the light curve of U Cephei two periodic terms are introduced to explain the asymmetry. One of them is the oscillation often called the periastron effect, but which Dugan regards as the effect of increased brightness of the advancing side of the smaller star. The second term has maxima at $\theta = 45°$ and $225°$ and, in combination with the ellipticity term, is designed to show the difference in direction between the major axes of the stars and the line of their centers. Dugan supposes that the stars are rotating in a shorter time than their revolution period and that the consequent retardation of the tides is shown not only by the asymmetry of the light curve but also by the observed increase in the revolution period. Partial success in representing in this way the departures from symmetry indicates to him that the theory is sound and that other factors require consideration, for example, the concentration of light near the center of the advancing hemisphere of the smaller star.

For the sake of comparison the Missouri observations are treated in the same way. The intensity l of the light outside eclipse is represented by the expression:

$$l = a - b \cos \theta - c \cos 2\theta + d \sin 2\theta + f \sin \theta$$

in which a is the rectified intensity which is constant between eclipses, $2b$ is the difference

TABLE II

PHOTOGRAPHIC OBSERVATIONS OF U CEPHEI

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O−C (uniform)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O−C (uniform)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag.			2420	days		mag.
357	1	423.7795	1.7352	6.81	−.04	529	12	744.6799	1.0518	6.96	+.06
	2	.7830	.7387	6.90	+.05		13	.6826	.0545	6.95	+.05
	3	.7865	.7422	6.91	+.06		14	.6854	.0573	6.98	+.08
							15	.6882	.0601	6.91	+.01
391	1	451.6017	2.1363	6.98	+.07		16	.6910	.0629	6.90	.00
	2	.6052	.1398	6.93	+.02		17	.6938	.0657	6.92	+.02
	3	.6087	.1433	6.94	+.03		18	.6965	.0684	6.99	+.09
	4	.6122	.1468	6.97	+.06						
	5	.6156	.1502	6.98	+.07	530	1	744.7000	1.0719	6.95	+.05
							2	.7028	.0747	6.88	−.02
392	1	452.5427	0.5845	6.91	.00		3	.7056	.0775	7.00	+.10
	2	.5490	.5908	6.96	+.05		4	.7083	.0802	6.92	+.02
	3	.5524	.5942	6.90	−.01		5	.7111	.0830	6.86	−.04
	4	.5559	.5977	6.98	+.07		6	.7139	.0858	6.84	−.06
	5	.5594	.6012	6.93	+.02		7	.7167	.0886	6.91	+.01
	16	.5976	.6394	6.91	.00		8	.7194	.0913	6.89	−.01
	17	.6010	.6428	6.87	−.04		9	.7222	.0941	6.91	+.01
	18	.6045	.6463	6.94	+.03		10	.7250	.0969	6.95	+.05
							11	.7278	.0997	6.94	+.04
393	1	452.6115	0.6533	6.93	+.02		12	.7306	.1025	6.87	−.03
	2	.6149	.6567	6.94	+.03		13	.7333	.1052	6.91	+.01
	3	.6184	.6602	6.97	+.06		14	.7361	.1080	6.87	−.03
	16	.6635	.7053	6.98	+.07		15	.7389	.1108	6.95	+.05
	17	.6670	.7088	6.94	+.03		16	.7417	.1136	6.89	−.01
	18	.6705	.7123	6.96	+.05		17	.7444	.1163	6.89	−.01
	19	.6740	.7158	6.96	+.05		18	.7472	.1191	6.91	+.01
	20	.6774	.7192	6.98	+.07						
						531	1	744.7507	1.1226	6.89	−.01
516	1	694.7653	0.9929	6.96	+.05		2	.7535	.1254	6.93	+.03
	2	.7681	.9957	6.89	−.02		3	.7562	.1281	6.87	−.03
	3	.7708	.9984	6.84	−.07		4	.7590	.1309	6.88	−.02
	4	.7736	1.0012	6.96	+.05		8	.7701	.1420	6.92	+.02
	5	.7764	.0040	6.93	+.02		9	.7729	.1448	6.96	+.06
	6	.7792	.0068	6.87	−.04		10	.7757	.1476	6.94	+.04
	7	.7819	.0085	6.83	−.08		11	.7785	.1504	6.98	+.08
	8	.7847	.0123	6.87	−.04		12	.7812	.1531	6.81	−.09
	9	.7875	.0151	6.84	−.07		13	.7840	.1559	6.90	.00
	10	.7903	.0179	6.84	−.07		14	.7868	.1587	6.94	+.04
	11	.7931	.0207	6.91	.00						
	12	.7958	.0234	6.89	−.02	537	1	748.6646	0.0508	9.53	−.03
	13	.7986	.0262	6.85	−.06		2	.6729	.0591	9.29	+.03
							3	.6812	.0674	8.97	+.09
528	1	743.6931	0.0650	8.94	−.03		4	.6882	.0744	8.51	−.08
	2	.7014	.0733	8.74	+.10		5	.6938	.0800	8.41	.00
	3	.7097	.0816	8.34	−.02		6	.6993	.0855	8.22	−.02
	4	.7180	.0899	8.13	.00		7	.7049	.0911	8.10	+.01
	5	.7264	.0983	7.94	+.02		8	.7097	.0959	7.96	−.03
529	1	744.6493	1.0212	6.87	−.03	543	1	750.8312	2.2175	6.98	+.06
	2	.6521	.0240	6.97	+.07		2	.8340	.2203	7.04	+.12
	3	.6549	.0268	6.98	+.08		3	.8368	.2231	6.96	+.04
	4	.6576	.0295	6.92	+.02		4	.8396	.2259	6.86	−.06
	5	.6604	.0323	6.92	+.02		5	.8424	.2287	7.02	+.10
	6	.6632	.0351	6.95	+.05		6	.8451	.2314	7.04	+.12
	7	.6660	.0379	6.89	−.01		7	.8479	.2342	6.99	+.07
	8	.6688	.0407	6.92	+.02		9	.8604	.2467	7.07	+.15
	9	.6715	.0434	6.92	+.02		10	.8632	.2495	7.01	+.09
	10	.6743	.0462	6.87	−.03						
	11	.6771	.0490	6.87	−.03	544	1	753.5743	2.4678	9.64	+.08

TABLE II—Continued

PHOTOGRAPHIC OBSERVATIONS OF U CEPHEI

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O−C (uniform)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O−C (uniform)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag.			2420	days		mag.
554	2	753.5840	2.4368	9.70	+.14	573	1	775.5514	2.0100	6.80	−.08
	3	.5938	2.4466	9.62	+.06		2	.5542	.0128	6.81	−.07
	4	.6035	0.0042	9.58	+.02		3	.5570	.0156	6.82	−.06
	5	.6132	.0139	9.55	−.01		4	.5597	.0183	6.82	−.06
	6	.6229	.0236	9.61	+.05		5	.5625	.0211	6.80	−.08
	7	.6326	.0333	9.60	+.04						
						576	1	775.6125	2.0711	6.86	−.03
555	1	753.6438	0.0445	9.64	+.08		2	.6153	.0739	6.86	−.03
	2	.6535	.0542	9.55	+.06		3	.6180	.0766	6.87	−.02
	3	.6632	.0639	9.09	+.06		4	.6208	.0794	6.88	−.01
	4	.6729	.0736	8.77	+.14		5	.6236	.0822	6.88	−.02
	5	.6826	.0833	8.34	+.02						
	6	.6924	.0931	8.09	+.04	586	1	777.8708	1.8365	6.91	.00
	7	.7021	.1028	7.84	+.01		2	.8750	.8407	6.89	−.02
							3	.8778	.8435	6.89	−.02
556	1	753.7111	0.1118	7.68	.00		4	.8806	.8463	6.90	−.01
	2	.7180	.1187	7.56	−.03		5	.8833	.8490	6.89	−.02
	3	.7250	.1257	7.54	+.04						
	4	.7319	.1326	7.42	+.01	594	1	779.6889	1.1618	7.02	+.11
	5	.7389	.1396	7.32	−.02		2	.6917	.1646	6.98	+.08
	6	.7444	.1451	7.19	−.09		3	.6944	.1673	6.89	−.01
	7	.7486	.1493	7.17	−.07		4	.6972	.1701	6.87	−.04
	8	.7528	.1535	7.14	−.06		5	.7000	.1729	6.93	+.02
	9	.7569	.1576	7.11	−.06		6	.7028	.1757	6.96	+.05
							7	.7056	.1785	6.94	+.03
557	1	753.7618	0.1626	7.09	−.04		8	.7083	.1812	6.96	+.05
	2	.7660	.1668	7.05	−.05		9	.7111	.1840	6.88	−.03
	3	.7701	.1709	7.07	.00		10	.7139	.1868	6.88	−.03
	4	.7743	.1751	6.95	−.10		11	.7167	.1896	6.87	−.05
	5	.7785	.1793	6.96	−.07		12	.7194	.1923	6.86	−.06
	6	.7826	.1834	6.92	−.09		13	.7222	.1951	6.89	−.03
	7	.7868	.1876	6.95	−.04		14	.7257	.1986	6.92	.00
	8	.7910	.1918	6.96	−.02		15	.7285	.2014	6.90	−.02
	9	.7951	.1959	6.93	−.03		16	.7312	.2041	6.94	+.02
	10	.7993	.2001	6.94	.00		17	.7340	.2069	6.89	−.03
							18	.7368	.2097	6.90	−.02
558	1	753.8035	0.2043	6.91	−.03		19	.7396	.2125	6.93	+.01
	2	.8062	.2066	6.91	−.04		20	.7424	.2153	6.86	−.06
	3	.8090	.2094	6.88	−.06						
	4	.8118	.2122	6.93	−.01	595	1	779.7458	1.2187	6.90	−.03
	5	.8146	.2150	6.91	−.02		2	.7486	.2215	6.96	+.03
	6	.8174	.2178	6.88	−.05		3	.7514	.2243	6.98	+.06
	7	.8201	.2205	6.88	−.05		4	.7542	.2271	6.96	+.03
	8	.8229	.2233	6.84	−.09		5	.7569	.2298	7.01	+.08
	9	.8257	.2261	6.87	−.06		6	.7597	.2326	6.92	−.01
	10	.8285	.2289	6.91	−.02		7	.7625	.2354	6.95	+.01
	11	.8312	.2316	6.88	−.05		8	.7653	.2382	6.96	+.02
	12	.8340	.2344	6.94	+.01		9	.7681	.2410	6.99	+.05
	13	.8368	.2372	6.94	+.01		10	.7708	.2437	6.96	+.02
	14	.8396	.2400	6.93	.00		11	.7736	.2465	6.93	−.01
	15	.8424	.2428	6.95	+.03		12	.7764	.2493	6.96	+.02
	16	.8451	.2455	6.88	−.04		13	.7792	.2521	6.96	+.02
	17	.8479	.2483	6.88	−.04		14	.7819	.2548	6.99	+.05
							15	.7847	.2576	6.92	−.02
569	1	773.5521	0.0106	9.54	−.02		16	.7875	.2604	6.91	−.03
	2	.5632	.0217	9.48	−.08		17	.7903	.2632	6.92	−.02
	3	.5743	.0328	9.46	−.10		18	.7931	.2660	6.88	−.06
	4	.5854	.0439	9.47	−.09		19	.7958	.2687	6.93	−.02
							20	.7986	.2715	6.93	−.02

TABLE II—Continued

PHOTOGRAPHIC OBSERVATIONS OF U CEPHEI

Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O−C (uniform)	Plate No.	Exp. No.	Julian Day G.M.T.	Helioc. Phase	Photg. Mag.	O−C (uniform)
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
		2420	days		mag.			2420	days		mag.
596	1	779.8056	1.2785	6.88	−.07	604	1	780.9118	2.3847	7.76	+.02
	2	.8083	.2812	6.92	−.03		2	.9229	.3958	7.93	−.02
	3	.8111	.2840	6.93	−.02		3	.9340	.4069	8.16	−.06
	4	.8146	.2875	6.91	−.04		4	.9451	.4160	8.51	−.06
	5	.8174	.2903	6.88	−.07						
	6	.8198	.2927	6.90	−.05	609	1	800.5667	2.0972	6.87	−.03
	7	.8219	.2948	6.92	−.03		2	.5694	.0999	6.89	−.01
	8	.8240	.2969	6.86	−.09		3	.5722	.1027	6.84	−.06
	9	.8261	.2990	6.91	−.04		4	.5750	.1055	6.85	−.05
	10	.8285	.3014	6.93	−.02		5	.5778	.1063	6.84	−.06
	11	.8312	.3041	6.98	+.03			2421			
	12	.8340	.3069	6.95	.00	700	1	050.8000	0.5450	6.80	−.09
	13	.8368	.3097	6.94	−.01		2	.8028	.5478	6.86	−.05
	14	.8396	.3125	6.95	.00		3	.8056	.5506	6.88	−.03
	15	.8424	.3153	6.93	−.02		4	.8083	.5533	6.82	−.09
	16	.8451	.3180	6.95	.00		5	.8111	.5561	6.89	−.02
	17	.8479	.3208	6.89	−.06						
	18	.8507	.3236	6.93	−.02	707	1	051.7944	1.5394	6.87	−.01
	19	.8535	.3264	6.92	−.03		2	.7972	.5422	6.88	.00
	20	.8562	.3291	6.95	.00		3	.8000	.5450	6.92	+.04
	21	.8590	.3319	6.92	−.03		4	.8028	.5478	6.90	+.02
							5	.8056	.5506	6.87	−.01
601	1	780.7712	2.2441	6.96	+.04		6	.8083	.5533	6.91	+.03
	2	.7747	.2476	7.00	+.08						
	3	.7781	.2510	6.94	+.02	710	1	051.8569	1.6019	6.78	−.08
	5	.7851	.2580	6.98	+.06		2	.8597	.6047	6.82	−.04
	6	.7885	.2614	7.00	+.08		3	.8625	.6075	6.84	−.02
	7	.7920	.2649	7.00	+.08		4	.8653	.6103	6.87	+.01
	8	.7955	.2684	6.98	+.06		5	.8680	.6130	6.87	+.01
	9	.7990	.2719	6.99	+.07						
	10	.8024	.2753	7.01	+.09	738	1	395.6306	1.3573	7.03	+.08
	11	.8059	.2788	6.94	+.02		2	.6333	.3600	6.95	.00
	12	.8094	.2823	7.07	+.14		3	.6364	.3631	7.08	+.13
	13	.8128	.2857	6.95	+.02		4	.6936	.3663	6.95	.00
	14	.8163	.2892	6.98	+.04		5	.6424	.3691	7.02	+.07
							6	.6451	.3718	6.91	−.04
602	1	780.8215	2.2944	7.13	+.17		7	.6479	.3746	6.86	−.09
	2	.8271	.3000	7.09	+.12		8	.6507	.3774	6.93	−.02
	3	.8326	.3055	7.10	+.11		9	.6535	.3802	6.89	−.06
	4	.8382	.3111	7.10	+.09		10	.6562	.3829	6.91	−.04
	5	.8438	.3167	7.12	+.08		11	.6590	.3857	6.96	+.01
	6	.8493	.3222	7.08	+.01		12	.6618	.3885	7.02	+.07
	7	.8549	.3278	7.21	+.10		13	.6646	.3923	7.00	+.05
	8	.8604	.3333	7.23	+.08		14	.6674	.3941	6.98	+.03
							15	.6701	.3968	6.91	−.03
603	1	780.8680	2.3409	7.25	+.04		16	.6729	.3996	6.93	−.01
	2	.8764	.3493	7.36	+.08		17	.6757	.4024	6.95	+.01
	3	.8847	.3576	7.43	+.05		18	.6785	.4052	6.94	.00
	4	.8931	.3660	7.53	+.06						
	5	.9014	.3743	7.65	+.06						

TABLE III

NORMAL MAGNITUDES OF U CEPHEI

No.	Helioc. Phase	Observed Magnitude	No. Obs.	Computed Magnitude (uniform)	O−C	No.	Helioc. Phase	Observed Magnitude	No. Obs.	Computed Magnitude (uniform)	O−C
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
	days				mag.		days				mag.
1	0.0126	9.538	4	9.560	−.022	26	1.1884	6.911	12	6.918	−.007
2	0.0299	9.557	3	9.560	−.003	27	1.2227	6.935	12	6.927	+.008
3	0.0464	9.547	3	9.560	−.013	28	1.2613	6.934	15	6.941	−.007
4	0.0566	9.420	2	9.390	+.030	29	1.2958	6.911	8	6.949	−.038
5	0.0654	9.000	3	8.972	+.028	30	1.3194	6.933	10	6.948	−.015
6	0.0738	8.673	3	8.632	+.041	31	1.3689	6.958	9	6.944	+.014
7	0.0826	8.328	4	8.328	.000	32	1.3941	6.956	9	6.941	+.015
8	0.0914	8.107	3	8.095	+.012	33	1.5464	6.892	6	6.862	+.030
9	0.0990	7.913	3	7.913	.000	34	1.6075	6.836	5	6.858	−.022
10	0.1187	7.593	3	7.593	.000	35	1.7387	6.873	3	6.858	+.015
11	0.1391	7.310	3	7.332	−.022	36	1.8432	6.891	5	6.864	+.027
12	0.1558	7.128	4	7.185	−.057	37	2.0248	6.818	6	6.885	−.067
13	0.1730	7.008	4	7.060	−.052	38	2.0874	6.870	7	6.900	−.030
14	0.1974	6.925	8	6.958	−.033	39	2.1329	6.927	7	6.907	+.020
15	0.2205	6.889	7	6.929	−.040	40	2.2259	6.984	7	6.922	+.062
16	0.2400	6.914	7	6.931	−.017	41	2.2512	6.994	7	6.924	+.070
17	0.5689	6.889	9	6.912	−.023	42	2.2771	6.990	8	6.927	+.063
18	0.6428	6.913	7	6.911	+.002	43	2.3083	7.103	6	7.000	+.103
19	0.7123	6.964	5	6.911	+.053	44	2.3306	7.220	2	7.130	+.090
20	1.0039	6.888	9	6.912	−.024	45	2.3451	7.305	2	7.252	+.053
21	1.0257	6.901	10	6.911	−.010	46	2.3618	7.480	2	7.423	+.057
22	1.0532	6.923	12	6.912	+.011	47	2.3795	7.705	2	7.660	+.045
23	1.0822	6.914	11	6.912	+.002	48	2.4014	8.045	2	8.090	−.045
24	1.1166	6.896	11	6.912	−.016	49	2.4180	8.510	1	8.578	−.068
25	1.1560	6.928	11	6.912	+.016	50	2.4776	9.653	3	9.560	+.093

in intensity between the hemispheres of the larger star turned toward and away from its brighter companion, c is the ellipticity constant, d is a constant which with c serves to determine the tidal retardation, $2f$ is that part of the difference in intensity between periastron and apastron which is not taken up in the reflection constant. After conversion from magnitudes to intensities, each normal place furnishes an equation of condition of the form obtained by differentiating the above expression. These, equally weighted, yield the following normal equations:

$$\begin{aligned}
+ 14.000\,\delta a + 0.413\,\delta b + 4.875\,\delta c - 1.990\,\delta d - 1.971\,\delta f &= -0.0171 \\
+ 4.562\,\delta b - 0.298\,\delta c + 1.184\,\delta d + 0.992\,\delta f &= -0.0674 \\
+ 5.654\,\delta c + 0.029\,\delta d - 0.790\,\delta f &= -0.0271 \\
+ 8.346\,\delta d + 0.094\,\delta f &= -0.1755 \\
+ 9.439\,\delta f &= -0.1671
\end{aligned}$$

On solving these equations we have:

Preliminary	Correction	Final	Probable Error	Dugan
$a = 0.9300$	$- 0.0059 =$	0.924	± 0.008	0.958
$b = 0.0250$	$- 0.0048 =$	0.020	± 0.012	0.022
$c = 0.0150$	$- 0.0024 =$	0.013	± 0.013	0.020
$d = 0.0300$	$- 0.0215 =$	0.008	± 0.009	0.021
$f = 0.0000$	$- 0.0184 =$	$- 0.018$	± 0.008	+ 0.008
$m_0 = 6^m.818$	$+ 0^m.086 =$	$6^m.904$	$\pm 0^m.009$	$6^m.879$

Regarding the lack of agreement of the constants d and f with the Princeton values, greater confidence should naturally be placed in Dugan's results, for his observations are more numerous than mine and better distributed. The negative sign of the constant f is consistent with its interpretation as a periastron effect. However, with this sign it serves to counteract the effect of the d-oscillation near primary minimum, so that very little progress is made toward the representation of the asymmetry on that part of the curve.

The terms involving c and d in the original expression now become: $- 0.0126 \cos 2\theta + 0.0085 \sin 2\theta$, and by combination: $- 0.0152 \cos (2\theta + 34°)$. The tidal retardation is therefore 17°; that is to say, minimum areas are presented to the earth when $\theta = - 17°$ and $+ 163°$. Dugan's values are $- 23°.7$ and $+ 156°.3$. The probable error of a normal magnitude is ± 0.025 mag. This completes the discussion of the light curve independent of the eclipses. The solution for the remaining elements of the system follows the method of Russell and Shapley.[8] Formulae and tables in their series of papers will be distinguished as usual by italics.

The effects of the periodic terms whose constants we have evaluated are now removed from the normal intensities during the eclipses by use of the formula:

$$R = \frac{l + b(1 + \cos\theta) + c' \cdot l \cos(2\theta + 34°) - f \cdot l \sin\theta}{a + b}$$

where R is the rectified intensity and $c' = 0.015$ is the new ellipticity constant resulting from the combination of the terms in c and d. The disks of the two stars are first assumed uniformly luminous. The loss of light at primary minimum is 0.875. Since this eclipse is known to be total, we have for the light of the large and small stars respectively $L_1 = 0.125$ and $L_2 = 0.875$. The value of L_1 is for the brighter side of the large star, for in the rectification the light of the fainter side is increased to equality with the brighter hemisphere. A real solution requires a central eclipse; thus $i = 90°$. From the light curve and by use of *Table IV*, k is found to be 0.595. By formula (h'), which for a central eclipse reduces to

$$\sin^2\theta' = a_1^2(1 - z\cos^2\theta')(1 + k)^2$$

where $z = 4c' = 0.060$, the semi-major axis of the large star is 0.320. The remaining

[8] H. N. Russell, *Astrophysical Journal*, **35**, 315, 1912; **36**, 54, 1912; H. N. Russell and H. Shapley, *Astrophysical Journal*, **36**, 239, 385, 1912.

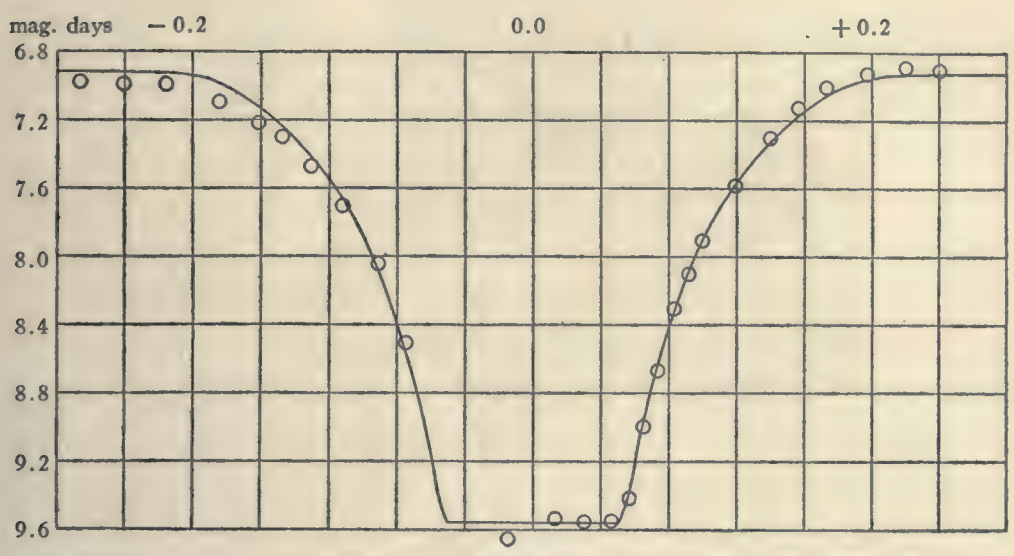

FIGURE 1. NORMAL MAGNITUDES AND THEORETICAL LIGHT CURVE OF U CEPHEI DURING PRIMARY MINIMUM

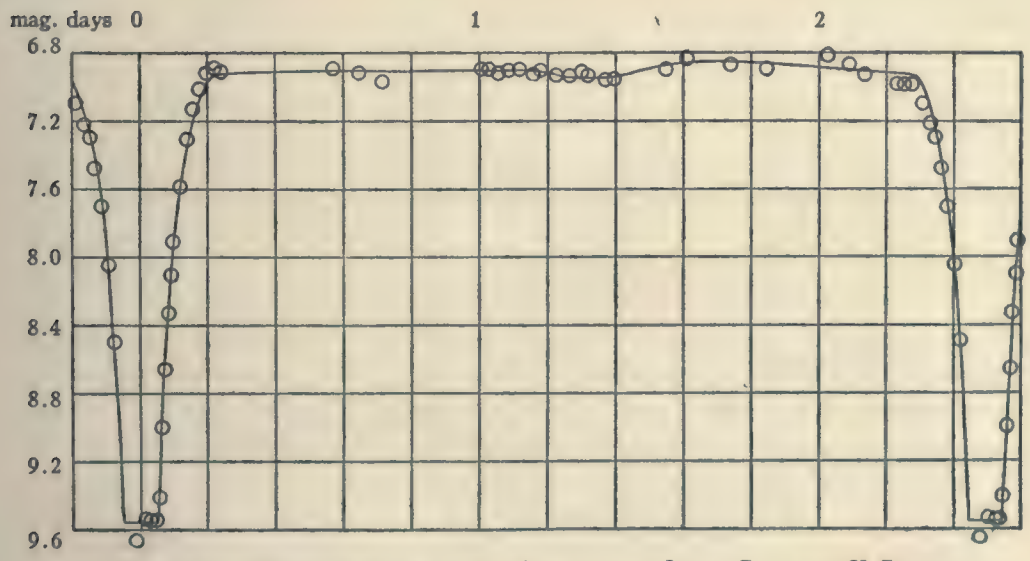

FIGURE 2. NORMAL MAGNITUDES AND THEORETICAL LIGHT CURVE OF U CEPHEI

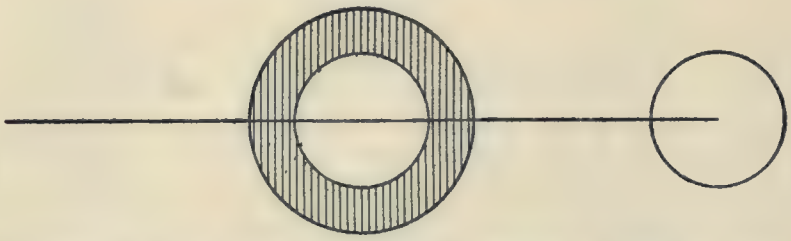

FIGURE 3. THE SYSTEM OF U CEPHEI AT SECONDARY MINIMUM AND AT GREATEST ELONGATION

TABLE IV

ELEMENTS OF THE SYSTEM OF U CEPHEI

		Uniform	Shapley	Dugan
			Uniform	Partially Darkened
Period of revolution	P	2.492884 days	2.492884 days	2.492884 days
Daily motion in longitude	μ	144°4111		
Epoch of primary minimum, G.M.T.	T	J.D. 2407890.3182	890.2992	variable
Displacement of secondary minimum	$t_2-t_1-\frac{1}{2}P$	+0.10 days	+0.1 days:	0†
Semi-duration of eclipse, primary	.....	0.2072 days	0.221 days	0.213 days‡
Semi-duration of total eclipse	.....	0.0503 days	0.0447 days	0.039 days‡
Inclination of orbit	i	90°	90°	86°4
Ratio of radii	k	0.595	0.63	0.62
Ratio of major and minor axes	b/a	0.969	1.00	0.956
Semi-major axis, larger star	a_1	0.320	0.324	0.322
Semi-major axis, smaller star	a_2	0.190	0.205	0.200
Semi-minor axis, larger star	b_1	0.310		0.308
Semi-minor axis, smaller star	b_2	0.184		0.191
Density of smaller star*	ρ_2	0.249⊙	0.185⊙	0.214⊙
Density of larger star*	ρ_1	0.025⊙	0.017⊙	0.022⊙
Maximum obscuration	α_0	1.00	1.00	1.00
"Radiation effect" constant	b	0.020		0.022
"Periastron effect" constant	f	−0.018		+0.008
Light of smaller star	L_2	0.875	0.890	0.838
Light of larger star, brighter side	L_1	0.125		0.162
Light of larger star, fainter side	L_1-2b	0.085		0.118
Ratio of surface intensities	J_b/J_f	19.8	20	13.5
Magnitude of system	.....	6.904	6.78	6.879
Magnitude of smaller star	.....	7.025		6.9
Magnitude of larger star, fainter side	.....	9.560	9.177	9.1
Magnitude at primary minimum	.....	9.560	9.177	9.1
Magnitude at secondary minimum	.....	6.949	6.829	6.953

* Assuming equal masses, corrected for difference in brightness. See *Princeton Contrib.* No. 3, p. 121.

† "Perhaps shifted slightly toward following primary minimum."

‡ Means. See *Princeton Contributions*, No. 5, p. 27.

elements are readily derived.

On the supposition that the stars are completely darkened at the limb, a central eclipse is again required and k is found equal to 0.65.

The elements of the system, from the uniform solution, appear in Table IV. Those from the darkened solution are not included in the table because the light curve does not represent the observations so well, which is likely to be the case for a central eclipse, as Shapley has pointed out. In the darkened solution the semi-duration of the principal eclipse is longer, 0.216 days, while that of totality is shorter, being 0.0436 days. The ratio of the radii, k, is 0.65. While the dimensions of the large star remain nearly the same in the darkened solution, the small star is made considerably larger, its major and minor semi-axes being 0.207 and 0.203, and its density is reduced to 0.184. The ratio of the surface intensities becomes 16.6 in the darkened solution. The greater depth of the secondary minimum, 0.072 mag., is not accordant with the photographic observations.

As the table shows, the new elements and those by Shapley and Dugan differ in general only a little, so that our knowledge of the relative dimensions and light of the system may be said to be accurate. Since the spectroscopic elements are not yet determined, the absolute dimensions are unknown. The outstanding feature of the photographic elements is the greater depth of the primary minimum. This, however, was to be expected, since Miss Cannon[9] observed a class K spectrum for the large star.

The light curve from the uniform solution is shown in Figures 1 and 2. For reasons which I presented in Bulletin No. 29, parts of the curve are not well covered by observations. On this account and because of the marked asymmetry of the light variation, the accuracy of the representation compares unfavorably with that for other systems we have studied at this observatory. The probable error of a single observation of the variable star is $\pm$ 0.036 mag.

The system of U Cephei is represented in Figure 3, at secondary conjunction and at greatest elongation. It is composed of two elongated stars, the larger one having a radius about one-third the distance between their centers and the smaller one about 60 per cent as great.

A displacement of the secondary minimum indicates eccentricity of the orbit, and a brightening of the system after this minimum suggests a periastron effect. A reflection effect is clearly shown.

The eclipse is total and central at primary minimum. The greater depth of this minimum in photographic light indicates that the larger star is the redder; its relative color index is about 0.4 mag.

The light variation is unusually asymmetrical and it is not well represented by the theoretical curve, especially near the beginning and end of principal eclipse. The darkened solution is less satisfactory than the uniform solution.

[9] A. J. Cannon, *Popular Astronomy*, **25**, 314, 1917.

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END OF VOLUME II

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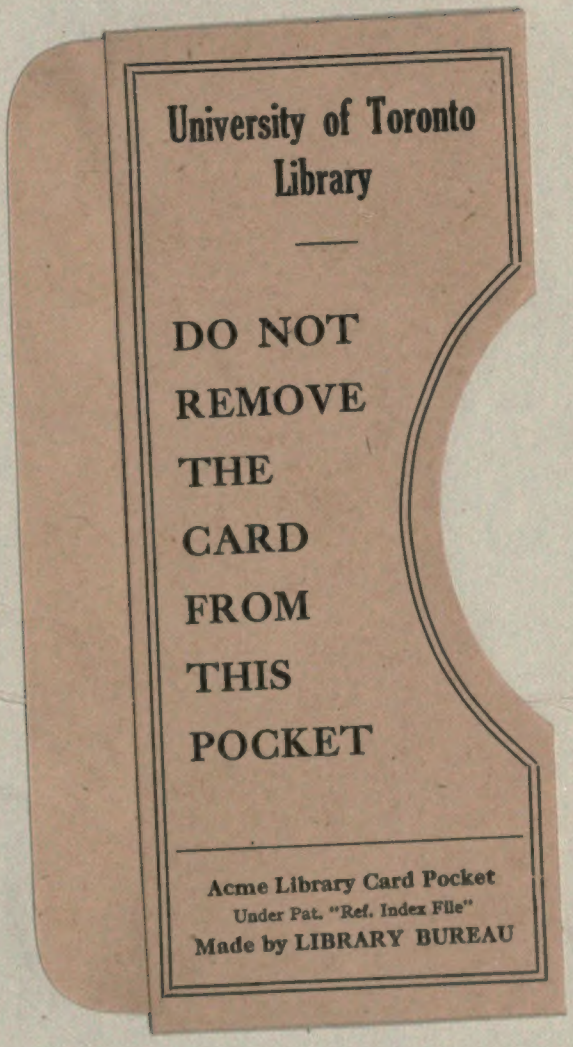
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